

HOMEWORK 7, ACTS 4306

For each problem, you need to choose a correct answer among 5 given answers.

1. Let T_1 be the time between a car accident and reporting a claim to the insurance company. Let T_2 be the time between the report of the claim and payment of the claim. The joint density function $f(x, y)$ of T_1 and T_2 is constant over the region $0 < t_1 < 6$, $0 < t_2 < 6$, $t_1 + t_2 < 10$, and zero otherwise. Determine $E(T_1 + T_2)$, the expected time between a car accident and payment of the claim.

- [A] 4.9 [B] 5.0 [C] 5.7 [D] 6.0 [E] 6.7

2. A diagnostic test for the presence of a disease has two possible outcomes: 1 for disease present and 0 for disease not present. Let X denote the disease state of a patient, and Y denote the outcome of the diagnostic test. The joint probability function of X and Y is given by:

$$P(X = 0, Y = 0) = .800; \quad P(X = 1, Y = 0) = .050;$$

$$P(X = 0, Y = 1) = .025; \quad P(X = 1, Y = 1) = .125.$$

Calculate $\text{Var}(Y|X = 1)$.

- [A] .13 [B] .15 [C] .20 [D] .51 [E] .71

3. A car dealership sells 0,1, or 2 luxury cars on any day. When selling a car, the dealer also tries to persuade the customer to buy an extended warranty for the car. Let X denote the number of luxury cars sold in a given day, and let Y denote the number of extended warranties sold.

$$P(X = 0, Y = 0) = 1/6; \quad P(X = 1, Y = 0) = 1/12; \quad P(X = 1, Y = 1) = 1/6;$$

$$P(X = 2, Y = 0) = 1/12; \quad P(X = 2, Y = 1) = 1/3; \quad P(X = 2, Y = 2) = 1/6.$$

What is the variance of X ?

- [A] .47 [B] .58 [C] .83 [D] 1.42 [E] 2.58

4. Let X and Y be continuous random variables with joint density function

$$f(x, y) = (8/3)xyI(0 \leq x \leq 1)I(x \leq y \leq 2x).$$

Calculate the covariance of X and Y .

- [A] .04 [B] .25 [C] .67 [D] .80 [E] 1.24

5. Let X and Y denote the values of two stocks at the end of a five-year period. X is uniformly distributed on the interval $(0, 12)$. Given $X = x$, Y is uniformly distributed on the interval $(0, x)$. Determine $\text{Cov}(X, Y)$ according to this model.

- [A] 0 [B] 4 [C] 6 [D] 12 [E] 24

6. The stock prices of two companies at the end of any given year are modeled with random variables X and Y that follow a distribution with joint density function

$$f(x, y) = 2xI(0 < x < 1)I(x < y < x + 1).$$

What is the conditional variance of Y given $X = x$?

- [A] 1/12 [B] 7/6 [C] $x + 1/2$ [D] $x^2 - 1/6$ [E] $x^2 + x + 1/3$

7. A device contains two circuits. The second circuit is a backup for the first, so the second is used only when the first has failed. The device fails when and only when the second circuit fails. Let X and Y be the times at which the first and second circuits fail, respectively. X and Y have joint probability density function

$$f(x, y) = 6e^{-x}e^{-2y}I(0 < x < y < \infty).$$

What is the expected time at which the device fails?

- [A] .33 [B] .50 [C] .67 [D] .83 [E] 1.50

8. An insurance policy pays a total medical benefit consisting of two parts for each claim. Let X represent the part of benefit that is paid to the surgeon, and let Y represent the part that is paid to the hospital. The variance of X is 5000, the variance of Y is 10,000, and the variance of the total benefit, $X + Y$, is 17,000. Due to increasing medical costs, the company that issues the policy decides to increase X by a flat amount of 100 per claim and to increase Y by 10% per claim. Calculate the variance of the total benefit after these revisions have been made.

- [A] 18,200 [B] 18,800 [C] 19,300 [D] 19,520 [E] None of the answers

9. Let X denote the size of a surgical claim and let Y denote the size of the associated hospital claim. An actuary is using a model in which $\text{Var}(Y|X = x) = 4x^2$, $E(Y|X = x) = x$, and X is uniformly distributed on $(2, 5)$. Calculate the variance of Y .

- [A] 48.5 [B] 53.8 [C] 57.4 [D] 61.5 [E] None of these

10. An insurance policy is written to cover a loss X where X has density function $f(x) = (3/8)x^2I(0 \leq x \leq 2)$. The time (in hours) to process a claim of size x , where $0 \leq x \leq 2$, is uniformly distributed on the interval from x to $2x$. Calculate the probability that a randomly chosen claim on this policy is processed in three hours or more.

- [A] .17 [B] .25 [C] .32 [D] .58 [E] .83