HOMEWORK 3, STAT 6331

1. Let \((X, Y)\) denote an iid sample from Geometric\((p)\), that is, the pmf of \(X\) is

\[
P(X = k | p) = p(1 - p)^{k-1}, \quad k = 1, 2, \ldots
\]

Consider the statistic \(T = X - Y\), and show that it is not sufficient.

Hint: It suffices to consider a particular value of \(T\), say \(T = 0\) for the sake of simplicity, and prove that \(P((X, Y) = (i, j)|T = 0, p)\) depends on \(p\).

2. Suppose that a family \(\{f_\theta(x), \theta \in \Omega\}\) of pdfs or pmfs is specified. It is known that \(\Omega := \{\theta_0, \theta_1\}\), that is, the parameter \(\theta\) takes on only two values.

The likelihood ratio statistic (typically written as \(\Lambda(x)\) or \(\lambda(x)\)) is defined as

\[
T(X) = \frac{f_{\theta_1}(X)}{f_{\theta_0}(X)}.
\]

Prove that the likelihood ratio is SS (Sufficient Statistic).

3. Let \(\underline{X} := (X_1, X_2, \ldots, X_n)\) be iid sample from a uniform distribution on \((\theta - 1/2, \theta + 1/2)\), that is, \(X_1 \sim U(\theta - 1/2, \theta + 1/2)\) or we can say that \(X_1\) has the uniform pdf \(f(x|\theta) = I(x \in (\theta - 1/2, \theta + 1/2))\). Find a two-dimensional SS.

4. Prove that if a sufficient statistic \(T\) is a function of \(U\), then \(U\) is sufficient. And if \(T\) is a function of \(U\) which is not sufficient, neither is \(T\).

Problems below are from the text, p.300

5. Exerc. 6.1
6. Exerc. 6.2
7. Exerc. 6.3
8. Exerc. 6.5
9. Exerc. 6.6
10. Exerc. 6.7