

Name (Please Print):

QUIZ 4 STAT5352

*Instruction: Each problem/question is of equal value (2 points).*

1. Consider a joint pdf

$$f^{XY}(x, y) = Cy^{-1}e^{-(y+x/y)}I(x > 0)I(y > 0).$$

Find:  $E(X|Y = y)$ .

2. Consider a joint pdf

$$f^{XY}(x, y) = C(y + 2x^2)I(0 < x < y < 1).$$

Find: (a)  $E(X|Y = y)$ .(b)  $E(Y|X = x)$ .

3. The joint density of  $X$  and  $Y$  is given by

$$f^{X,Y}(x, y) = C(x + 2y^2)I(x + 2y \leq 1)I(x > 0)I(y > 0).$$

Compute  $E\{X^2|Y = y\}$ .

4. The joint density of  $X$  and  $Y$  is given by

$$f^{X,Y}(x, y) = y^{-1}e^{-y}I(0 < x < y)I(0 < y < \infty).$$

Compute  $E\{X^3|Y = y\}$ .

3. Three types of batteries A,B and C are tested by three laboratories.

Working times until failure are as follows:

A: 72, 74, 76.

B: 70, 72, 74.

C: 76, 78, 74,

Use  $\alpha = .01$  and  $\alpha = .005$  to test the null hypothesis that these compressors have the same mean failure time.

4. Sales (in thousands of dollars) during 14 days are

13, 14, 13, 10, 13, 14, 19, 17, 15, 16, 11, 11, 16, 14.

There is a historical opinion that the median sale is 13.5. The manager thinks that now sales are improved. Use  $\alpha = .1$ , and test this conjecture using: (a) Binomial Table. (b) Normal approximation.

5. Consider a linear regression problem based on  $n$  pairs of observations  $\{(Y_l, X_l), l = 1, \dots, n\}$  with  $Y$  and  $X$  being response and predictor, respectively. Consider a least square linear regression approach and prove the

classical anova relation  $SST = SSR + SSE$  with mentioning an identical relation for corresponding degrees of freedom. Hint: write down a system of normal equations, then write down definitions for  $SST$ ,  $SSR$  and  $SSE$ , and then use the normal equations to prove the anova relation.