Problem 1 – classical wave equation

a) Show that \( u(x,t) = \sin(kx - \omega t) \) satisfies the classical wave equation by directly using
the function \( \sin(kx - \omega t) \) in the wave equation.

b) Show that \( u(x,t) = \sin(kx - \omega t) \) satisfies the classical wave equation by using the
trigonometric identity \( \sin(A - B) = \sin A \cos B - \cos A \sin B \).

c) Show that \( u(x,t) = e^{i(kx - \omega t)} \) satisfies the classical wave equation by using the Euler
identity \( e^{i\theta} = \cos \theta + i \sin \theta \).

d) Show that \( u(x,t) = e^{i(kx - \omega t)} \) satisfies the classical wave equation by directly differen-
tiating the function \( e^{i(kx - \omega t)} \).

Problem 2 – different wavelength components

a) Show that \( u(x,t) = \sin(k_1x) \cos(\omega_1 t) - \cos(k_2x) \sin(\omega_2 t) \) is not a classical wave if
\( k_2 = 2k_1 \) and \( \omega_1 = \omega_2 \).

b) Show that \( u(x,t) = \sin(k_1x) \cos(\omega_1 t) - \cos(k_2x) \sin(\omega_2 t) \) is a classical wave if \( k_2 = 2k_1 \)
and \( \omega_2 = 2\omega_1 \). What is the propagation speed of this wave?

Problem 3 – Taylor series

For (a), (b), and (c) you can look up the answers using any resource.

a) Write down, up to (and including) 7th powers of \( x \), the Taylor series for \( \sin x \).

b) Write down, up to 7th powers of \( x \), the Taylor series for \( \cos x \).

c) Write down, up to 7th powers of \( x \), the Taylor series for \( e^x \).

d) Write down, up to 7th powers of \( x \), the Taylor series for \( e^{ix} \) by using your answer (c).

e) By comparing your answer (d) to the Euler formula \( e^{i\theta} = \cos \theta + i \sin \theta \) show how you
could identify the \( \sin x \) and \( \cos x \) Taylor series (assuming you didn’t know them).
Problem 4 – Schrödinger Equation

For a particle in free space \((V = 0)\), the angular frequency \(\omega\) and the wave number \(k\) of its associated wave function are related by

\[
\hbar \omega = \frac{\hbar^2 k^2}{2m} \tag{1}
\]

a) Verify that, if a monochromatic wave of the form \(\psi = e^{i(kx - \omega t)}\) is substituted into the Schrödinger time dependent equation (also called the Schrödinger wave equation), the above relation is reproduced.

b) Show that \(\psi = \cos(kx - \omega t)\) fails to satisfy the Schrödinger time dependent equation.