Supply Chain Coordination with a Risk-Averse Retailer
and a Risk-Neutral Supplier

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Abstract

We investigate how a supply chain, formed by a risk-neutral supplier and a risk-averse retailer, can be coordinated with a supply contract. We demonstrate that the standard wholesale, buy-back or revenue-sharing contracts do not coordinate such a channel. We propose a definition of coordination of the supply chain, and design an easy-to-implement risk-sharing contract that offers the desired downside protection to the retailer as well as channel coordination both in terms of expected profit and risk.

1 Introduction

Coordinating supply chains has been a major issue in supply chain management (SCM) research. Supply contracts have attracted many researchers in the field, and have generated a significant body of literature. It is known that these contracts can coordinate supply chains for expected performance measure. The coordination is accomplished by having the up-stream agent (supplier) offer a downside protection to the down-stream agent (retailer). As a result, the retailer orders an appropriately greater amount under the contract than he would otherwise.

Most of the SCM literature focuses on the criteria of minimizing expected cost or maximizing expected profit. Thus, research in supply chain coordination is devoted to designing supply contracts that can improve the expected value of a given performance measure. While a number of papers have argued that the coordination is achieved by sharing of the downside risks among risk-neutral agents, the notion of coordination becomes unclear if the supply chain consists of agents with different attitudes toward risk (Tayur et al., 1999).
In the field of economics and finance, agents are often assumed to be risk averse, and they maximize a concave utility of wealth (von Neumann and Morgenstern, 1953). An important operational approach to deal with risk aversion is that of mean-variance analysis due to Markowitz (1959). There have been some attempts in the inventory and supply chain literature to deal with risk aversion. Here we mention these attempts briefly, while providing a fuller discussion of those related to this paper in the next section. Lau (1980), Bouakiz and Sobel (1992), Eeckhoudt et al. (1995), Chen and Federgruen (2000), and Buzacott, Yan and Zhang (2002) consider the inventory problem of a single risk-averse decision maker. Agrawal and Seshadri (2000a) consider a single risk-averse firm’s price and inventory decision jointly. Gaur and Seshadri (2003) address the problems of optimal inventory planning and hedging of inventory risk. Lau and Lau (1999), Webster and Weng (2000), Agrawal and Seshadri (2000b), Tsay (2002), and Gan, Sethi and Yan (2003) address some issues in supply chains consisting of risk-averse agents.

The mean-variance approach works best when the retailer’s profit is normally distributed. But this is not the case in the newsvendor problem. Therefore, a newsvendor may consider his downside risk to be more important than simply the variance of his profit. In such situations, Telser (1955) considers a safety-first firm’s optimal decision when the firm is subject to a downside risk constraint. Markowitz (1959) also recognizes the importance of downside risk measures. There are a number of different measures of downside risk. Examples are semivariance computed from mean return (SVM), semivariance computed from a target return (SVT), and value at risk computed from a specific fractile of the return distribution (VaR).

We begin with a newsvendor who maximizes his expected profit subject to a downside risk, and derive his optimal decision. This problem is similar to that in Telser (1955). We show that there exists a critical value, such that if the order quantity is less than this value, the downside risk is 100%. Above the critical value, the downside risk is a monotone increasing function of the order quantity. We also develop an upper bound for the downside risk in cases lacking an explicit expression for the performance measure. With results developed for the risk-averse newsvendor in hand, we study the downside risk for a class of supply contacts, including revenue-sharing and buy-back contracts. We show that for a supply chain coordinated by a buy-back or a revenue sharing contract, the retailer, the supplier, and the
supply chain have the same downside risk. Finally, if the downside risk is larger than a level the retailer is willing to take, then he would order less than the optimal quantity in the standard newsvendor problem.

Then the natural question that arises is how to measure the supply chain’s performance when the retailer orders less. As we know, in the risk-neutral case, we measure it by the channel’s expected profit. But such a method does not work in the case when the supply chain consists of risk-averse agents. Gan, Sethi and Yan (2003) use Pareto-optimality as a criterion to measure the supply chain’s performance in such cases. They define each agent’s payoff as a function of a random variable that represents his profit, and they propose that a supply chain can be treated as coordinated if no agent’s payoff can be increased without impairing someone else’s payoff.

In this paper, we study a supply chain consisting of a risk-neutral supplier and a risk-averse retailer with a downside risk constraint. We employ the general definition of supply chain coordination in Gan, Sethi and Yan (2003), and we obtain the specific coordinating conditions for the problem under consideration.

In this paper we construct a risk-sharing contract which is a contract consisting of two parts: the initial contract, and a clause which ensures a limited portion of the unsold items to be fully refundable. We demonstrate that incorporation of the additional clause coordinates the supply chain defined above.

The remainder of the paper is organized as the follows. In Section 2 we review the literature. In Section 3 we model risk aversion by a downside risk constraint. We then obtain optimal ordering decisions for a risk-averse newsvendor facing a downside constraint. In Section 4 we use these results to measure the downside risks of the agents using two well-known coordinating contracts designed for risk-neutral agents. We show that these contracts cannot always coordinate a supply chain with a risk-averse retailer and a risk-neutral supplier. In Section 5 we define the coordination of such a supply chain, and propose a new risk-sharing contract that achieves the defined coordination. We also provide a detailed discussion of incentives and risks for both the agents. In Section 6 we discuss the limitations of the proposed risk-sharing contract, and provide some remedies to overcome them. The paper is concluded in Section 7.
Supply contracts have attracted a great deal of attention in the SCM research recently. The book edited by Tayur, Ganeshan and Magazine (1999) contains a number of chapters addressing supply contracts. The recent survey article by Cachon (2002) summarizes existing research and provides up-to-date results. Therefore, we limit ourselves to providing a summary of articles closely related to our research.

Pasternack (1985) studies optimal pricing and return policies (buy-back contract) in a supply chain consisting of one supplier and one retailer. He proves that a contract in which the supplier offers the retailer partial credit for all unsold goods can coordinate the channel.

Lau and Lau (1999) study a supply chain consisting of a monopolistic supplier and a retailer. The supplier and the retailer employ a return policy, and each of them has a mean-variance objective function. Lau and Lau find the optimal wholesale price and return credit for the supplier to maximize his utility. However, they do not consider the issue of improving the supply chain’s performance (improving both agents’ utilities).

Chen and Federgruen (2000) re-visit a number of basic inventory models using the mean-variance approach. They exhibit how a systematic mean-variance trade-off analysis can be carried out efficiently and how the resulting strategies differ from those obtained in the standard analyses.

Webster and Weng (2000) investigate risk-free return policies in the sense that they satisfy the following conditions: (1) the retailer’s profit is increased and (2) the manufacturer’s profit is at least as large as the profit when no returns are allowed. Thus, these return policies are preferred by the manufacturer regardless of the manufacturer’s attitude toward risk.

Agrawal and Seshadri (2000a) consider how a risk-averse retailer, whose utility function is assumed to be increasing and concave in wealth, chooses the order quantity and the selling price in a single-period inventory model. They show that in comparison to a risk-neutral retailer, a risk-averse retailer will charge a higher price and order less if a change in price affects the scale of demand; whereas a risk-averse retailer will charge a lower price if a change in price only affects the location of the demand distribution.

Agrawal and Seshadri (2000b) consider a single-period model in which multiple risk-averse retailers purchase a single product from a common vendor. They demonstrate that
a risk-neutral distributor, who is responsible for the ordering decisions of the retailers, can offer a menu of mutually beneficial contracts to the retailers. Every contract in the menu has two parameters: (i) a fixed payment that the distributor makes to the retailer; and (ii) a unit price that the distributor charges the retailer for every unit sold. The menu of contracts simultaneously induces every risk-averse agent to select a unique contract from it, maximizes the distributor’s profit, and raises the order quantities of the retailers to the expected value maximizing (newsvendor) quantities.

Buzacott, Yan and Zhang (2001) model a commitment and option contract for a risk-averse newsvendor with a mean-variance objective. The contract, also known as a take-or-pay contract, belongs to a class of volume flexible contracts, where the buyer reserves a capacity with initial information and adjusts the purchase at a later stage when some new information update becomes available. They compare the performance of strategies developed for risk-averse and risk-neutral objectives. They conclude that the former objective can be an effective approach when the quality of information revision is not high. Their study indicates that it is possible to reduce the risk (measured by the variance of the profit) by six-to eight-fold, while the loss in the expected profit is almost invisible. On the other hand, the strategy developed for the expected profit objective can only be considered when the quality of information revision is high. They show furthermore that these findings continue to hold in the expected utility framework. The paper points out a need for modeling approaches dealing with downside risk considerations.

Tsay (2002) studies how risk aversion affects both sides of the supplier-retailer relationship under various scenario of relative strategic power, and how these dynamics are altered by the introduction of a return policy. The sequence of play is as follows: first the channel leader announces a return policy, and then the retailer chooses order quantity without knowing the demand. After observing the demand, the retailer chooses the price and executes on any relevant terms of the distribution policy as appropriate (e.g., returning any overstock as allowed). Tsay shows that behaviors under risk aversion are qualitatively different from those under risk neutrality. He also quantifies the penalty for errors in estimating a channel partner’s risk aversion, and the penalty can be substantial.

Cachon and Lariviere (2002) study a revenue-sharing contract and show that it coordinates a supply chain with multiple competing retailers and that the contract is independent
of the price of the product. They also show that for every buy-back contract, there always exists an equivalent revenue-sharing contract.

Gan, Sethi and Yan (2003) propose a definition of supply chain coordination with risk-averse agents. Their definition generalizes the standard one in the risk-neutral case. They show that coordinating such a supply chain is to find a Pareto-optimal solution acceptable to each agent by choosing the channel’s appropriate external joint action and internal allocation of the channel’s gain. They examine coordinating contracts for the supply chain consisting of one risk-averse supplier and one risk-averse retailer in several special cases.

3 A Newsvendor with Downside Risk Constraint

In this section we first introduce the concept of downside risk. We then study the optimal decision making problem of a risk-averse newsvendor subject to a downside risk constraint. Bounds are developed in cases when analytical expressions of the expected performance measure are not available.

We characterize the downside risk of a newsvendor as the probability that his realized profit is less than or equal to his specified target profit. Let \( \alpha \) be the target profit and \( X \) be the random demand with distribution \( F(\cdot) \) and density \( f(\cdot) \). Let \( \Pi(q, X) \) denote the newsvendor’s profit when the order quantity is \( q \). Thus, the downside risk of the newsvendor is

\[
P \{ \Pi(q, X) \leq \alpha \}. \tag{1}
\]

The newsvendor wants to choose an order quantity \( q \) so as to maximize his expected profit \( E(\Pi(q, X)) \), while specifying that his actual profit should not fall below his target profit level of \( \alpha \) with a probability exceeding a specified \( \beta \). That is, his decision problem is:

\[
\max_{q \geq 0} E(\Pi(q, X)), \tag{2}
\]

s.t. \( P\{\Pi(q, X) \leq \alpha \} \leq \beta \), \( \tag{3} \)

where \( \Pi(q, X) = p \min\{q, X\} - wq \). Note that for risk-aversion pairs \((\alpha^1, \beta^1) \) and \((\alpha^2, \beta^2) \), if \( \alpha^1 \leq \alpha^2 \) and \( \beta^1 \geq \beta^2 \), then the second pair means a higher aversion to risk than does the first.

The objective (2) without the downside risk constraint (3) is the standard newsvendor
problem, for which the optimal order quantity is

\[ \hat{q} = F^{-1}\left(\frac{p - w}{p}\right). \] (4)

For the risk-averse newsvendor represented by (2) and (3), we have the following result.

**Proposition 3.1** For any target profit level \( \alpha \), there is a critical order quantity

\[ q^0 = \frac{\alpha}{p - w} \] (5)

such that for an order quantity \( q \leq q^0 \), the downside risk is one, and for \( q > q^0 \), the downside risk is \( F\left(\frac{\alpha + wq}{p}\right) \), which is an increasing function of \( q \).

**Proof.** If \( q \leq q^0 \), then \( \Pi(q, X) = p \min\{q, X\} - wq \leq pq^0 - wq^0 = \alpha \). Therefore, \( P\{\pi(q, x) \leq \alpha\} = 1 \). If \( q > q^0 \), then clearly \( P\{\{\Pi(q, X) \leq \alpha\} \cap \{X > q\}\} = 0 \) and, furthermore,

\[ P\{\Pi(q, X) \leq \alpha\} = P\{pX - wq \leq \alpha\} = P\left\{X \leq \frac{\alpha + wq}{p}\right\} = F\left(\frac{\alpha + wq}{p}\right) \] (6)

\[ \square \]

It is obvious that the downside risk in (6) increases in \( q \). The next result providing a solution for the risk-averse newsvendor problem (2) and (3) follows easily from Proposition 3.1.

**Proposition 3.2** For the newsvendor with the risk-aversion pair \((\alpha, \beta), \beta > F(q^0)\), the optimal order quantity is

\[ q^* = \begin{cases} \hat{q}, & \text{if } F\left(\frac{\alpha + wq}{p}\right) \leq \beta, \\ \frac{p F^{-1}\left(\frac{\beta - \alpha}{w}\right)}{w}, & \text{if } F(q^0) < \beta < F\left(\frac{\alpha + wq}{p}\right), \end{cases} \] (7)

where \( \hat{q} \) is given in (4). When \( F(q^0) < \beta < F\left(\frac{\alpha + wq}{p}\right) \), the downside risk constraint (3) is binding at \( q^* \), i.e.,

\[ P\{\Pi(q^*, X) \leq \alpha\} = \beta. \]

Proposition 3.2 implies that if the order quantity \( \hat{q} \) satisfies the downside risk constraint (3), then the risk-averse newsvendor orders \( \hat{q} \). Otherwise, he orders the maximum \( q \) which satisfies (3). It is illuminating to depict in Figure 1 the result in Proposition 3.2. We have three cases depending on the value of \( \beta \).

(i) If \( \beta \leq F(q^0) \), there is no feasible solution.
(ii) If $F(q^0) < \beta \leq F\left(\frac{\alpha + w\hat{q}}{p}\right)$, the optimal order quantity is $\frac{pF^{-1}(\beta - \alpha)}{w}$.

(iii) If $\beta > F\left(\frac{\alpha + w\hat{q}}{p}\right)$, the optimal order quantity is $\hat{q}$.

Chen and Federgruen (2000) discuss a newsvendor model in the mean-variance framework. It is interesting to contrast our results to theirs. They find that the risk measured by variance of the profit increases from zero to $p\sigma^2$, as the order quantity increases from zero to infinity, where $\sigma^2$ is the variance of the demand.

An important special case of a risk-averse newsvendor arises when the target profit level is set at the expected profit of the corresponding risk-neutral newsvendor. Then $\alpha = \pi(\hat{q}) = E(\Pi(\hat{q}, X))$, and the downside risk is $F\left(\frac{\pi(\hat{q}) + w\hat{q}}{p}\right)$. We denote $F\left(\frac{\pi(\hat{q}) + w\hat{q}}{p}\right)$ as the Natural Downside Risk (NDR), which represents a newsvendor’s downside risk when the order quantity is $\hat{q}$ and the target profit is $\pi(\hat{q})$. A formula for NDR can be derived from the observation that NDR is equivalent to the probability that the demand is less or equal to the average sales. That is,

$$NDR = P\{\Pi(\hat{q}, X) \leq \pi(\hat{q})\} = P\{\min(\hat{q}, X) \leq E[\min(\hat{q}, X)]\}$$

$$= P\{X \leq \hat{q} - \int_0^{\hat{q}} F(x) dx\} = F[\hat{q} - \int_0^{\hat{q}} F(x) dx]. \quad (8)$$

For single-mode distribution functions, it is also possible to develop the following bounds for NDR.
Proposition 3.3 \, NDR is bounded below by

\[
\begin{cases}
F\left(\frac{q^*(p+w)}{2p}\right), & \text{if } m > q^*, \\
F\left(q^* - \frac{mF(m)}{2} - \frac{(p-w)(q^*-m)}{p}\right), & \text{otherwise},
\end{cases}
\]

where \( m \) is the unique mode of the density \( f(\cdot) \).

Proof. There are two cases to consider. In case 1, the order quantity \( q^* < m \). Since demand is non-negative, \( F(0) = 0 \). Also, \( f(x) \) is an increasing function for \( x \leq m \). Thus, \( F(x) \) is convex for \( x \leq m \), and we have

\[
\frac{F(x)}{x} \leq \frac{F(q^*)}{q^*}.
\]

Therefore,

\[
F(q^*) - \int_0^{q^*} F(x) \, dx \geq F(q^*) - \int_0^{q^*} \frac{x F(q^*)}{q^*} \, dx
= F(q^*) - \frac{q^*(p-w)}{2p} = F\left(\frac{q^*(p+w)}{2p}\right).
\]

In case 2, we have \( m \leq q^* \). Thus, \( F(x) \leq \frac{x F(m)}{m} \) for \( x \leq m \), and \( F(x) \leq \frac{p-w}{p} \) for \( m \leq x \leq q^* \). Therefore,

\[
F(q^*) - \int_0^{q^*} F(x) \, dx \geq F(q^*) - \int_0^{m} \frac{xF(m)}{m} \, dx - \frac{p-w}{p}(q^*-m)
= F(q^*) - \frac{mF(m)}{2} - \frac{(p-w)(q^*-m)}{p}.
\]

Example 3.1 Let \( p = 5, w = 3 \), and the demand be normally distributed with \( \mu = m = 10 \) and \( \sigma = 5 \). The optimal order quantity for the supply chain is

\[
q^* = F^{-1}\left(\frac{p-w}{p}\right) = 8.73.
\]

Since \( m > q^* \), we have from (9), a lower bound \( F\left(\frac{q^*(p+w)}{2p}\right) = F(6.99) = 0.27 \) for NDR. The actual NDR is \( F(q^*) - \int_0^{q^*} F(x) \, dx = F(7.35) = 0.30 \).

4 NDR in Supply Contracts with Risk-Neutral Agents

In this section we consider a supply chain with a risk-neutral supplier and a risk-neutral retailer. Such a chain is known to be coordinated by buy-back or revenue-sharing contracts.
with appropriately chosen parameters. While the agents are risk-neutral, it is nevertheless possible to evaluate the natural downside risk of the agents under these contracts that coordinate the chain. We conclude the section by showing that buy-back or revenue-sharing contracts cannot coordinate the supply chain in some circumstances. We introduce the following notation:

agent $i = \text{retailer, supplier, and supply chain when } i = r, s, sc$, respectively

- $w = \text{the unit wholesale price}$
- $p = \text{the unit retail price}$
- $c = \text{the unit production cost}$
- $b = \text{the unit buy-back price}$
- $\phi = \text{the retailer’s share of the total profit of the chain}$

$\Pi_i(q, X) = \text{the agent’s profit when the order is } q \text{ and the demand is } X, i = r, s, sc$

$\pi_i(q) = E(\Pi_i(q, X)) = \text{the expected profit of agent } i \text{ when the order is } q, i = r, s, sc$

4.1 NDR in a Buy-Back Contract

In a buy-back contract, the supplier receives an income of $w$ for each unit that the retailer purchases, and the supplier pays the retailer an amount $b$ for each unit of unsold product at the end of the selling season. Since a buy-back contract with appropriately chosen $w$ and $b$ can coordinate the supply chain, the retailer chooses the optimal order quantity $\hat{q}$, which is optimal for the channel acting as a newsvendor. In the buy-back contract, the NDRs of the supplier, retailer and the supply chain are characterized in the following proposition.

**Proposition 4.1** In a buy-back contract that coordinates the supply chain, the retailer, supplier and the chain have the same NDR, which is equal to

$$F \left( \frac{\pi_{sc}(\hat{q}) + c\hat{q}}{p} \right).$$

where $\hat{q}$ is defined in (4).

**Proof.** In a coordinated buy-back contract, Pasternack (1985) shows that $\frac{p-b}{p} = \frac{w-b}{c}$. Denoting this ratio by $\phi$, we can write

$$p - b = \phi p, \ w - b = \phi c,$$

(14)
\[ \pi_r(\hat{q}) = pE[\min(\hat{q}, X)] - w\hat{q} + bE[\max(\hat{q} - x, 0)] \]
\[ = (p - b) \left( \hat{q} - \int_0^{\hat{q}} F(x) dx \right) - (w - b)\hat{q} \]
\[ = \phi \pi_{sc}(\hat{q}). \]  

(15)

For the coordination action, it is possible for us to write the profit function of the retailer and supplier as a fraction of the supply chain profit, that is \( \Pi_r(\hat{q}, X) = (p - b) \min(\hat{q}, X) - (w - b)\hat{q} = \phi \Pi_{sc}(\hat{q}, X) \) and \( \Pi_s(\hat{q}, X) = (1 - \phi)\Pi_{sc}(\hat{q}, X) \). Therefore, we have

\[ P\{\Pi_r(\hat{q}, X) \leq \pi_r(\hat{q})\} = P\{\Pi_s(\hat{q}, X) \leq \pi_s(\hat{q})\} = P\{\Pi_{sc}(\hat{q}, X) \leq \pi_{sc}(\hat{q})\}. \]

From (6) we see that the right hand expression in the above is the same as (13).

\[ \square \]

**4.2 NDR in a Revenue-Sharing Contract**

In a revenue-sharing contract that coordinates the supply chain, the supplier receives an income \( w = \phi c \) for each unit that the retailer purchases in addition to \( (1 - \phi) \) fraction of the revenue from the retailer. Since the order quantity is \( \hat{q} \), the profits of the retailer and the supplier are \( \phi \Pi_{sc}(\hat{q}, X) \) and \( (1 - \phi)\Pi_{sc}(\hat{q}, X) \), respectively, when the demand is \( X \). Thus,

\[ P\{\Pi_r(\hat{q}, X) \leq \pi_r(\hat{q})\} = P\{\Pi_s(\hat{q}, X) \leq \pi_s(\hat{q})\} = P\{\Pi_{sc}(\hat{q}, X) \leq \pi_{sc}(\hat{q})\}. \]  

(16)

Therefore, the retailer, the supplier and the chain have the same NDR, which is

\[ F\left( \frac{\pi_{sc}(\hat{q}) + c\hat{q}}{p} \right). \]  

(17)

**Remark 4.1** Note that the profits of each agent in revenue-sharing and buy-back contracts are identical for every demand realization if the contract parameters satisfy \( p - b = \phi p \) and \( w - b = \phi c \) (see Cachon and Lariviere, 2002). In this case, both the revenue-sharing and the buy-back contracts provide the same downside risk for the retailer and the supplier.

**Remark 4.2** In a wholesale price contract, the retailer faces a newsvendor problem and the supplier receives an income \( w \) for each unit that the retailer purchases. In this case, the retailer chooses an optimal order quantity \( q_r \) based on the newsvendor solution. The
supplier’s profit $\Pi_s(q, X) = wq$, regardless of the demand realization $X$. Thus, the supplier has no downside risk. On the other hand, the retailer’s NDR is $F \left( \frac{\pi_r(q) + wq}{p} \right)$. Therefore, in a wholesale price contract, the retailer bears a downside risk, while the supplier is risk-free in the context of NDR.

In a revenue-sharing contract, a risk averse retailer subject to a natural downside risk constraint solves the following problem:

$$\max_{q \geq 0} E(\Pi_r(q, X)),$$

s.t. $P\{\Pi_r(q, X) \leq \phi \pi_{sc}(\hat{q})\} \leq \beta$. \hfill (18)

Then from Proposition 3.2, the retailer’s optimal order quantity is

$$q_r^* = \begin{cases} 
\hat{q}, & \text{if } F \left( \frac{\pi_{sc}(\hat{q}) + c\hat{q}}{p} \right) \leq \beta, \\
\frac{PF^{-1}(\beta) - \phi \pi_{sc}(\hat{q})}{c}, & \text{if } F(q^0) < \beta < F \left( \frac{\pi_{sc}(\hat{q}) + c\hat{q}}{p} \right). 
\end{cases} \hfill (19)$$

Clearly, if $F \left( \frac{\pi_{sc}(\hat{q}) + c\hat{q}}{p} \right) > \beta$, the retailer orders less than $\hat{q}$, and the channel will not be coordinated.

In view of Remark 4.1, the same result holds for the buy-back contract as well.

5 A Risk-Sharing Contract (RSC)

5.1 Definition of Coordination of a Supply Chain

Consider a supply chain consisting of a retailer and a supplier, who have individual preferences toward risk. We let $\alpha_i$ denote the target profit level and $\beta_i$ denote the maximum risk agent $i$ is willing to take for his actual profit to fall below the target $\alpha_i$, $i = r, s, sc$. In the supply chain, the retailer makes a single purchase of a product from the supplier at the beginning of a period and sells the product at the end of the period. Before the retailer chooses his order quantity $q$, the supplier and the retailer agree to a supply contract.

In this paper, we assume that the supplier is risk-neutral with the objective of optimizing his expected profit, i.e., $\alpha_s = -\infty, \beta_s = 1$, and the retailer is risk-averse with parameters $(\alpha_r, \beta_r)$. The risk-neutrality assumption on the part of the supplier is reasonable because he is able to diversify his risk by serving a number of independent retailers, which is quite often the case in practice. Since the retailers are independent, the supply chain can be divided into
a number of sub-chains, each consisting of one supplier and just one retailer. In this case, therefore, it is enough to study a supply chain consisting of one supplier and one retailer.

We adopt the following definition of coordination proposed in Gan, Sethi and Yan (2003).

**Definition 5.1 Supply Chain Coordination.** A contract coordinates a supply chain if under the contract,

1. the agents’ reservation payoff constraints are satisfied, and
2. the agents’ joint action under this contract is Pareto-optimal.

Gan, Sethi and Yan (2003) prove that if the supplier is risk-neutral and the retailer is subject to a downside risk constraint as in this paper, then the joint action of the supplier and the retailer is Pareto-optimal if, and only if, the supply chain’s profit is maximized and the downside risk constraint is satisfied. For our special problem, Definition 5.1 can be reduced to the following definition.

**Definition 5.2** The supply chain is coordinated if the following conditions are satisfied:

1. the retailer and the supplier get payoffs not less than their respective reservation payoffs,
2. the retailer’s downside risk constraint is satisfied, and
3. the supply chain’s expected profit is maximized.

### 5.2 Design of the Risk-Sharing Contract

Let us begin with an initial contract viewed as the status quo for the supply chain. Let \( \Pi_i^0(q, X) \) be the agent’s profit under the initial contract when the order quantity is \( q \) and the demand is \( X \), and let \( \pi_i^0(q) \) be the agent’s expected profit when the order quantity is \( q \), \( i = r, s \). Let \( q^* \) be the optimal order quantity of the retailer with the risk aversion parameters \((\alpha_r, \beta_r)\) under the initial contract. The retailer, as we see in Section 4, will order less than \( \hat{q} \) if he is sufficiently risk averse. In the case when \( q^* = \hat{q} \), the system is already coordinated. Without loss of generality, we assume that \( q^* < \hat{q} \) in the remainder of this section. Our requirement on the initial contract is that under the contract, the retailer’s downside risk increases with his order quantity \( q \) when \( q^* \leq q \leq \hat{q} \), and his expected profit does not decrease with the channel’s profit. Then we have

\[
P\{\Pi_r^0(q^*, X) \leq \alpha_r\} = \beta_r.
\]
Obviously, the initial contract can be a buy-back contract, a revenue-sharing contract, or a wholesale price contract. It is natural to let their reservation payoffs be the same as those in the initial contract.

Since the supplier is risk neutral, intuitively it is possible for the supplier to induce the retailer to order \( \hat{q} \) by providing the required downside protection for the retailer. In return, the supplier gets higher expected profit.

We begin now with the construction of a risk-sharing contract, which involves adding a return policy to the initial contract. Specifically,

i) if the retailer’s order quantity \( q \) is less than or equal to \( q^* \), the initial contract is executed,

ii) if \( q \) is greater than \( q^* \) but not greater than \( \hat{q} \), then in addition to the initial contract with an order quantity \( q^* \), the retailer pays the wholesale price \( w' \) for each unit in excess of \( q^* \), and he receives a full refund of \( w' \) for each unsold unit, and

iii) if \( q \) is greater than \( \hat{q} \), the terms of the contract are the same as that in (ii) except that the number of unsold item that are fully refundable cannot exceed \( \hat{q} - q^* \).
Next we obtain the retailer's profit trajectory for \( q \in [0, \hat{q}] \) in these segments under the constructed risk-sharing contract. Let \( \Pi^0_i(q, X) \) be the agent \( i \)'s profit under the initial contract, when the order quantity is \( q \) and the demand is \( X \), \( i = r, s, sc \).

i) If \( q < q^* \), then the retailer’s profit trajectory is \( \Pi^0_i(q, X) \). In this case, the retailer’s expected profit is \( E(\Pi^0_i(q, X)) \).

ii) If \( q^* \leq q \leq \hat{q} \), then the retailer’s profit trajectory is

\[
\begin{align*}
\Pi^0_i(q^*, X) & \quad \text{if } X \leq q^*, \\
\Pi^0_i(q^*, X) + (p - w')(X - q^*) & \quad \text{if } q^* \leq X < q, \\
\Pi^0_i(q^*, X) + (p - w')(q - q^*) & \quad \text{if } X \geq q,
\end{align*}
\]

(21)

where \( (p - w')(X - q^*) \) represents the revenue \( p(X - q^*) \) on the additional sale of \( (X - q^*) \) items plus the refund \( w'(X - X) \) on the \( (q - X) \) unsold items minus the wholesale cost \( w'(q - q^*) \) paid on the additional \( (q - q^*) \) items, and \( (p - w')(q - q^*) \) represents the revenue \( p(q - q^*) \) on the additional sale of \( (q - q^*) \) items minus the wholesale cost \( w'(q - q^*) \) paid on the additional \( (q - q^*) \) items. In this case, the retailer’s expected profit is

\[
E(\Pi^0_i(q, X)) + (p - w') \left[ \int_{q^*}^{q} (x - q^*) f(x) dx + (1 - F(q))(q - q^*) \right].
\]

(22)

iii) If \( \hat{q} < q \), then similar to case (ii), the retailer’s profit trajectory is

\[
\begin{align*}
\Pi^0_i(q^*, X) - w'(q - \hat{q}) & \quad \text{if } X \leq q^*, \\
\Pi^0_i(q^*, X) + (p - w')(X - q^*) - w'[(q - X) - (\hat{q} - q^*)] & \quad \text{if } q^* \leq X < q, \\
\Pi^0_i(q^*, X) + (p - w')(q - q^*) & \quad \text{if } X \geq q.
\end{align*}
\]

(23)

In this case, since

\[
P(\Pi_r(X) \leq \alpha_r) \\
\geq P \{\Pi_r(X) \leq \alpha_r \} \cap \{X \leq q^*\} = P \{\Pi^0_r(q^*, X) - w'(q - \hat{q}) \leq \alpha_r\} \\
> P \{\Pi^0_r(q^*, X) \leq \alpha_r\} = \beta_r,
\]

the retailer’s downside risk constraint is not satisfied. Therefore, the retailer would not order a quantity \( q \) such that \( q > \hat{q} \).

When the initial contract is a revenue-sharing contract and the retailer orders \( \hat{q} \), his profit trajectory in RSC with respect to the demand realization can be illustrated by ABCD in Figure 2. When the realized demand is less than \( q^* \), the supplier takes a loss due to full refunding of \( (\hat{q} - q^*) \) units of the excess inventory. As a result, the retailer’s profit is the same as that in the initial contract. When the demand is larger than \( q^* \), the supplier takes
back all the unsold inventory. Therefore, the retailer’s profit increases linearly at the rate of 
\( (p - w') \) as the realized demand increases from \( q^* \) to \( \hat{q} \), and his profit remains constant as 
the realized demand increases beyond \( \hat{q} \).

It is easy to see that if the retailer enters into a risk-sharing contract and orders \( \hat{q} \), then 
the expected profits of the retailer and the supplier change by the amount

\[
(p - w') \left[ \int_{q^*}^{\hat{q}} (x - q^*) f(x) dx + (1 - F(q))(q - q^*) \right]
\]

(24)

and

\[
w' \left[ \int_{q^*}^{\hat{q}} (x - q^*) f(x) dx + (1 - F(q))(q - q^*) \right] - c(q - q^*),
\]

(25)

respectively. To ensure that the retailer and the supplier have incentives to enter into a risk-
sharing contract, (24) and (25) have to be greater than or equal to 0. Thus, we get a lower bound 
and an upper bound for \( w' \) in the following Proposition. Note that the increased profits of 
the retailer and the supplier are given in (24) and (25), respectively, if \( w' \) is between the 
bounds.

**Proposition 5.1** To ensure that the retailer and the supplier have incentives to enter into 
a risk-sharing contract, the lower bound and the upper bound of \( w' \) are

\[
c \quad \text{and} \quad \frac{c(q - q^*)}{\int_{q^*}^{\hat{q}} (x - q^*) f(x) dx + (1 - F(q))(q - q^*)}.
\]

(26)

**Proof.** The results follows directly from

\[
(p - w') \left[ \int_{q^*}^{\hat{q}} (x - q^*) f(x) dx + (1 - F(q))(q - q^*) \right] \geq 0
\]

(27)

and

\[
w' \left[ \int_{q^*}^{\hat{q}} (x - q^*) f(x) dx + (1 - F(q))(q - q^*) \right] - c(q - q^*) \geq 0.
\]

(28)

Now, we proceed to prove that our risk-sharing contract coordinates the supply chain.

**Theorem 5.1** For any risk-aversion pair \( (\alpha_r, \beta_r) \), the risk-sharing contract coordinates the 
supply chain if \( w' \) in the contract is between the bounds given in Proposition 5.1.

**Proof.** We prove that three conditions of Definition 5.2 are satisfied.
(i) According to (22), the retailer gets an expected profit more than his reservation payoff. From Proposition 5.1, the supplier’s reservation profit is satisfied.

(ii) If the retailer orders \( q^* \), then according to (20), the downside risk is

\[
P\{\Pi_r^0(q^*, X) \leq \alpha_r \} = \beta_r.
\]  

(29)

With the risk-sharing contract, for any order quantity \( q, \) \( q^* \leq q \leq \hat{q} \), the retailer’s profit from (21) is

\[
\begin{align*}
\Pi_r^0(q, X) & \quad \text{if } X \leq q^*, \\
\Pi_r^0(q^*, X) + (p' - w')(X - q^*) & \quad \text{if } q^* \leq X < q, \\
\Pi_r^0(q^*, X) + (p' - w')(q - q^*) & \quad \text{if } X \geq q.
\end{align*}
\]  

(30)

It is obvious that the retailer’s profit is larger than \( \alpha_r \), when the realized demand is larger than \( q^* \). The retailer’s downside risk is

\[
P\{\Pi_r(q, X) \leq \alpha_r \} = P \{ \Pi_r^0(q^*, X) \leq \alpha_r \} = \beta_r.
\]  

(31)

Therefore, the retailer’s downside risk constraint is satisfied under the risk-sharing contract.

(iii) From (22), the expected profit of the retailer is increasing for \( q^* \leq q \leq \hat{q} \). In addition, the retailer’s ordering a quantity exceeding \( \hat{q} \) violates his downside risk constraint, since the contract specifies the maximum return quantity as \( (\hat{q} - q^*) \). Therefore, the retailer would order \( \hat{q} \), which maximizes his expected profit subject to his downside risk constraint. When the retailer orders \( \hat{q} \), the supply chain’s profit is also maximized.

According to Definition 5.2, the supply chain is coordinated.

\[\Box\]

**Remark 5.1** The risk-sharing contract is not the only coordinating contract. Consider the following two-part tariffs contract: the supplier sets the wholesale price equal to the exogenous retail price, and then gives the retailer a lump-sum side payment not less than the retailer’s expected profit \( E(\Pi_r^0(q^*, X)) \) in the initial contract. The tariff contract will coordinate the supply chain. However, in this contract the supplier takes all the risk, and this risk may be quite large. More specifically, in our risk-sharing contract, the retailer takes as much risk as he possibly can without affecting the total expected channel profit. Thus, the supplier’s risk in the risk-sharing contract is much less than that in the tariff contract.
5.3 Incentive and Risk Analysis

Let $E(\Pi_0^s(q^*, X))$ denote the supplier’s expected profit in the initial contract. Let $\Delta_r(w')$ and $\Delta_s(w')$ represent the incremental profits of the retailer and the supplier given by (24) and (25), respectively. Then the following table summarizes the retailer’s expected profits, his downside risks, and the supplier’s expected profits in the initial and the risk-sharing contracts.

<table>
<thead>
<tr>
<th>Contract</th>
<th>Retailer</th>
<th>Supplier</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Expected profit</td>
<td>Downside risk</td>
</tr>
<tr>
<td>Initial Contract</td>
<td>$E(\Pi_0^r(q^*, X))$</td>
<td>$\beta_r$</td>
</tr>
<tr>
<td>Risk-Sharing</td>
<td>$E(\Pi_0^r(q^*, X)) + \Delta_r(w')$</td>
<td>$\beta_r$</td>
</tr>
</tbody>
</table>

In the risk-sharing contract, the supply chain’s expected profit increases because the order quantity increases to the channel’s optimal order quantity $\hat{q}$. The retailer’s profit increases by the amount $\Delta_r(w')$, but his downside risk does not increase because he gets a full refund for all unsold units above the quantity $q^*$. The supplier assumes more risk in the risk-sharing contract because of the refund policy, but he also gets the additional expected profit of $\Delta_s(w')$ in return for assuming that risk. Because of the risk neutrality of the supplier, it is better for the supplier to assume some risk in return for an increased expected profit.

6 Limitation of RSC and Discussion

Our construction of the risk-sharing contract assumes that the retailer communicates his risk-aversion parameters and the demand distribution to the supplier. The important question to examine here is whether the retailer has an incentive to cheat the supplier by telling that he is more risk-averse than he actually is, and by exaggerating his demand distribution.

In what follows, we give an example to show that the retailer can cheat and increase his profit.

**Example 6.1** Let $p = 5$, $c = 2 \alpha_r = 27$, and $\beta_r = 0.4$. Let the initial contract be a buy-back contract with the wholesale price $w = 3$, and the return credit $b = 2.5$. Let the demand $X$ have a uniform distribution $U[5,15]$. Then, $q^*_r = 9$ and $\hat{q} = 11$. Suppose the retailer and the supplier have equal bargaining power, and they agree that each gets a half of the profit, then $w'$ should be $4.5$ and $\pi_{sc}(11) = 24$, $\pi_s(11) = 12$, and $\pi_r(11) = 12$. If the retailer cheats the supplier by announcing $X \sim U[7,17]$, $\alpha_r = 27$, and $\beta_r = 0.2$, then $q^*_r = 9$,
\( \hat{q} = 13 \), and \( w' = 3.74 \) according to the negotiated profit splitting rule. But under the true demand, the expected profits of the supply chain, the supplier and the retailer are 23, 10, and 13, respectively, and the retailer’s downside risk constraint is satisfied.

In Example 6.1, we see that the retailer’s profit increases, whereas the profits of the supplier and the supply chain decrease. Thus, we face a moral hazard problem. Such problems have been extensively studied in the game theory literature. One way to remedy this situation is to invest resources to find the truth. Another way is to have the retailer’s proportion of the channel profit depend on \((\alpha_r, \beta_r)\) in such a way that the proportion increases with \( q^*(\alpha_r, \beta_r) \). This would reduce the retailer’s incentive to cheat. A more detailed analysis of the moral hazard problem arising in the risk-sharing contract is a topic of further research.

In our analysis, we have assumed the supply chain to be risk neutral even though one of the agents, namely, the retailer, is risk averse. Some may argue that in this case, the supply chain considered as a single entity should have some aversion toward risk. More generally, if a supply chain consists of a risk-averse supplier with parameter \((\alpha_s, \beta_s)\) and a risk-averse retailer with parameter \((\alpha_r, \beta_r)\), imputing the risk-aversion pair \((\alpha_{sc}, \beta_{sc})\) for the supply chain is a matter of concern. Moreover, the optimal order quantity for the channel might be the solution of the channel’s problem that maximizes its profit under a downside risk constraint with appropriately selected parameters \((\alpha_{sc}, \beta_{sc})\). In this case, a contract could be said to coordinate the supply chain consisting of a risk-averse supplier and a risk-averse retailer, if the actions taken by the agents under the contract maximize the profit of the supply chain with the downside risk parameters \((\alpha_{sc}, \beta_{sc})\), and satisfy the retailer’s and the supplier’s downside risk constraints. Clearly, the coordinating contract of this type depends on the setting of the downside risk parameters \((\alpha_{sc}, \beta_{sc})\) for the supply chain. Thus, such a contract has an element of arbitrariness, and moreover, it may not be Pareto optimal.

7 Conclusion

We have treated the issue of supply chain coordination consisting of a risk-averse retailer and a risk-neutral supplier. We show that the well-known revenue-sharing and buy-back contracts may not coordinate a supply chain with a risk-averse retailer. By adding a return policy to the initial contract, we construct a risk-sharing contract. We demonstrate that the risk-sharing contract coordinates the supply chain with a retailer subject to a downside risk
References


