SOLUTIONS FOR HOMEWORK 8, STAT 4382

1. Read text.

2. Read text.

3.ab Exerc. 3.8.1. Here $X_0 = 1$, $X_{n+1} = \sum_{i=1}^{X_n} \xi_i$, where ξ_i are iid RVs that are "almost" Bernoulli, that is

$$P(\xi = 0) = .5, \quad P(\xi = 2) = .5.$$

Why almost? Because $\xi/2$ is Bernoulli with the probability of S equal to .5.

Now we can make some preliminary calculations. Write,

$$\mu = E(\xi) = 0(.5) + 2(.5) = 1,$$

$$\sigma^2 = Var(\xi) = E(\xi^2) - (E\{\xi\})^2 = [0(.5) + 2^2(.5)] - 1^2 = 2 - 1 = 1.$$

Using these results we get:

a)

$$E(X_n) = \mu^n = 1$$

b)

$$Var(X_n) = \sigma^2 \mu^{n-1} n = n.$$

5. Problem 3.8.2. This is a nice problem. We have

$$Z = \sum_{n=0}^{\infty} X_n$$

and

$$X_n = \sum_{i=1}^{X_{n-1}} \xi_i$$

with $\mu = E(\xi) < 1$ and $X_0 = 1$.

Note that

$$E(Z) = E(\sum_{n=0}^{\infty} X_n) = \sum_{n=0}^{\infty} \mu^n$$
$$= \frac{1}{1-\mu}.$$

In the last line I used the geometric sum's formula.

6. Exerc. 1.2. For the considered transition matrix we have the system

$$\pi_0 = .6\pi_0 + .3\pi_1 + .4\pi_2$$

$$\pi_1 = .3\pi_0 + .3\pi_1 + .1\pi_2$$

$$\pi_2 = .1\pi_0 + .4\pi_1 + .5\pi_2$$

$$\pi_0 + \pi_1 + \pi_2 = 1.$$

Choose two of the three equations, combine with the last, solve them and the answer is in the end of the text.

7. Exerc. 1.3. Here the (sufficient) system of 3 equations is (but you can choose any two of the first three)

$$\pi_0 = .1\pi_0 + .2\pi_1 + .3\pi_2$$

$$\pi_1 = .2\pi_0 + .2\pi_1 + .6\pi_2$$

$$\pi_0 + \pi_1 + \pi_2 = 0.$$

You solve the system and get $\pi_1 = 3/13$ (I just took this from the text because, as a professor, I am not supposed to solve these systems but you should...) In any case, then the average time that the system spends in state 1 is 3/13.

8. Problem 1.9. The limiting distribution satisfies equations

$$\pi_0 = (1/4)\pi_2$$

$$\pi_1 = (1/4)\pi_2$$

$$\pi_2 = \pi_0 + (1/4)\pi_2 + (1/2)\pi_3$$

$$\pi_3 = \pi_1 + (1/4)\pi_2 + (1/2)\pi_3$$

$$\pi_0 + \pi_1 + \pi_2 + \pi_3 = 1.$$

Solve them and get the answer: $\pi_0 = \pi_1 = 1/10$, $\pi_2 = \pi_3 = 2/5$.