SOLUTION FOR HOMEWORK 11, STAT 4372

Welcome to your 11th homework. Here you have an opportunity to solve classical credibility-related problems. Actuarial exams typically contain several questions on the topic. Before solving them, look through pages that I explained you in class.

1. Problem 20.5 Solution: Because N is Poisson(λ) and can be approximated by a normal distribution, we get $Z = (N - \lambda)/\lambda^{1/2}$ being standard normal. Then it is given that

$$\mathrm{P}(|Z| < .03\lambda^{1/2}) = .975.$$

From the Normal table $\mathrm{P}(|Z| < 2.24) = .975$, and this yields the answer $\lambda = 5,582$. Please note how large λ is.

2. Problem 20.7 Given: $X = \sum_{i=1}^{N} Y_i$, N is Poisson(θ), $E(Y) = 1,500$; $Var(Y) = 7,500^2$, and $\Pr(|X - E(X)| < .06E(X)) = .0$. Find $E(N)$.

Solution: First of all, remember that in class I pointed on this-type problem in your text (page 560) and explained all the tricks around it. So read that page again and then be comfortable with this type of problems.

Note that $Z = (X - E(X))/(Var(X))^{1/2}$ is standard normal, and then

$$\Pr\Big(\frac{|X - E(X)|}{(Var(X))^{1/2}} < \frac{.06E(X)}{(Var(X))^{1/2}}\Big) = .9.$$

This yields $.06E(X)/(\mathrm{Var}(X))^{1/2} = 1.645$, the last number is from the Normal Table. Now, using our technique for the aggregated loss X I calculate $E(X) = \theta 1500$ and $\mathrm{Var}(X) = \theta(1,500^2 + 7,500^2)$; remember how I did it in class using conditional expectation and variance formulae.

Plug in and get

$$\frac{(.06)\theta(1500)}{\theta^{1/2}(1,500^2 + 7,500^2)^{1/2}} = 1.645.$$

Answer: $\theta = 19,544$.

3. Problem 20.8 Solution: This is a curious problem that I told you in class about. In the previous problem we calculated the standard 19,544, and here we have only 6,000 observations. This yields $Z = 6000/19544 = .55$. Then

$$P_C = ZY + (1 - Z)M = 16,001,330.$$

4. Problem 20.9 Given: The familiar aggregated/compounded model $X_1, \ldots, X_n$ where $X_i = Y_{i1} + \ldots + Y_{iN_i}$ where N_i are Poisson(θ). Further, $r = .05$, $\sigma^2 = Var(N) = \theta$, $\xi = E(N) = \theta$.

Solution: Remember the formula for the standard

$$n = (y_p/r)^2(\sigma/\xi)^2.$$

Thus for the standard number of claims $800 = n\theta = (y_p/.05)^2$ which yields $y_p = 2^{1/2}$.

Then a direct integration gives you

$$E(Y) = \int_0^{100} (.0002)y(100-y)dy = 100/3,$$

and similarly $Var(Y) = 555.55$. Then the new standard for full credibility is (similarly to case 2 on page 560 and as I explained for previous problems)

$$(2^{1/2}/.1)^2[1 + 555.55/(100/3)^2] = 300.$$

5. Problem 20.13 Solution: This is a similar problem. So I show only calculations.Write,

$$E(X) = .5(1) + .3(2) + .2(10) = 3.1.$$

Then

$$E(X^2) = .5(1)^2 + .3(2)^2 + .2(10)^2 = 21.7.$$

And combining $Var(X) = 21.7 - 3.1^2 = 12.09$. This yields the standard for full credibility

$$(1.645/.1)^2(1 + 12.09/3.1^2) = 611.04.$$

Answer: The standard for full credibility is $n = 612$.

6. Problem 20.38 Given: $Z = n/(n+k)$. $Z = .5$ for $n = .5$. Find Z for $n = 3$.

Solution: From $.5/(.5+k) = .5$ we get $k = .5$. Answer: $Z = 3/(3+.5) = 6/7$.

7. Problem 20.40 (a,b) Solution: (a). I use notation of the text (p.584-...)and just follow the steps. Write:

$$\mu(\lambda) = \lambda, \quad v(\lambda) = \lambda,$$

$$\mu = E(\Lambda) = \int_1^{\infty} 4\lambda^{-4} d\lambda = 4/3, \quad v = E(\Lambda) = 4/3,$$

$$a = Var(\Lambda) = \int_1^{\infty} 4\lambda^{-3} d\lambda - 16/9 = 2/9,$$

$$k = (4/3)/(2/9) = 6, \quad Z = 3/(3+6) = 1/3.$$

We did all the necessary calculations for the Bühlmann credibility premium

$$P_C = Z\bar{X} + (1-Z)\mu = (1/3)(1) + (2/3)(4/3) = 11/9.$$

(b) In the same way

$$\mu = \int_0^1 \lambda d\lambda = 1/2, \quad v = \mu = 1/2,$$

$$a = \int_0^1 \lambda^2 d\lambda - (1/2)^2 = 1/12, \quad k = (1/2)(1/12) = 6, \quad Z = 3/(3+6) = 1/3.$$

This yields the Bühlmann credibility premium

$$P_C = (1/3)(1) + (2/3)(1/2) = 2/3.$$

8 Problem 20.44 (a,b). Solution: (a) Write

$$\Pr(N=0) = \int_1^3 e^{-\lambda}(.5)d\lambda = (e^{-1} - e^{-3})/2 = .16$$

(b) Using our traditional notation for the Bühlmann model,

$$\mu = v = E(\Lambda) = \int_1^3 \lambda(.5)d\lambda = 2,$$

$$a = Var(\Lambda) = \int_1^3 \lambda^2(.5)d\lambda - 2^2 = 1/3,$$

$$k = 2/(1/3) = 6, \quad Z = 1/(1+6) = 1/7,$$

$$P_C = (1/7)(1) + (6/7)(2) = 13/7.$$

9. Problem 20.61 Solution: Here I use the Bühlmann-Straub model. Remember the discussion in class and use pp. 588-589. Write:

$$\mu = (.6)(2000) + (.3)(3,000) + (.1)(4,000) = 2,500,$$

$$v = 1,000^2, \;\; a = (.6)(2000)^2_{.}3(3000)^2 + .1(4000)^2 - (2500)^2 = 450,000,$$

$$Z = \frac{80}{80 + (1,000,000)/450,000} = .973,$$

$$\bar{X} = \frac{24,000 + 36,000 + 28,000}{80} = 1,100.$$

Using these data we calculate the credibility premium

$$P_c = (.973)(1,111) + (.027)(2500) = 1138.$$