Section 1.1
Computer Problems

21) continued

integer n; **real** x, y, z
for $n = 1$ **to** 10
  $x \leftarrow (n-1)/2$
  $y \leftarrow n^2/3.0$
  $z \leftarrow 1.0 + 1/n$
  output x, y, z
end for

c. What values would the following assignment statements produce?

integer i, j; **real** $c, f, x, half$
$x \leftarrow 10/3$
$i \leftarrow$ integer$(x + 1/2)$
$half \leftarrow 1/2$
$j \leftarrow$ integer$(half)$
$c \leftarrow (5/9)(f - 32)$
$f \leftarrow 9/5c + 32$
output $x, i, half, j, c, f$

d. Discuss what is wrong with the following pseudocode segment:

real *area, circum, radius*
$radius \leftarrow 1$
$area \leftarrow (22/7)(radius)^2$
$circum \leftarrow 2(3.1416)radius$
output *area, circum*

22. Criticize the following pseudocode for evaluating $\lim_{x\to 0} \arctan(|x|)/x$. Code and run it to see what happens.

integer i; **real** x, y
$x \leftarrow 1$
for $i = 1$ **to** 24
  $x \leftarrow x/2.0$
  $y \leftarrow \arctan(|x|)/x$
  output x, y
end for

23. Carry out some computer experiments to illustrate or test the programming suggestions in Appendix A. Specific topics to include are these:

a. When to avoid arrays.

b. When to limit iterations.

c. Checking for floating-point equality.

d. Ways for taking equal floating-point steps.

e. Various ways to evaluate functions.

Hint: Comparing single and double precision results may be helpful.

24. (**Easy/Difficult Problem Pairs**) Write a computer program to obtain the power form of a polynomial from its roots. Let the roots be $r_1, r_2, \dots, r_n$. Then (except for a scalar factor) the polynomial is the product

$$p(x) = (x - r_1)(x - r_2)\cdots(x - r_n).$$

Find the coefficients in the expression

$$p(x) = \sum_{j=0}^{n} a_j x^j$$

Test your code on the **Wilkinson polynomials** in Computer Exercises 3.1.10 and 3.3.9. Explain why this task of getting the power form of the polynomial is *trivial*, whereas the inverse problem of finding the roots from the power form is quite difficult.

25. A **prime number** is a positive integer that has no integer factors other than itself and 1. How many prime numbers are there in each of these open intervals: $(1, 40)$, $(1, 80)$, $(1, 160)$, and $(1, 2000)$? Make a guess as to the percentage of prime numbers among all numbers.

26. Mathematical software systems such as Maple, Mathematica, and MATLAB are able to do both numerical calculations and symbolic manipulations. Verify symbolically that a nested multiplication is correct for a general polynomial of degree ten.

27. In MATLAB, the `rat` function finds a rational fraction approximation (numerator and denominator) within a certain tolerance to a given floating-point number. For example, `[a,b]=rat(pi, 8000e-6)` return `a=22` and `b=7`. However, the relative error between $19/6$ and π is 0.007981306248670 in format `long`, which is less than the tolerance 0.008. What's going on here? In terms of absolute and relative errors, is $19/6$ or $22/7$ the better approximation to π?

28. Use mathematical software to reproduce the three solutions to Example 1.1.2.
Hint: In MATLAB, use commands `str2nun(num2str (x,4))` for rounding to four significant decimal digits as well as `format long`.

29. Explain the results from coding and executing the following pseudocode using mathematical software such as in MATLAB with `format long`: