Opportunistic Beamforming with Limited Feedback

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***Abstract*— Transmit-side channel state information (CSI) can improve the quality of communication over fading channels. One way this information can be exploited is via (opportunistic) multi-user diversity. Also, in a multi-antenna scenario, CSI allows the transmitter to propagate along the eigendirections of the channel, thus achieving beamforming gain or array gain. In the face of these possibilities, the question we raise is this: if we can have only limited (reliable) feedback for channel state information, what form should this information take, and how should it be used. In a previous work, we have shown that most of the gain of a single-antenna multi-user diversity system can be captured by feedback of only one bit. In this paper, we extend that system to a multi-antenna transmitter. We propose and analyze a compound strategy that uses one bit for multi-user diversity and any further information for beamforming. We show that this method not only enjoys good throughput, but also has desirable fairness properties. Simulations confirm our assertions.**

I. Introduction

Perhaps one of the more ingenious ways of using CSI is opportunistic beamforming [1] of Viswanath et al. This method applies to a multi-user system, where the users estimate and feedback their instantaneous downlink SNR to the base station. Then the base station will transmit to the "best" user. Because the channels between users and the base station is variable, loosely speaking, each user receives data when their channel "peaks". In order for users to have reasonable delay, they must not be made to wait too long. To improve the delay profile [1] proposes an i.i.d. random phase on one of the two transmit antennas, thus creating an artificial fast fading.

There is also another way of using channel state information in a multi-antenna transmitter, namely (deterministic) beamforming. For example, one may use a conventional round robin scheduling between users, and when each user's turn comes, transmit along the eigen-direction of the channel to that user (either with adaptive power control or with adaptive rate control). It has been suggested [2] (Exercises) that in certain scenarios, in terms of overall network throughput, deterministic beamforming is inferior to opportunistic (random) beamforming.

Each of these methods, in the pure form, require unlimited reliable feedback. Sanayei and Nosratinia [3] have addressed the question of opportunistic scheduling with limited feedback, showing that only one bit of feedback per user is sufficient to capture most of the gain of multi-user diversity. Also, there exists a good amount of work on quantized beamforming (a nice summary of work in quantized beamforming is presented in [4]).

In this paper we are interested not so much in the superiority of one method vs. the other, but rather in how they can coexist and augment each other *in a practical scenario of limited feedback.* In other words, we ask the following question: in the presence of limited feedback, what combination of the two methods (opportunistic multi-user vs. deterministic beamforming) should we use, how can this combination be accomplished, and how well does it perform. At this point we should also address the notion of channel hardening with increasing number of transmit antennas. First, the channel hardens only in the asymptote of large number of antennas. Second, it has recently been shown [5] that channel hardening due to multiple antennas is not always detrimental to multi-user diversity, which is also confirmed by our research.

We use the following notation: $\mathbb{E}[\ ]$ refers to expected value of a random variable, $\gamma \approx 0.577$ is the Euler-Mascheroni constant and $e \approx 2.718281$ is the base of natural logarithm. We use $a_n \overset{\circ}{=} b_n$ to denote the asymptotic equivalence of a_n and b_n defined as: $\lim_{n\to\infty} \frac{a_n}{b_n} = 1$.

II. System Model

We consider a network of K users each having one antenna for receiving data from the base-station. The base station has M antennas. For k^{th} user we assume the linear time invariant flat fading model:

$$y_k(t) = h_k^T \cdot x_k(t) + n_k(t)$$

where $y_k(t) \in \mathbb{C}$ is the received signal and $x_k(t) \in \mathbb{C}^{M\times 1}$ is the transmitted signal for user k at time t and the transmit power is limited by ρ, i.e. $\mathbb{E}[\|x\|^2] \leq \rho$, $n_k(t)$ is a i.i.d. circularly symmetric complex Gaussian noise distributed according to $n_k(t) \sim \mathcal{CN}(0,1)$ and h_k is a column vector whose i^{th} element, $h_{i,k}$ represents the channel gain between i^th transmit antenna at the base-station and the receive antenna of the k^{th} user.

We assume the antennas at the base-station are not correlated, therefore $h_k \sim \mathcal{CN}(0, I_M)$, also we assume channel vectors across different users are independent. We assume that for k^{th} user the channel vector h_k is perfectly known at the receiver but not necessarily known at the base-station and for each users there exits a feedback channel with limited rate that can be used to securely convey the channel state information to the base-station.

III. Opportunistic Scheduling with Limited Feedback of CSI

For the case where the base station has one antenna ($M = 1$), Sanayei and Nosratinia [3] proposed an algorithm for downlink scheduling that only requires one bit of feedback from each user to the base-station. The scheduling algorithm is as follows:

The base-station sets a threshold α for all users. Each user compares the absolute value of their channel gain to this threshold. Whenever the channel gain exceeds the threshold, a "1" will be transmitted to the base station; otherwise a "0" will be transmitted. The base station receives feedback from all users and then randomly picks a user whose feedback bit was set to one for data transmission. If all the feedback bits received by the base-station are zero, then no signal is transmitted in that interval.[1]

In [3] it is proved that subject to judicious choice of the threshold, the above scheduling algorithm leads to the same sum-rate capacity growth as that of a scheduling with full knowledge of CSI at the base-station.

It was also shown that one of the advantages of scheduling using 1-bit feedback is improvement in fairness. In a slow-fading environment, scheduling with full CSI (always choosing the user with the maximum channel gain) leads to excessive delays. The 1-bit scheduling scheme has a much better delay and fairness profile simply because in each time interval the base-station can choose from a pool of *eligible users* (those above the threshold) thus the base-station can prioritize the eligible users based on their previous utilization and on average this leads to better fairness.

Fig. 1 compares the sum-rate capacity of the 1-bit scheduling and full CSI scheduling schemes for different values of SNR. The closeness of the curves in Fig. 1 suggests that when the available rate in the feedback channel is more than one bit, using the extra feedback bits for quantizing the channel gains does not lead to a significant capacity improvement. *Therefore it is reasonable to use any extra feedback, over and above one bit, for other purposes*. Aside from multiuser diversity, there are other ways of using transmit-side channel state information, perhaps the most obvious being beamforming (array gain). We propose that any excess channel state information, over and above one bit, can be used to exploit beamforming gain.

There exists prior work on beamforming with limited feedback of channel knowledge. Reference [4] provides a comprehensive survey of feedback methods. When there are multiple antennas at the base-station, we suggest to combine our 1-bit opportunistic scheduling [3] and beamforming with limited feedback [6][7]. We suggest that a combination of our 1-bit method and the limited feedback beamforming enjoys both the multiuser diversity and the transmit diversity and it has better performance over opportunistic (random) beamforming, assuming the same rate feedback is available for both methods.

[1]In this case the base station can also randomly pick a user for data transmission, although for large number of user this has vanishing advantage over no transmission when all the received feedback bits are "0".

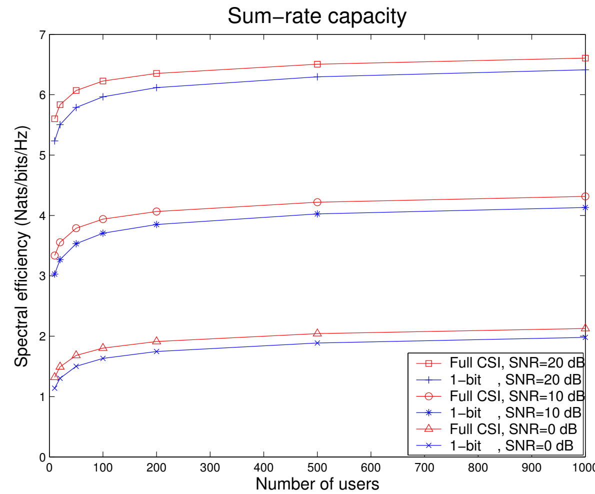

Fig. 1. Comparison of sum-rate capacity for limited and full CSI feedback scheduling for different values of SNR ($M = 1$)

When there are multiple antennas at the base-station and the rate of the feedback channel is limited to be L bits per channel realization, we can use the limited feedback methods for exploiting the beamforming gain [4]. A beamforming codebook $\mathcal{U} = \{u_1, \cdots, u_{2^L}\}$ ($u_1, \cdots, u_{2^L}$ are the beamforming vectors) is shared by all users and the bate-station. The k^{th} user finds the beamformer $\widehat{u}_k$ that leads to the highest gain, i.e.

$$\widehat{u}_k = \arg\max_{u \in \mathcal{U}} |h_k^T u|^2$$

then it compares the corresponding channel gain $\eta_k = |h_k^T \widehat{u}_k|^2$ to the threshold value α advertised by the base-station. If the channel gain was above the threshold, The user sends its L-bit feedback information to the base-station, otherwise it does not transmit any feedback information. Thus the reception of L bits from user k by the base-station indicates that

1) The user k is *eligible* for transmission
2) The base-station should use the beamforming vector $\widehat{u}_k \in \mathcal{U}$ for transmission to user k.

For scheduling, the base-station randomly selection one of the eligible users and when there is no eligible user in the network, it does not transmit to any user [2]

IV. Opportunistic Transmit Antenna Selection

When the beamforming vector is of size M (hence $L = \log_2 M$) then the best choice the code-book is to take u_l's as columns of identity matrix M.

$$\mathcal{U} = \left\{ \begin{pmatrix} 1 \\ \vdots \\ 0 \end{pmatrix}, \cdots, \begin{pmatrix} 0 \\ \vdots \\ 1 \end{pmatrix} \right\}$$

[2]The scheduling to users with favorable channels may also be implemented via round robin. In long run, both these strategies have the same average throughput per user. However, the round-robin version may be more appealing from a fairness point of view.

In this case, the beamforming is equivalent to antenna selection in the base-station (since only one antenna at a time is active). So what user k does to find the element of her channel vector h_k whose absolute value is maximum among all, i.e.

$$\widehat{i} = \arg \max_{1 \leq i \leq M} |h_{i,k}|^2$$

and then compare $|h_{\widehat{i},k}|^2$ with the threshold value α advertised by the base station. If $|h_{\widehat{i},k}|^2 > \alpha$, then the feedback bits $b_1 b_2 \cdots b_M$ (the binary digits of $\hat{i}$) are transmitted to the base station, otherwise, no feedback is sent. Upon receipt of this information from eligible users, the base station randomly chooses one user for transmission from among all users whose feedback information has been successfully received. If the feedback bits for selected user, are $b_1 b_2 \cdots b_M$, the base station uses its $\hat{i}^{\ th}$ antenna for transmission, where $\hat{i} = \overline{b_1 b_2 \cdots b_M}$.

A. Sum-Rate Capacity

In this section we evaluate the sum-rate capacity for the opportunistic antennas selection with *one bit* quantization. The equivalent channel gain for the users j is

$$\eta_j = \max_{1 \leq i \leq M} |h_{i,j}|^2$$

under Rayleigh fading assumption, each channel gain $|h_{i,k}|^2$ is exponentially distributed thus the CDF of η_j is $F(x) = (1 - e^{-x})^M$. let $p = \Pr[\eta_k > \alpha]$ since the channel gains are all mutually independent, the probability of having k feedback bits equal to one obeys a binomial law, i.e.

$$p_k = \binom{K}{k} p^k (1-p)^{K-k} \tag{1}$$

the ergodic capacity upon having k user above the threshold is:

$$\begin{aligned} \overline{C}_k &= \sum_{i=1}^{k} Pr[\text{the } i^{th} \text{ best user is selected}]\ C_i \\ &= \frac{1}{k} \sum_{i=1}^{k} C_i \end{aligned} \tag{2}$$

where $C_i = \int_0^\infty \log(1 + \rho x) dF_i(x)$ and $F_i(x)$ is the CDF of the i^{th} highest equivalent channel gain. In other words if $\{X_1, \ldots, X_K\}$ is a permutation of $\{\eta_1, \ldots, \eta_K\}$ such that $0 \leq X_K \leq \ldots \leq X_1$, then $F_i(x) = Pr[X_i < x]$. When the channel gains are iid, it can be shown that [8]:

$$F_i(x) = \sum_{l=0}^{i-1} \binom{K}{l} (F(x))^{K-l} (1 - F(x))^l \tag{3}$$

where $F(x) = (1 - e^{-x})^M$ is the CDF of $|\eta_j|^2$ for $j = 1, \ldots, K$. Thus the sum-rate capacity of the network with limited feedback can be formulated as:

$$C_{LF} = \sum_{k=1}^{K} p_k \overline{C}_k \tag{4}$$

Using developments similar to [3],

$$C_{LF} = \int_0^\infty \log(1 + \rho x) dF_\pi \tag{5}$$

where $F_\pi = \sum_{i=1}^{K} \pi_i F_i$ is a mixture probability measure of all order statistics of the family with parent CDF $F(.)$ and $\{\pi_i\}_{i=0}^{K}$ is a discrete probability measure defines as

$$\pi_i = \frac{1}{Kp} \sum_{k=i}^{K} p_k, \quad i = 1, \ldots, K \tag{6}$$

B. Optimal Threshold

The sum-rate capacity is a function of ρ, p and K. On the other hand, because the equivalent channel gains η_j are the extreme value of M i.i.d exponential ransom variables , the threshold α is uniquely determined by p from the following formula:

$$\alpha = -\log\left(1 - (1-p)^{\frac{1}{M}}\right) \tag{7}$$

In order to find the optimal threshold we choose p such that the sum-rate capacity C_{LF} is maximized. The cost function $C_{LF}(p)$ is a weighted sum of functions of the form $p^k(1-p)^{K-k}$ which are all concave over the interval $[0, 1]$, hence C_{LF} is a concave function of p. Therefore it has a unique maximum over the interval $[0, 1]$. To calculate the value of p that maximizes the sum-rate capacity, we must solve $\frac{\partial C_{LF}(p)}{\partial p} = 0$ for p. By differentiating Eq. (4) with respect to p we get:

$$\sum_{k=1}^{K} (k - Kp) p_k \overline{C}_k = 0 \tag{8}$$

A closed form solution to this equation is in general not tractable. However, a numerical solution is possible with $O(K)$ complexity.

C. Asymptotic Analysis

In this section we investigate the opportunistic scheduling proposed in Section IV in the asymptote of large number of users, in particular we show the proposed opportunistic scheme has the same capacity growth of *coherent opportunistic beamforming with full CSI* where the base-station has full channel knowledge of all users and the users with the highest channel norm is selected for transmission. The sum-rate capacity of this scheme provides an upper bound on the all opportunistic beamforming methods that user partial knowledge of the channel. For this scheduling scheme, the sum-rate capacity is obtain as follows:

$$C_{Full\_CSI} = \mathbb{E}[\log(1 + \rho \max_{1 \leq k \leq K} \|h_k\|^2)] \tag{9}$$

The following Lemma is useful for our asymptotic analysis:

Lemma 1: Let $\{X_i\}_{i=1}^{n}$ be a sequence of positive iid random variables with finite mean μ_n and finite variance σ_n^2, also $\mathbb{E}[\log^2(X_n)] < \infty$, if $\lim_{n\to\infty} \frac{\sigma_n}{\mu_n} = 0$, then:

$$\log(\mathbb{E}[X_n]) - \mathbb{E}[\log(X_n)] \longrightarrow 0$$

as $n \to \infty$.

Proof: See [3].

Lemma 1 states that if the probability measure associated with the random variable X_n is well concentrated around its mean value for large n, then Jensen's inequality for $\log()$ is asymptotically tight. Note that X_n can be either a *discrete* or a *continuous* random variable.

Theorem 1: The sum-rate capacity of coherent opportunistic beamforming with full CSI available at the base-station scales as

$$C_{Full\_CSI} \overset{\circ}{=} \log\log K + \log\rho$$

Proof: The random variable $Y_k = \|h_k\|^2$ is distributed according to χ^2_{2M} and let $Z_K = \max_{1\leq k\leq K} Y_k$. Using the classical results in extreme value theory, it is shown in [9], [10] that the mean and the variance of this random variable have the following asymptotic behavior

$$\mu_K = \mathbb{E}[Z_K] \overset{\circ}{=} \log K + \log\left(\frac{K^{M-1}}{(M-1)!}\right) + \gamma \quad (10)$$

$$\sigma_K^2 = \mathbb{E}\left[(Z_K - \mathbb{E}[Z_K])^2\right] \overset{\circ}{=} \frac{\pi^2}{6} \quad (11)$$

where $\gamma \approx 0.577$ is the Euler-Mascheroni constant. Therefore $\frac{\sigma_K}{\mu_K} \to 0$ as $K \to \infty$ thus Z_k satisfies the condition in Lemma 1 and we can invoke Lemma 1

$$\begin{aligned} C_{Full\_CSI} &= \mathbb{E}[\log(1+\rho Z_K)] \\ &\overset{\circ}{=} \log(1+\rho\mathbb{E}[Z_K]) \\ &\overset{\circ}{=} \log\left(1+\rho(\log K + \log\left(\frac{K^{M-1}}{(M-1)!}\right)+\gamma)\right) \\ &\overset{\circ}{=} \log\log K + \log\rho. \end{aligned} \quad (12)$$

The next theorem explains that the sum-rate capacity for our proposed scheme (Eq. 5) also has the same capacity growth as the scheduling with full CSI.

Theorem 2: The sum-rate capacity of Opportunistic transmit antenna selection scales the same as scheduling with full CSI (coherent beamforming), i.e.

$$\lim_{K\to\infty} \frac{C_{LF}}{C_{Full\_CSI}} = 1$$

Sketch of the proof: For probability measure F_π (defined after Eq. (5)) we prove $\frac{\sigma_\pi}{\mu_\pi} \to 0$, thus we can invoke Lemma 1 to show that C_{LF} scales the same as C_{Full_CSI}, the details will appear in [11].

V. Quantized Opportunistic (Random) Beamforming

Recall that the main premise of this paper is a balance/tradeoff between two main methods of using transmit CSI, in the limited feedback regime. One of them is deterministic beamforming, and the other is the opportunistic beamforming of Viswanath et al [1]. There exists some work on the quantization of feedback for deterministic beamforming, but to our knowledge, little work exists in the public-domain literature on the quantization effects in opportunistic beamforming. As a baseline for comparing our algorithms, in this section we develop a limited-feedback (quantized) version of the opportunistic beamforming.

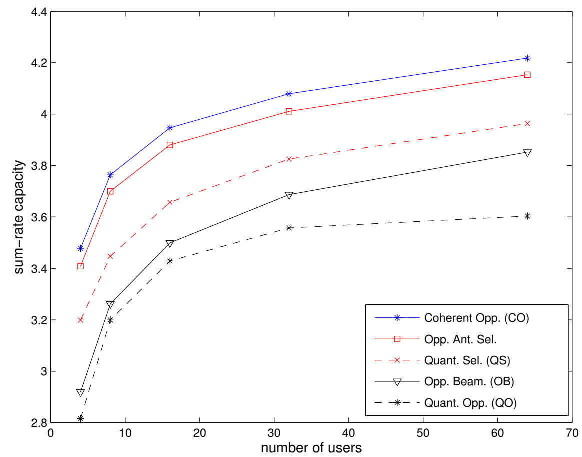

Fig. 2. Comparison of the throughput of various methods at SNR=10dB

In opportunistic beamforming, each user sends back to the base station its instantaneous receive SNR. For the quantized opportunistic beamforming, only a discrete index can be transmitted. The calculation of optimal quantization for this purpose is intractable due to the complicated error function involved, however, precise quantization is not critical because the boundaries of the quantization bins are important only to the point that they give the same ordering of the users as given by optimal quantization. Therefore a reasonable but approximate quantization performs well with high probability.

The approximate quantization that we developed for our simulations and comparison purposes is chosen so that the quantization index takes all possible values with equal probability. Denote the CDF of SNR as $F_\gamma(x)$. We consider L quantization bits, thus 2^L quantization levels. The quantization bins are defined by

$$F_\gamma\left(\frac{\ell}{2^L}\right) \leq \gamma < F_\gamma\left(\frac{\ell+1}{2^L}\right) \qquad \ell = 0,\ldots,2^L-1$$

VI. Comparisons

A. Throughput

Figure 2 shows a comparison of the throughput of various methods considered in this paper. The best performance is that of coherent opportunistic beamforming (CO), where the multiple-antenna transmitter has full knowledge of the (vector) channel of all users, and in each time interval beamforms towards the best user. Although this strategy has the best overall throughput, it suffers in two ways: first, it requires the highest amount of feedback compared with other methods, especially compared with the limited-feedback methods proposed in this paper. Second, in terms of fairness it is one of the worst techniques, as shown in the next subsection.

With a slight loss of throughput, one may use opportunistic antenna selection, where everything is like the previous case, except instead of full beamforming, the transmitter selects one of the antennas. In other words, with the knowledge of

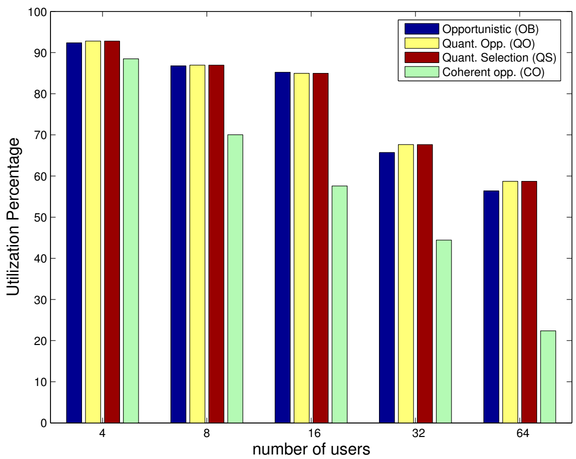

Fig. 3. Normalized fairness

user (vector) channels, the transmitter picks the best transmit antenna and user among all possible such pairs.

Opportunistic beamforming (OB) as suggested by Viswanath et al [1] has performance below that of opportunistic antenna selection.

Finally, each of the last two cases can be quantized, as explained elsewhere, each resulting in some loss of throughput. The quantized versions shown in Figure 2 are done with only one bit per user, that selects the best antenna for that user. We note that using this one bit as antenna selection is only due to the fact that we have used two transmit antennas. The more general case is as explained below.

We have also generated results for 4-bit per user feedback. In that case, 4 bit is used to select a beamformer out of a codebook of beamformers, in the manner suggested by the work of Love et al [6] on Grassmannian beamforming. These results have been omitted due to space constraints.

B. Fairness

v The simplest form of scheduling in a multi-user system is a deterministic round robin, which in fact has the best delay profile: every user is guaranteed a slot within a finite time. When there is flexibility on delay, for example in data communication, other modes of scheduling can be used that result in higher throughput, for example opportunistic beamforming. Even in the case of a winner-take-all strategy, however, one does not want to lose sight of the delay of users who do not "win" the competition for the channel, and this has been addressed for example in [1]. In this section, we define a metric for fairness and compare the methods of interest (in the limited feedback regime) according to this metric.

Assume that we apply opportunistic beamforming N times to a group of users. In traditional opportunistic beamforming, when the exact SNR of all users is known at the base station, only one user (with the highest SNR) is eligible for transmission. In the limited-feedback versions, more than one user may be eligible for reception, among which one may choose randomly or according to other algorithms (e.g., whoever has been waiting the longest, etc.)

For our metric, in each realization of the channel, we note the ineligible users. In a sense these are our *underprivileged* users. Since the decision of privileged/unprivileged is a binary decision, we can model it with a 0-1 Bernoulli random variable. After N transmissions, we look at the most underprivileged user (the one who was eligible the least number of times) and calculate the expected value of the number of times it has been eligible.[3] This is a measure of the utilization of the worst user. We then normalize the utilization factor above by the utilization of round robin scheduling, to remove the effects of N and the number of users. We show a comparison of the fairness of various methods in Figure 3.

The four methods compared here are opportunistic beamforming with unlimited feedback (OB), quantized opportunistic beamforming (QO), our limited feedback scheme (QS), and coherent opportunistic beamforming (CO) where the base station knows all the (vector) channel gains of all users, and at each time interval deterministically beamforms towards the best user.

As observed in Figure 3, in terms of fairness alone, OB and its quantized version QO perform quite well, as well as our method QS. Deterministic beamforming does not do as well, especially when the number of users is high. The small variations between OB and QO, we believe, are due to the finite window effect and possibly also to the sample size of the simulation.

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[3]This is an order statistic which is difficult to calculate in closed form, because the random variables corresponding to different users are dependent, but it can be calculated numerically.