Math 6318 Homework
Dr. Minkoff

[1] Using linear lagrange finite elements, obtain a solution to

$$\begin{cases} -\frac{d}{dx}\left((1+x)\frac{du}{dx}(x)\right) + u = 0, \quad 0 < x < 1 \\ u(0) = 1 \\ u(1) = 0 \end{cases}$$

Be sure you use adequate quadrature to exactly integrate all terms. Refine the mesh and plot the solution with different mesh values on the same plot. Clearly label all solutions.

[2] Consider the heat equation with inhomogeneous boundary conditions:

$$\begin{cases} \rho c \frac{\partial u}{\partial t} - k\frac{\partial^2 u}{\partial x^2} = f(x,t), \quad 0 < x < l, \quad t > 0 \\ u(x, t_0) = \Psi(x), \quad 0 < x < l \\ u(0,t) = g(t), \quad t > t_0 \\ u(l,t) = h(t), \quad t > t_0. \end{cases}$$

Define

$$p(x,t) = g(t) + \frac{x}{l}(h(t) - g(t))$$

and

$$v(x,t) = u(x,t) - p(x,t).$$

What initial-boundary value problem does v satisfy?