SOLUTION FOR HOMEWORK 12, STAT 4372

This is the LAST homework — simulations!

1. Problem 21.6 Solution: The general theory is simple. Variance of $\bar{X} = \sum_{l=1}^{n} X_l$ for iid observations is $Var(\bar{X}) = \sigma^2/n$ where $\sigma^2 = Var(X)$.

As a result, for this problem we need $\sigma/n^{1/2} = .05$ which yields

$$n = \sigma^2/(.05)^2.$$

We do not know σ so we estimate it using the given data by a sample variance estimate, (note that the sample mean is 3)

$$S_5^2 = (5-1)^{-1} \sum_{l=1}^{5} (l-3)^2 = 2.5$$

Plug-in this sample variance and get $n = 1000$.

2. Problem 21.7 Solution: Here you need to have and use the Normal Table — do use it because it is necessary to understand how to use it here.

Here, for the normal random variable simulation, we need to find the solution x of the equation

$$u = \Phi((x - 15,000)/2000)$$

where Φ is the cdf of standard normal RV available in the table. For $u = .5398$ the solution is

$$\frac{x - 15,000}{2000} = .1$$

which yields $X_1 = 15,200$. Absolutely similarly, for the second month $u = .1151$ we get

$$-1.2 = \frac{x - 15,000}{2000}$$

which yields $X_2 = 12,600$. For the third month $u = .0013$ and this implies

$$-3 = \frac{x - 15,000}{2000}$$

with solution $X_3 = 9000$. For the fourth month $u = .7881$ so

$$.8 = \frac{x - 15,000}{2000}$$

with $X_4 = 16,600$.

Now we have a deductible 10,000, so the premiums are $Y_i = (X_i - 10,000)_+$ with $Y_1 = 5,200$, $Y_2 = 2,600$, $Y_3 = 0$ and $Y_4 = 6,600$. The total is 14,400.

3. Problem 21.9 Solution: Well, here a graphic can help but I will describe it step by step in any case.

Use Binomial Table, software or Formula and calculate $f_X(x)$ for X being Binomial($theta = .03318, n = 100$). But do not rush to calculate for all x — begin with smallest and see when the sum "strikes" u.

Indeed, the simulated value is $u = .18$. Then the answer is the solution of $u = F_X(x)$. Note that X is discrete with jumps at 0,1, etc. Now, $f_X(0) = .034$, $f_X(1) = .1175$ and $f_X(2) = .2$. Bingo! If you graph the cdf you will see that the solution to the equation is $x = 2$. This is the simulated number of deaths.

4. Problem 21.11 Solution: The empirical distribution assigns probability .5 to each point. The mean of the RV is 2 and the variance is 1. Using a tree diagram you can check that there are four possible bootstrap samples. They are

$$(1,1),\ \ (3,1)\ \ (1,3)\ \ (3,3).$$

Then you calculate for each pair the value of the estimator which are 0,1,1,0. The MSE (mean squared error) is (remember that the variance is 1)

$$\frac{(0-1)^2 + (1-1)^2 + (1-1)^2 + (0-1)^2}{4} = .5.$$

5. Problem 21.16 Solution: The random variables (charges) X are simulated by inverse method, that is

$$u = 1 - e^{-X/1000}.$$

Solution is

$$X = -1000 \ln(1-u).$$

This yields the simulated values 356.7, 2525.7, 1204 and 83.3.

The corresponding reimbursements (100 deductible with 80% reimbursement and 1000 limit) are 205.3 (this is $.8(356.7 - 100)$), 1000, 883.2 and 0. The average of them is 522.