Chem 3322 homework #6 – due March 11, 2019

Problem 1 – reduced mass

Please read the following as background for this question

http://www.eng.fsu.edu/~dommelen/quantum/style_a/nt_mred.html

(note: this question is actually problem 5-5 and 5-6 from your textbook)

a) Consider two particles of masses \( m_1 \) and \( m_2 \) in one dimension, interacting through a potential that depends only upon their relative separation. Given that the force acting upon the \( j \)th particle is \( f_j = -(\partial V / \partial x_j) \), show that \( f_1 = -f_2 \). What law is this?

b) Newton’s equations for the two particles are

\[
m_1 \frac{d^2 x_1}{dt^2} = -\frac{\partial V}{\partial x_1} \quad \text{and} \quad m_2 \frac{d^2 x_2}{dt^2} = -\frac{\partial V}{\partial x_2}
\]

Now introduce center of mass and relative coordinates by

\[
X \equiv \frac{m_1 x_1 + m_2 x_2}{M} \quad \text{and} \quad x \equiv x_1 - x_2
\]

where \( M = m_1 + m_2 \), and solve for \( x_1 \) and \( x_2 \) to obtain

\[
x_1 = X + \frac{m_2}{M} x \quad \text{and} \quad x_2 = X - \frac{m_1}{M} x
\]

Show that Newton’s equations in these coordinates are

\[
m_1 \frac{d^2 X}{dt^2} + \frac{m_1 m_2}{M} \frac{d^2 x}{dt^2} = -\frac{\partial V}{\partial x}
\]

and

\[
m_2 \frac{d^2 X}{dt^2} - \frac{m_1 m_2}{M} \frac{d^2 x}{dt^2} = +\frac{\partial V}{\partial x}
\]

c) Now add these two equations to find

\[
M \frac{d^2 X}{dt^2} = 0
\]

Interpret this result.
d)

Now divide the first equation by \( m_1 \) and the second by \( m_2 \) and subtract to obtain

\[
\frac{d^2 x}{dt^2} = - \left( \frac{1}{m_1} + \frac{1}{m_2} \right) \frac{\partial V}{\partial x}
\]  

(7)

or

\[
\mu \frac{d^2 x}{dt^2} = - \frac{\partial V}{\partial x}
\]  

(8)

where \( \mu = \frac{m_1 m_2}{m_1 + m_2} \) is the reduced mass. Interpret this result, and discuss how the original two-body problem has been reduced to two one-body problems.

e)

Extend the result of (a-d) to three dimensions. Note that in three dimensions the relative separation is given by

\[
r_{12} = \left[ (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 \right]^{1/2}
\]  

(9)