Capacity Maximizing Algorithms for Joint Transmit-Receive Antenna Selection

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***Abstract*— In this paper, we study and compare two algorithms for transmit/receive antenna selection in MIMO channels. Generally the only way to assure optimality in antenna selection is exhaustive search, however, this constitutes an unacceptable burden on the receiver. We describe two suboptimal algorithms in this paper. First, a separable Tx/Rx successive selection, which has excellent performance but has complexity that grows as a quadratic function of the number of antennas. Second, an algorithm that entertwines transmit and receive side selection, which has linear complexity in the number of antennas, and only a small performance penalty compared with the first algorithm. we validate the results using simulations.[1]**

I. Introduction

In a rich scattering environment, multiple antennas at transmit and receive sides can significantly improve the spectral efficiency. It has been known that the capacity of the system scales linearly with the minimum number of transmit and receive antennas [1], [2], and this has spurred a great flurry of research in recent years.

However, this extended capacity is obtained using complex signal processing techniques at both ends. Furthermore, multiple antennas require multiple RF chains which are quite costly. Therefore cost and complexity are two major factors that may limit the use of multiple antennas in future communication systems.

On the other hand, in each realization of the channel, some of the antennas (either at the transmit side or the receive side) may be in deep fade. With increasing number of antennas, the probability that at least some of them are experiencing deep fading increases. Thus, a natural and practical solution offers itself: to select a subset of the available antennas so that (a) the effective size of the channel gain matrix is reduced, thus the processing requirements are simplified, and (b) the number of RF chains, which often is the main driver of the unit cost, are also reduced [3], [4].

Finding the optimal subset of antennas requires an exhaustive search, which is usually computationally unaffordable. Finding sub-optimal algorithms for antenna selection has been the subject of several efforts, a summary of which can be found in [3], [4]. Previous work on antenna selection cover either the receive selection [5], [6] or the transmit selection[7], [8]. But to the best of our knowledge no algorithm has been previously proposed for joint transmit/receive selection.

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In this paper, we consider the problem of joint transmit and receive selection in MIMO channels. We propose and study two methods. We first study a natural extension of greedy selection algorithms that have been studied in [5], [6] for receive side and in [7], [8] for transmit side. We first select the receive antennas via a greedy algorithm, and then transmit antennas selection is performed on the sub-channel resulting from receive antenna selection. The cost function of this algorithm is capacity, therefore it is assumed that signaling and outer coding will be designed to take advantage of the selected antennas.[2] This algorithm has quadratic complexity in the number of antennas.

Motivated to reduce the computational complexity further, we investigate a different approach. First, receive selection is done via a simple norm-based ranking, and then the usual incremental successive selection is performed on the reduced channel gain matrix to select the transmit antennas. Since the successive selection is used only once, this algorithm has only linear computational complexity in the number of antennas, with a performance that is very close to the previous algorithm.

The organization of the paper is as follows: Section II describes the system model and problem setup. In Section III we propose two algorithms for joint transmit/receive antenna selection. In Section V we compare the performance of the two algorithms via Monte-Carlo simulation and finally, Section VI concludes the paper.

In this paper we use the following notations: $\mathbb{E}[\ ]$ is the expected value operator, $A^\dagger$ represents the complex conjugate transpose of the matrix A, $A(i,:)$ refers to the i^{th} row of the matrix A, $\mathcal{A} \setminus \mathcal{B} = \{x : x \in \mathcal{A} \ \&\ x \notin \mathcal{B}\}$

II. System Model

We assume a narrow-band frequency flat linear time invariant model for the wireless channel between N_t transmit and N_r receive antennas. The signal model is:

$$y(t) = Hx(t) + n(t) \ , \tag{1}$$

where $y(t)$ represents the $N_r \times 1$ received vector sampled at time t, and $x(t)$ represents the $N_t \times 1$ vector transmitted by

[2]Capacity achieving coding and signaling may be non-trivial, since the selected channel is no longer Gaussian.

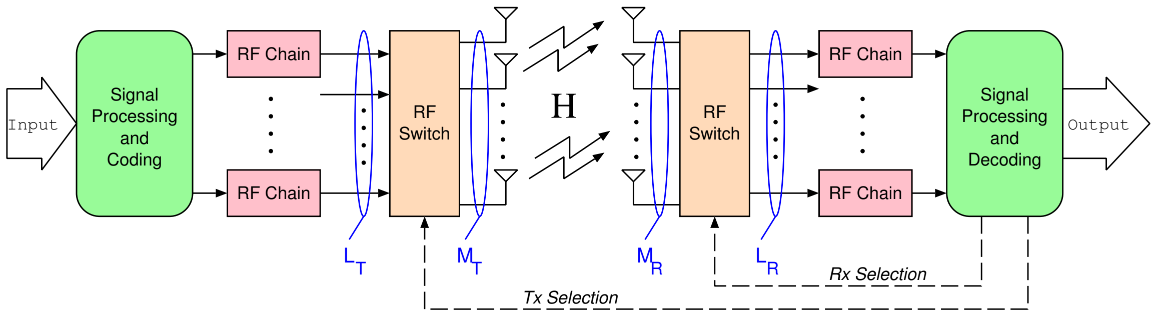

Fig. 1. Joint transmit/receive antenna selection

the antennas with power constrain $\mathbb{E}[x^H x] \leq \rho$, where ρ is the average SNR (per channel use), $n(t)$ is the $N_r \times 1$ additive circularly symmetric complex Gaussian noise vector with zero mean and covariance matrix equal to I_{N_r} (the $N_r \times N_r$ identity matrix) and H is the $N_r \times N_t$ channel matrix, whose ij-th element is the scalar channel between the i-th receive and j-th transmit antenna. We assume that the elements of H are independent and have complex Gaussian distribution with zero mean and unit variance. We also assume that H is perfectly known at the receiver but it is not necessarily known at the transmitter. For transmit antenna selection, we assume there is a rate-limited feedback channel from receiver to transmitter so that a subset of transmit antennas can be selected by the receiver and furthermore we assume that the feedback channel is without error or delay.

III. Antenna Selection Algorithms

The complexity of MIMO transceiver algorithms increases exponentially with the number of transmit and/or receive antennas. This requires powerful signal processors at both sides which is costly especially at the mobile unit. Also in order to use all antennas at both sides, multiple RF chains are required and this requirement immensely increases the cost of the mobile unit. It is a well-known fact [3] that antenna selection provides a solution to the cost and complexity problem in MIMO systems. However, finding optimal set of transmit and receive antennas requires an exhaustive search over all the possibilities, which has very high computational complexity. Successive selection algorithms [7],[8],[6],[5], [9] provide a solution to this problem, in particular the incremental successive selection algorithm (ISSA) [8],[6] seems very attractive when the number of selected antennas are much less than the total number of antennas.

A. Incremental Successive Selection Algorithm (ISSA)

The input of the algorithm consists of the channel realization H, overall SNR ρ, number of selected antennas L . The algorithm determines the selection, which are indices of the channel matrix. This is equivalent to knowing the selected channel submatrix $\widetilde{H}$.

Input: ρ, L, H

1) Let $\mathcal{C}_1 = \{\text{all columns of } H\}$ and $\widetilde{P}_1 = I_N$.
2) choose $\widetilde{h}_1 = \underset{h \in \mathcal{C}_1}{\arg\max}\ ||h||_2$, $\widetilde{H}_1 = \widetilde{h}_1$.
3) for $i = 2 : L$
   a) $\mathcal{C}_i = \mathcal{C}_{i-1} \setminus \{\widetilde{h}_{i-1}\}$
   b) $\widetilde{P}_i = I - \widetilde{H}_{i-1}(\frac{1}{\rho} I + \widetilde{H}_{i-1}^{\dagger}\widetilde{H}_{i-1})^{-1}\widetilde{H}_{i-1}^{\dagger}$
   c) $\widetilde{h}_i = \underset{h \in \mathcal{C}_i}{\arg\max}\ h^{\dagger}\widetilde{P}_i h$
   d) $\widetilde{H}_i = [\widetilde{H}_{i-1}\ \widetilde{h}_i]$
4) $\widetilde{H} = \widetilde{H}_L$

Output: $\widetilde{H}$

Note that the above algorithm selects the columns of H, in such way that each step guarantees *the minimal loss in the capacity due to selection*. The complexity of this algorithm is $O(\max\{N_t, N_r\} N_t L)$ [6]. The same algorithm can be used for row selection (receive antenna selection) but instead of H, $H^{\dagger}$ is input to the algorithm.

IV. Joint Transmit-Receive Selection

Optimal joint transmit-receive selection involves an exhaustive search among $\binom{N_t}{L_t}\binom{N_r}{L_r}$ possible subsets where L_t and L_r are the number selected antennas at transmit and receive side, respectively. Hence the complexity of optimal joint selection is $O({N_t}^{L_t}{N_r}^{L_r})$ which is too much for practical applications. Therefore we would like to devise low-complexity sub-optimal subset selection algorithms.

In the following we compare two sub-optimal antenna selection algorithms for jointly selecting the transmit and receive antennas. We assume that $L_r = L_t = L$, the rationale behind this assumption is for a given L_r, to have the maximum spatial multiplexing gain we should have $L_t \geq L_r$. Let h_1 be an arbitrary column of $\widetilde{H}$ then $\widetilde{H} = [\widetilde{H}_1\ h_1]$. For high SNR we can approximate the capacity as follows [7],[8]:

$$\begin{aligned} C(\widetilde{H}) &\approx L_r \log_2(\rho/L_t) + \log_2(\det(\widetilde{H}\widetilde{H}^\dagger))] \\ &= L_r \log_2(\rho/L_t) + \log_2(\det(\widetilde{H}_1\widetilde{H}_1^\dagger + h_1 h_1^\dagger))] \\ &= L_r \log_2 \rho - L_r \log_2 L_t \\ &\quad + \log_2(\det(\widetilde{H}_1\widetilde{H}_1^\dagger)) \\ &\quad + \log_2(1 + h_1^\dagger(\widetilde{H}_1\widetilde{H}_1^\dagger)^{-1} h_1) \end{aligned} \tag{2}$$

On the other hand

$$C(\widetilde{H}_1) \approx L_r \log_2(\frac{\rho}{L_t - 1}) + \mathbb{E}[\log_2(\det(\widetilde{H}_1\widetilde{H}_1^\dagger))]$$

thus we have

$$\begin{aligned} \Delta C &\approx C(\widetilde{H}) - C(\widetilde{H}_1) \\ &\approx -L_r \log_2(\frac{L_t}{L_t - 1}) \\ &+ \log_2(1 + h_1^\dagger(\widetilde{H}_1\widetilde{H}_1^\dagger)^{-1} h_1) \end{aligned} \tag{3}$$

the negative term in Eq. (3) corresponds to the increase in power per antenna due to antenna selection, and the positive term corresponds to the fact that there are more paths for signal transmission when we use L_t antennas instead of $L_t - 1$. But when there are enough degrees of freedom (large L_t), then h_1 can be chosen such that the logarithmic gain is dominated by the negative term in Eq. (3) and this means that there is something to gain by judiciously discarding one of the columns of $\widetilde{H}$, by and by an inductive argument we deduce that $L_r = L_t$ is a near optimal assumption. Moreover, this assumption is convenient for many conventional space-time signaling scheme such as orthogonal space-time codes, linear receivers and VBLAST.

A. Separable Transmit/Receive Successive Selection

An intuitive way for joint transmit/receive selection is through separating the transmit and receive selection problems. In this approach we can use the ISSA algorithm introduced in Section III-A for first selecting the best L_t transmit antennas and then selecting the best L_r receive antennas. Recall that ISSA(ρ, N, M, H) is the algorithm defined in the previous section which takes SNR per antenna, number of transmit and receive antennas, and t he channel realization, and returns the selected channel submatrix. We use this algorithm in a two-step process as follows.

Input: ρ, L_r, L_t, H

1) $H_1 = \text{ISSA}(\rho/L_t, L_t, H)$
2) $H_2 = \text{ISSA}(\rho/L_t, L_r, {H_1}^\dagger)$

Output: $\widetilde{H} = {H_2}^\dagger$

Note that the order of transmit and receive selection can be interchanged, but when $L_r = L_t = L$ and $N_t \approx N_r \gg L$. this has little impact on the average performance of the system. The complexity of this algorithm is $O(N^2 L)$, where $N = \max\{N_t, N_r\}$.

B. Successive Joint Transmit/Receive Selection

The previous algorithm has a computational complexity that grows as a quadratic function of the number of antennas. In this section we explore options for reducing this computational complexity. In particular we propose the following sub-optimal algorithm whose complexity is linear in the maximum number of transmit and receive antennas.

In this algorithm, we first select the maximum modulus element in the channel gain realization, H, call it element h_{i_1,j_1}. Then order all elements in column j_1 according to their modulus, and pick the top L elements in that column. The row indices of these elements are denoted $i_1, i_2, \ldots, i_L$. We form a submatrix using these rows, effectively selecting L receive antennas. In this submatrix, run the ISSA algorithm to pick the selected transmit antennas.

The flow of the algorithm is as follows.

Input: ρ, L, H

1) Find $(i_1, j_1) = \arg\max_{(i,j)} |h_{ij}|$
2) Find $(i_2, i_3, ..., i_{N_r})$ such that: $|h_{i_2,j_1}| \geq |h_{i_3,j_1}| \geq \cdots \geq |h_{i_{N_r},j_1}|$.
3) Form the $L \times N_t$ matrix, $G = \begin{bmatrix} H(i_1,:) \\ H(i_2,:) \\ H(i_L,:) \end{bmatrix}$
4) $\widetilde{H} = \text{ISSA}(\rho/L, L, G)$

Output: $\widetilde{H}$

The key idea of the new algorithm and its key difference with the previous one can be explained as follows: The previous algorithm attempted to make best (greedy) choices both in the row as well as columns of H, thus the previous algorithm was condemned to a quadratic complexity. The new algorithm tries to make easier (yet attractive) choices in one dimension in a way that reduces computational complexity order, and yet does not have a significant impact on performance. The computational complexity of this algorithm is $O(NL^2)$ which is only linear in $N = \max\{N_r, N_t\}$.

Note that in the second step, one could sort along the i_1 row, and form G by removing unwanted columns. But these two approach lead to the same average performance.

V. Simulation Results

For comparing the performance of the proposed algorithms, we assume a system with $N_t = N_r = 8$ and $L_r = L_t = 2$. This can be the case when there are many antennas at both side but the transmitter is restricted to use a low complexity scheme such as Alamouti code.

To compare the two algorithms, we performed a Monte-Carlo simulation using 2000 channel realizations. Fig. 2 depicts the ergodic capacity of selected channel for optimal selection and the two sub-optimal algorithm. As it can be seen, Algorithm I, performs slightly better than Algorithm II, but its computation complexity is quadratic whereas that of

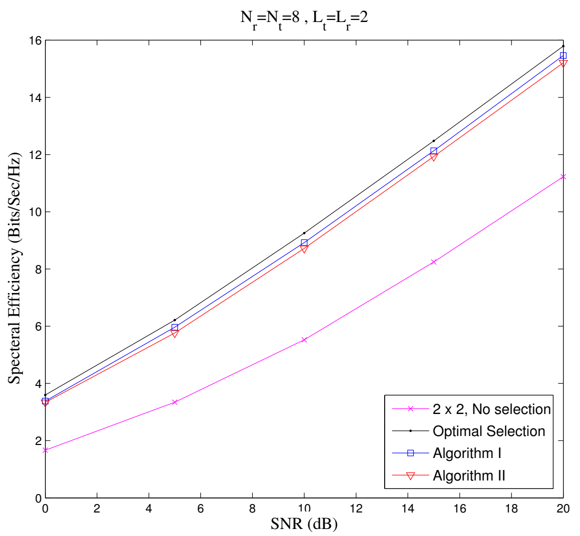

Fig. 2. Ergodic capacity

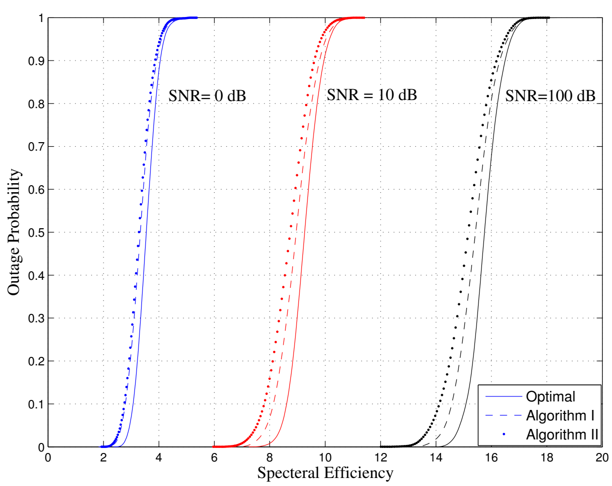

Fig. 3. Outage Probability

Algorithm I is only linear in N. It is also observed that the selected 2×2 system obtained from any of these algorithms has a much higher capacity than a 2×2 Rayleigh fading channel with no selection.

Fig. 3 shows the outage probability of the optimal selection and the two sub-optimal algorithms. In low SNR both Algorithm I and II perform almost the same and they are only slightly worse than the optimal selection in terms of outage probability. But as the SNR increases the difference between optimal and sub-optimal selection becomes more noticeable, although the two sub-optimal algorithm still perform very close to each other.

VI. Conclusion and Future Work

In this work, we examined the problem of joint transmit/receive antenna selection in MIMO systems. We compare two sub-optimum algorithms that have performances very close to the optimal selection. The first algorithm has a computational complexity that increases quadratically with the number of antennas, but the second one has a complexity that grows only linearly with the number of antennas. This is at the cost a small amount of capacity loss.

Future work may address channel estimation. With a limited number of RF chains at transmit and receive sides, accurate estimation of the full channel matrix may take time and thus waste data bandwidth. Since eventually the system does not use all the channel state information (a full use of that information would be equivalent to water-filling), it may not be necessary to estimate all channel gain coefficients with high precision. The trade off between channel estimation overhead and data transmission in a system equipped with antenna selection is an interesting topic for future research.

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