Audit and Remediation Strategies in the Presence of Evasion Capabilities

Shouqiang Wang
Naveen Jindal School of Management, The University of Texas at Dallas, Richardson, Texas, Shouqiang.Wang@utdallas.edu

Francis de Véricourt
ESMT European School of Management and Technology, Schlossplatz 1, D-10178 Berlin, Germany, francis.devericourt@esmt.org

Peng Sun
Fuqua School of Business, Duke University, Durham, NC 27708, USA, psun@duke.edu

In this paper, we explore how to uncover an adverse issue that may occur in organizations with the capability to evade detection. To that end, we formalize the problem of designing efficient auditing and remedial strategies as a dynamic mechanism design model. In this set-up, a principal seeks to uncover and remedy an issue that occurs to an agent at a random point in time, and that harms the principal if not addressed promptly. Only the agent observes the issue's occurrence, but the principal may uncover it by auditing the agent at a cost. The agent, however, can exert effort to reduce the audit's effectiveness in discovering the issue. We first establish that this set-up reduces to the optimal stochastic control of a piecewise deterministic Markov process. The analysis of this process reveals that the principal should implement a dynamic cyclic auditing and remedial cost-sharing mechanism, which we characterize in closed form. Importantly, we find that the principal should randomly audit the agent unless the agent's evasion capacity is not very effective and the agent cannot afford to self-correct the issue. In this latter case, the principal should follow pre-determined audit schedules.

Key words: dynamic mechanism design, stochastic optimal control, asymmetric information, moral hazard, voluntary disclosure, environmental issue, corporate responsability

History: October 2, 2023

---

1. Introduction

Organizations sometimes discover an issue that, if not addressed promptly, harms other parties. Rather than fixing the problem, however, these organizations may prefer to conceal the issue and exert effort to evade detection. For instance, when technology executives at the auto manufacturer Volkswagen discovered that their newly designed engine would not comply with the US Clean Air Act, they chose to develop a sophisticated software that evaded emission tests rather than investing in the creation of more expensive but effective emission equipment (Ewing 2016). Similarly, the Quaker Pet Group, one of Wal-Mart's largest suppliers, devised workarounds when the firm realized that one of its own suppliers would not pass Walmart's workplace inspections. Instead of switching to a more expensive but complying supplier, the Quaker Pet Group falsified its order forms, wrongly claiming that it sourced from a Walmart-certified facility (Clifford and Greenhouse 2013). Efforts to evade detection instead of fixing problems are also present in the biotechnology industry (see, for instance, the case of Theranos, Carreyrou 2018).

Audits serve as a common tool employed by firms and regulators to identify and address adverse issues.[1] While audits can be costly, regulators and businesses often utilize incentive mechanisms to promote compliance. An example of this is the U.S. Environmental Protection Agency's (EPA) Audit policy (EPA 2000), which grants disclosure benefits, typically in the form of reduced penalties, to companies that voluntarily report hazardous incidents. Another notable case involves Levi's, an American blue-jean retailer. Hadler (2013) states that if Levi's uncovers a supplier's failure to meet their requirements and attempts to conceal it, Levi's terminates the contract. However, if a supplier proactively discloses information about a problem, Levi's collaborates with them to find a solution.

Designing an effective audit policy incorporating these incentive mechanisms, however, remains a significant challenge and an open research question. In this paper, we provide managerial insights on how an organization can uncover and remedy an adverse issue that may randomly occur in another entity that can evade detection. To that end, we formalize the problem of designing efficient auditing and remedial strategies as a dynamic principal-agent problem in continuous time, in which the principal seeks to discover an adverse issue. In this set-up, the principal uses audits and disclosure benefits to enforce compliance. Importantly, the principal optimizes over a large set of policies, which includes all implementable audit schedules one can reasonably think of.

This formulation reduces to the optimal stochastic control of a piecewise deterministic Markov process (PDP), a class of stochastic processes that generalizes semi-Markov decision processes. We study the control of this PDP analytically, which yields new insights concerning auditing in the presence of evasion capabilities.

Overall, our analysis suggests that the principal should incentivize the agent to always disclose the issue as soon as it occurs, without taking evasive actions or self-correct. For this purpose, the principal audits the agent *randomly* and *periodically*, but offers to cover part of the agent's remedial cost when the agent voluntarily reports the issue. In essence, the policy ultimately motivates the agent to come clean. Further, we find that the audits become deterministic when the agent's evasion capacity is not too effective (i.e. it is imperfect and sufficiently costly) *and* the agent cannot afford to self-correct the issue. In this case, the audit follows a *pre-determined* schedule. In this sense, the agent's evasion capability affects the very nature of the auditing policy.

[1] For example, U.S. EPA has developed systematic audit protocols under various legislations (`https://www.epa.gov/compliance/audit-protocols`).

To be more specific, we consider a generic situation, in which a principal (e.g., a firm or an employer) seeks to discover and avoid the negative consequences of an adverse event (e.g., a quality or non-compliance issue), the occurrence of which is private information to the agent (e.g., a supplier, or a business unit). A key feature of our set-up is that the agent can exert effort to evade the principal's audits (e.g., by falsifying forms, developing circumventing software, establishing hidden accounts, or even self-correct without disclosure). However, evasion may not be perfect, in that an audit may still reveal the issue with positive probability after the agent has exerted evasive efforts. The lower the cost of evasive effort and the resulting detection probability are, the more effective the agent's evasion capability becomes.

When the agent discovers the issue, the agent prefers to conceal the problem from the principal rather than to incur the associated remedial costs for which the agent is liable. This, however, obstructs the timely correction of the problem, which, in turn, harms the principal. To uncover whether such an adverse event has occurred, the principal can decide at any time (possibly randomly) to audit the firm and charge a non-disclosure penalty if the adverse event is detected. Because audits are expensive and the agent can evade them, the principal may also offer to cover part of the remedial costs if the agent voluntarily reports the issue. In our set-up, the agent may also decide to self-correct the issue, possibly at a later time, without notifying the principal, in an attempt to avoid potential penalty or to defer the remedial cost.

The principal's objective is to identify an audit schedule and her contribution to the remedial cost that minimizes her total discounted cost. This cost includes the principal's share of the remedial costs, the audit costs and possible damages resulting from failing to address the issue promptly.

This dynamic agency setting involves not only an adverse selection problem due to the agent's private information on the timing of the adverse event, but also a moral hazard problem due to the agent's ability to evade audits or self-correct the issue. Further, audit schedules can take very general forms, as inter-audit times can be history-dependant and stochastic, following general probability distributions. As such, possible audit times may follow any deterministic schedule, random audits at deterministic times, random time between audits that follow any (well-behaved) probability distributions, or any combinations of the above.

We first establish a version of the revelation principle tailored to our dynamic setting, which states that inducing the agent to reveal the adverse event as soon as it occurs is always optimal for the principal. This means that the principal can restrict the search for the optimal strategy to those that remedy the issue without delay. This result allows us to reformulate the problem as an optimal stochastic control problem.

Given this, we first examine situations where the evasion technology is able to render the principal's audits completely ineffective, i.e., the evasion capability is perfect. In this case, the problem reduces to the optimal stochastic control of a one dimensional PDP, which is known to be hard (see, e.g., Davis et al. 1987). Nonetheless, we show that the optimal policy is a cyclic cost sharing and *random* auditing policy. Under this policy, the principal shares part of the remedial costs with the agent, the exact amount of which depends on the timing of the agent's disclosure. The principal adjusts the split of the remedial cost over time, following a cyclic pattern. In the beginning of a cycle, the agent's contribution of the remedial cost increases over time. If the agent's contribution reaches a maximum level (equal to the evasion cost) before any disclosure, the principal runs an audit after a exponentially distributed random time. If the audit does not reveal any issue, the cycle ends and the agent's contribution is reset to its minimum value to start a new cycle. Overall, the optimal policy alternates between deterministically changing payments and random audits.

We then study situations where the evasion technology is imperfect, so that an audit still reveals the issue with a positive detection probability even after the agent has taken an evasive action. In this case, we show that the optimal policy remains cyclic, but is either *random* or *deterministic*, depending on whether or not the agent can afford to correct the issue alone. In particular, if the remedial cost is within the agent's limited liability, the optimal policy maintains the same random cyclic structure observed in the case with perfect audits. If the remedial cost is higher than the agent's limited liability, however, the principal sometimes follows a simple deterministic cyclic audit schedule. This happens when the evasion cost is sufficiently high (i.e. higher than a specific threshold, which is decreasing in the post-evasion detection probability of audits). In this case, the policy adheres to a similar cyclic pattern, except that the principal runs an audit as soon as the agent's share of the remedial cost reaches its maximum level (the agent's limited liability), rather than after a random period.

Finally we study the case in which the evasion technology is imperfect but the evasion costs are below the aforementioned threshold. This creates mixed incentives for the agents since the agent's evasive action is imperfect but also inexpensive. In this case, the problem becomes the control of a two-dimensional of PDP governed by two sets of incentive compatible and state contraints. Optimally control this problem is generally intractable not only analytically but also numerically (see, e.g., Chehrazi et al. 2019, and also Remark A.1 in Appendix A for more details). In addition, and perhaps more importantly, even if one is able to compute the optimal policy, it may be too complex to implement in practice (see again Remark A.1 in Appendix A). We thus restrict the search for efficient auditing policies within a large class of tractable policies, which we refer to as *proportional policies*.

Under a proportional policy, the agent's payment upon self-reporting an issue is proportional to the expected penalty of getting caught. In particular, the previous optimal random and cyclic policies belong to this class. Proportional policies, however, do not need to be cyclic or deterministic;

they may allow for random audits, a mixture of random and deterministic audits, random audits with different or time-varying rates, and different time interval lengths between audits.

We then show that the optimal proportional policy is a cyclic *random* policy akin to the perfect evasion case, except that the initial cycle is of a different length than the following ones. Furthermore, the parameters of this policy can be obtain by solving a deterministic bivariate constrained optimization problem.

These analytical results allow us to numerically explore the impact of the agent's evasion capacity on the principal's audit policy. Our study reveals a non-monotone relationship between the evasion detection probability and audit frequency. Specifically, we find that the principal should audit first more and then less frequently as the agent's evasion capability becomes less effective. The total expected audit costs exhibits a similar structure.

Finally, we show that the structure of our policy continues to hold when the principal can inflict different penalties depending on whether the agent does not disclose the issue with or without evasive actions (Section 9.1), a third party, such as non-for-profit organization, can independently uncover the agent's violation (Section 9.2), the principal maximizes social welfare (Section 9.3), and the agent may be uninformed about the event's occurrence (Section 9.4).

2. Literature Review

Stochastic modeling of audits/inspections dates back to the reliability theory literature (Barlow and Proschan 1996, Parmigiani 1993), which mostly focuses on a single decision-maker's inspection policy to discover system breakdowns. Extending this framework to a game-theoretical setting, Kim (2015) examines two types of inspection schedules (i.e., a deterministic periodic schedule and an exponential random schedule) in order to incentivize voluntary disclosure. Wang et al. (2016) adopt a mechanism design framework with costly state verification, and show that a deterministic inspection schedule is optimal when used together with subsidies that are decreasing over time between inspections. A key component that distinguishes our paper is that we explicitly account for the agent's opportunistic behavior of evading the principal's audits (i.e., moral hazard). This evasion capability yields fundamentally different results. In particular, we show that the presence of a moral hazard problem can render the previous deterministic schedule sub-optimal. Instead, it is sometimes optimal for the principal to alternate between a fixed period with no audit followed by a random period with audit.

More recently, Varas et al. (2020) study how a principal inspects an agent whose production quality follows a two state Markov chain. Following Board and Meyer-Ter-Vehn (2013), the agent's effort increases the transition rate from low-quality state to high-quality state, and reduces the transition rate from high-quality state to low-quality state. The principal and the market forms a belief about the quality state over time, which captures the firm's reputation. The firm's payoff is linear in its reputation, and the principal's payoff is convex in the firm's reputation. The principal controls when to conduct costly inspection in order to fully review the firm's quality state of that moment. Interestingly, their optimal inspection schedule shares a similar structure as ours – each cycle starts with a fixed period of time with no insepction, followed by an exponentially distributed random time before an inspection occurs and finishes the cycle. However, the model and analysis of Varas et al. (2020) is quite different from ours. In particular, with no payment, their model does not rely on the promised utility framework, which is the foundation of our model. Instead, the belief probability is the only state variable in their model.

More generally, there has been an emerging literature in management science that combines incentive management with detections. Bakshi and Gans (2010), for example, study incentive programs that induce firms to improve security - and hence reduce inspection costs - against potential terrorist attacks in cargo shipment. Babich and Tang (2012) compare deferred payment and inspection mechanisms to address the moral hazard issue of "corner cutting" behaviors by suppliers. Hwang et al. (2006) study similar problems but compare the inspection versus certification mechanisms. Cho et al. (2015) study inspection and penalization strategies to combat child labor, rather than to ensure product quality. The aforementioned models are generally static in nature, and therefore do not address the timing of adverse events as we do. They also do not consider the ability to render inspections ineffective. Levi et al. (2019) study farmers' strategic adulteration behavior in response to quality uncertainty, supply chain dispersion, traceability, and testing sensitivity. Chen et al. (2020) study insepction policies for supply networks with different centrality measures. More recently, Kim and Xu (2023) propose a class of policies that randomize between deterministic and exponential audits to mitigate financial risks, and optimize over the policy parameters for given policy structures. An interesting work by Baliga and Ely (2016) examines the use of torture as a means of extracting information from a possibly informed agent who knows the timing of a future attact. With the principal's full commitment, their problem becomes a standard, static mechanism design, while our principal faces a dynamic adverse selection problem, because our agent knows the timing of the event only when it has happened.

In this stream of research, the only paper that considers deliberate audit evasion is Plambeck and Taylor (2016). One of their key insights is that too high a violation penalty may backfire by creating an incentive for the agent to actively evade the audit, which was first revealed in the economics literature of auditing in the presence of avoidance (Malik 1990). In our set-up, Malik's (1990) logic explains why the principal never requires the agent to incur a cost higher than his effort cost of evading audits. This upper limit on the agent's contribution toward the remedial costs turns out to be the main driver for the optimality of our random audit schedule.

Overall, our work contributes to the longstanding research on the economics of law enforcement initiated by Becker (1968) (see, e.g., Polinsky and Shavell 2000, for a review). Central to this area of study is the inquiry into the most efficient approach to minimize societal costs while ensuring compliance. What sets our work apart is its focus on a *dynamic* principal-agent framework, which incorporates both *adverse selection* and *moral hazard*.

In a static principal-agent model, Townsend (1979) initiated the paradigm of costly state verification, which, however, is restricted to only deterministic audits. Later, Mookherjee and Png (1989) generalized the analysis to allow random audits and provide conditions for random audits to be optimal. We consider a dynamic setting, which is closer to Ravikumar and Zhang (2012), who examine a tax auditing problem albeit without audit evasion behaviors.

In an information environment with costly state verification *or* multiple periods, Townsend (1988) pointed out that the usual version of the revelation principle (e.g., Dasgupta et al. 1979, Myerson 1979) is no longer automatically applicable. He extended the revelation principle separately to these two environments. We contribute to this literature by establishing a version of the revelation principle applicable to a private information environment (adverse selection) with hidden action (moral hazard), and costly state verification.

From a more technical perspective, we leverage existing recursive representation techniques (Spear and Srivastava 1987, Abreu et al. 1990, Ljungqvist and Sargent 2004) to tackle dynamic principal-agent problems (e.g., Sannikov 2008, Biais et al. 2010, Li et al. 2013). This approach helps reduce our original principal-agent problem to a stochastic optimal control of a *piecewise deterministic process* (PDP). Optimal control of PDPs, however, are often analytically intractable (e.g., Davis et al. 1987). We attack this problem using the verification approach via *quasi-variational inequalities* (Bensoussan and Lions 1982) and obtain a closed-form characterization of the optimal policy.

3. Model

Consider a principal-agent relationship in continuous time. The principal seeks to discover and avoid negative consequences of an adverse issue that occurs at and is privately known to the agent. In the context of environmental regulation, we can conceptualize the Environmental Protection Agency (EPA) as the principal, while a firm like Volkswagen assumes the role of the agent. Similarly, when considering supplier compliance matters in the private sector, we can envision influential retailers such as Walmart or Levi's as the principal, with suppliers like the Quaker Pet Group acting as the agent. The adverse issue emerges and comes to the agent's awareness at a random time T, which follows an exponential distribution with rate $\lambda > 0$. If not corrected with appropriate countermeasures, the consequences of this adverse event persist after time T and inflict a cost c

per unit of time on the principal. This cost captures the event's detrimental effects imposed upon society in the case of environmental violations, or the financial damages and potential harm to the retailer's reputation in the context of supplier non-compliance.[2]

A remedial action can bring an end to these damages at cost r, which covers the expenses involved in restoring the environmental impact, compensating victims, as well as adopting compliant equipment and repairing the retailer's reputation. This cost is less than the maximum (discounted) negative impact of the event, i.e., $r < c/\theta$, where $\theta > 0$ is the discount rate. (Otherwise, no party has any incentive to take the remedial action, and the problem becomes trivial.) Thus, the fact that the agent may need to bear (part of) the remedial cost discourages him from disclosing and fixing the issue, giving rise to a problem of *adverse selection*.

To determine whether the event has occurred, the principal can (possibly randomly) audit the firm at any time and charge a non-disclosure penalty if the issue is uncovered. The agent, however, can exert an effort to evade these audits through deception/falsification without addressing the issue. In other words, the principal also faces a problem of *moral hazard*, in addition to the adverse selection issue. Because of these incentive misalignments, and because audits are costly, the principal may alternatively provide the agent with incentives to voluntarily disclose the event.

Audit and evasion. Specifically, the principal can conduct an audit with a cost k at any time. Audit schedules can be very general and combine both "impulsive" and/or "intensive" audits. An impulsive audit takes place at time epoch t with probability $q_t^m \in [0,1]$, where we require only finitely many impulsive audit time epochs with $q_t^m > 0$ within any finite time interval. By contrast, an intensive audit occurs in time interval $[t, t+\Delta t)$ with probability $q_t^n \Delta t + o(\Delta t)$, where the audit rate $q_t^n \geq 0$. We denote the principal's *audit schedule* by $Q := (Q_t)_{t\in[0,\infty)}$, where $Q_t := (q_t^m, q_t^n)$. (A rigorous definition is provided in Appendix A.) This framework captures any type of reasonable auditing schedules. For example, the principal can decide to follow a deterministic auditing schedule, randomly audit at pre-specified times, randomly audit according to an arrival rate, or any combinations of the above. This allows the principal to consider all possible scheduling policies one can reasonably imagine. Despite this very rich set of policies, we demonstrate later in the paper that the optimal scheduling policy is easy to understand and implement.

A distinctive feature of our setting is that, the agent can exert an *evasive effort*, which is unobservable to the principal, at cost $h > 0$ to render the principal's audits less effective (e.g., Lacker and Weinberg 1989). The evasion action and its cost h corresponds to either a one-time occurrence

[2] The cost inflicted on the principal by the persistence of the issue can be indiscernible, making the principal unable to infer the emergence of the issue from such a cost. This situation is pervasive. For example, such costs may be confounded with other factors like demand or price fluctuations. These costs may also represent any risk that may materialize to the principal *in the future*, such as the reputation damage associated with a third party publicly revealing the issue.

or a continuous level of effort. For instance, the bulk of Volkswagen's evasive efforts consisted in developing a software to avoid detection, which, once developed, was install on each car at negligible cost. When the deceptive mechanism is not automatized and requires continuous effort, evasion cost h represents the agent's cumulative total expected discounted cost of effort.[3]

Evasive actions may not be perfect, however. We denote by $\beta \in [0,1)$ the probability that an audit reveals the issue given that the agent has exerted evasive efforts. Taken together, the pair (h,β) characterizes the agent's evasion technology, such that evasive effort h reduces the detection probability from one to β. Lower values of either h or β provide stronger incentives for the agent to evade the principal's audits.

As an alternative to taking evasive actions, the agent may voluntarily fix the issue at cost r without notifying the principal (i.e, take self-correction actions). Technically, this action is equivalent to a perfect evasion ($\beta = 0$) at cost r but which also terminates the incurrence of cost rate c. Note that the principal may still prefer the agent's disclosing to self-correcting the issue, in order to avoid running unnecessary but costly audits.

When the cost of evasion becomes so significant that it exceeds the remedial cost ($h > r$), the agent lacks any incentive to evade, and the problem simplifies to the one addressed in Wang et al. (2016). Therefore, we focus on the case where evasive actions are meaningful in the sense that:

$$h \le r. \tag{1}$$

Payment transfers. If the audit reveals the adverse event at time t, the principal charges the agent a fine $F_t \le F$, where F is the maximal possible penalty that the principal can inflict on the agent (see Harrington 1988, for a series of justification). In the case of a firm such as Walmart or Levi's, for instance, the penalty may consist in terminating the contract with the supplier, in which case F corresponds to the total opportunity cost associated with this loss of revenue (Hadler 2013). More generally, the agent is protected by limited liability, where F is the maximum penalty that the agent can bear.[4] We focus on the non-degenerate case where

$$h \le F.$$

Otherwise the agent cannot afford to evade. Together with (1), we define

$$\bar{h} := r \wedge F, \text{ such that } h \le \bar{h}, \tag{2}$$

[3] Modelling the agent's evasion as a binary decision lends analytical tractability. This modelling choice has also been widely adopted in dynamic moral hazard literature (e.g., Biais et al. 2010, Myerson 2012, Sun and Tian 2018).

[4] Note that environmental economists have argued against the principal enjoying a surplus beyond the remedial costs, i.e. $F > r$, which can be politically and legally prohibited (Harrington 1988). In this paper, we make no assumptions regarding whether F is larger or smaller than r In this case, inflicting F may force bankruptcy upon the agent (Clifford and Greenhouse 2013)

where we use notation $a \wedge b$ to represent $\min\{a, b\}$.

Alternatively, if the agent voluntarily discloses the issue at time t, the principal charges the agent a penalty $P_t \leq F$. We do not assume but will show that $0 \leq P_t \leq \bar{h}$ at optimality. Thus, the policy corresponds to a cost-sharing mechanism, where payments P_t and $r - P_t$ represent a breakdown of remedial cost shared between the agent and the principal, respectively. In particular, if $P_t < r$, then the agent strictly prefers self-disclosure (by paying P_t towards remediation) to self-correction, whereas he is indifferent between these two options if $P_t = r$. In practice, such disclosure incentives are typically implemented in the form of penalty reductions (EPA 2000) or remediation assistantship (Hadler 2013).

Time line. The sequence of events at any point in time is as follows (see Figure 1). The principal first designs and commits to a *policy* $\mathcal{P} := (F_t, P_t, Q_t)_{t\in[0,\infty)}$ that specifies her audit schedule Q_t and the agent's payments (F_t, P_t) upon detection and disclosure, respectively. We note that the policy is dynamic in that it is adaptive to the public history $\mathcal{I}_t$, which consists of all previous audit time epochs and audit results up to time t. If the issue occurs, the agent responds to the principal's policy by choosing whether and when to (i) disclose the issue, (ii) evade the audit, or (iii) self-correct. Once a disclosure or an audit detection occurs, the strategic interaction between the principal and the agent ends. Therefore, if time proceeds to time t, no audit must have detected any issue until t, and hence the public history $\mathcal{I}_t$ simply corresponds to all the audits' time epochs that have been run thus far.

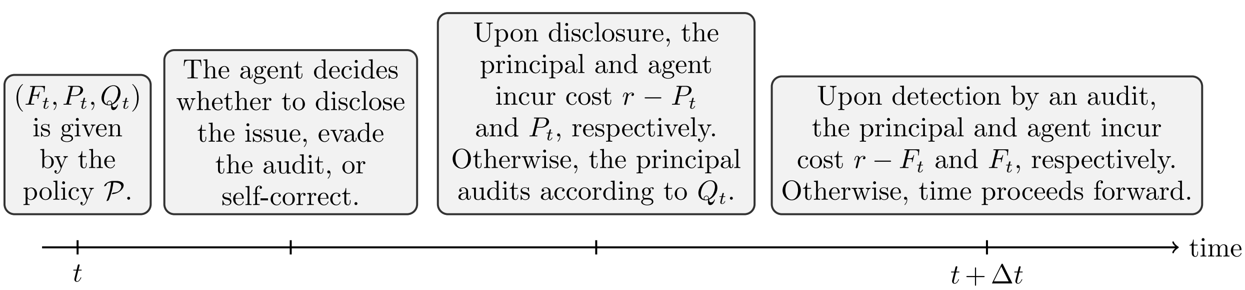


Figure 1 Sequence of events at any moment in time t ($\Delta t \approx 0$) if the issue has occurred (i.e., for $t \geq T$).

Threat utility. After taking an evasive action, the agent faces the risk of getting caught if evasion is imperfect with $\beta > 0$. We define a *threat utility* U_t to represent the agent's expected discounted cost from time t onwards after taking an evasive action (conditional on that the issue has occurred, i.e., $T \leq t$). Threat utility U_t is fully determined by the audit schedule Q_t, fine F_t, and post-evasion detection probability β (see (A.4) of Appendix A for a formal definition). In particular, when an evasion is perfect, i.e. $\beta = 0$, the agent does not face any threat utility and

$U_t = 0$ for $t \geq 0$. In contrast, when $\beta > 0$, the principal can adjust Q_t and F_t to increase U_t and incentivize the agent to comply. Doing so, however, may also increase the principal's auditing costs.

Agent's Cost. Let stopping time $\sigma(T) \geq T$ denote the instant when the agent takes an action after an issue has emerged at time T. In particular, the agent has not yet taken any action during time interval $[T, \sigma(T))$. If $\sigma(T) = T$, then the agent acts without any delay. At time $\sigma(T)$, the agent's costs for (i) disclosing the issue, (ii) evading audits, or (iii) self-correcting, are $P_{\sigma(T)}$, $h + U_{\sigma(T)}$, and r, respectively. Therefore, the agent chooses the lowest among them and inccurs a cost

$$\mathfrak{c}_{\sigma(T)} := P_{\sigma(T)} \wedge \left(h + U_{\sigma(T)}\right) \wedge r.$$

Next define $\tau(T) > T$ as the time epoch of the first audit after the issue has occurred at time T. If $\sigma(T) \leq \tau(T)$, the agent is not audited during $[T, \sigma(T)]$, and incurs a cost $\mathfrak{c}_{\sigma(T)}$ at time $\sigma(T)$. If $\sigma(T) > \tau(T)$, the agent has not yet evaded the audit and will be caught by an audit with certainty at time $\tau(T)$, which results in penalty $F_{\tau(T)}$. Thus, the agent's expected discounted cost of following strategy σ in response to the principal's policy $\mathcal{P}$ is equal to

$$C_a\left(\mathcal{P}, \sigma\right) = \mathbb{E}\left[e^{-\theta\sigma(T)} \mathbb{1}_{\{\sigma(T) \leq \tau(T)\}} \mathfrak{c}_{\sigma(T)} + \mathbb{1}_{\{\sigma(T) > \tau(T)\}} F_{\tau(T)} e^{-\theta\tau(T)} \,\middle|\, \mathcal{P}, \sigma\right] . \tag{3}$$

Principal's problem. Prior to the issue's occurrence at time T, the principal incurs a total discounted auditing cost of $k \int_0^T e^{-\theta t} dN_t$, where N_t represents the counting process for the total number of audits up to time t. After time T, the principal accrues a cost at rate c between T and $\tau(T) \wedge \sigma(T)$, which yields a total discounted cost of $c \int_T^{\sigma(T) \wedge \tau(T)} e^{-\theta t} dt$. If $\sigma(T) > \tau(T)$, the agent is caught by an audit at $\tau(T)$ with certainty, and the principal incurs an audit cost k as well as the net remedial cost $r - F_{\tau(T)}$. (The fine is the principal's income.) Otherwise (i.e., $\sigma(T) \leq \tau(T)$), three situations need to be considered. First, if $\mathfrak{c}_{\sigma(T)} = P_{\sigma(T)}$, the agent discloses the issue and the principal covers the remaining remedial cost, $r - P_{\sigma(T)}$. Second, if $\mathfrak{c}_{\sigma(T)} = r$, the agent self-corrects and the principal keeps incurring auditing costs (but no damage cost c) indefinitely afterwards. We denote the principal's total expected cost from $\sigma(T)$ onwards in this case as $W_{\sigma(T)}$, which is determined by control Q_t. Finally if $\mathfrak{c}_{\sigma(T)} = h + U_{\sigma(T)}$, the agent takes an evasive action at $\sigma(T)$, and we denote the resulting principal's total expected cost from $\sigma(T)$ onwards as $V_{\sigma(T)}$, which is determined by control Q_t and F_t. (See (A.6) and (A.5) of Appendix A for a formal definition of W_t and V_t, respectively.) Taken together, the principal's total discounted cost is given by

$$\begin{aligned} C\left(\mathcal{P}, \sigma\right) := \mathbb{E}\Bigg[& k\int_0^T e^{-\theta t} dN_t + c\int_T^{\sigma(T)\wedge\tau(T)} e^{-\theta t} dt + \mathbb{1}_{\{\sigma(T) > \tau(T)\}} e^{-\theta\tau(T)} \left(k + r - F_{\tau(T)}\right) \\ & + \mathbb{1}_{\{\sigma(T) \leq \tau(T)\}} e^{-\theta\sigma(T)} \left\{ \mathbb{1}_{\left\{P_{\sigma(T)} \leq \min\left\{r, h + U_{\sigma(T)}\right\}\right\}} \left(r - P_{\sigma(T)}\right) \right. \end{aligned}$$

$$+\mathbb{1}_{\left\{r<P_{\sigma(T)},r\leq h+U_{\sigma(T)}\right\}}W_{\sigma(T)}+\mathbb{1}_{\left\{h+U_{\sigma(T)}<\min\left\{r,P_{\sigma(T)}\right\}\right\}}V_{\sigma(T)}\Big\}\,\Big|\,\mathcal{P},\sigma\Big]. \tag{4}$$

Overall, the principal's problem consists in designing policy $\mathcal{P}$ that minimizes total expected discounted cost $C(\mathcal{P},\sigma)$ while accounting for the agent's strategic responses. Formally, the principal's *optimal policy* $\mathcal{P}^\star$ is determined as the solution to the following problem,

$$C^\star := \min_{\mathcal{P}} \; C(\mathcal{P},\sigma), \quad \text{subject to} \quad C_a(\mathcal{P},\sigma) \leq C_a(\mathcal{P},\sigma') \text{ for all } \sigma', \tag{5}$$

whereby $C^\star$ denotes the principal's optimal expected total discounted cost. Under the optimal policy $\mathcal{P}^\star$, the agent's optimal total expected discounted cost is given by $C_a^\star := \min_\sigma C_a(\mathcal{P}^\star,\sigma)$, and a best response strategy is a stopping time $\sigma^\star$ (when the agent either discloses or evades) such that $C_a(\mathcal{P}^\star,\sigma^\star) = C_a^\star$.

4. Problem Reformulation

The generality of our framework allows for a large variety of possible auditing policies, which can potentially induce complex disclosure and evasion strategies from the agent. Nonetheless, in this section, we establish that inducing the agent to report the issue without any delay nor evasion is optimal for the principal. In other words, the principal can restrict the search for the optimal policy within the set of incentive-compatible policies that always induce the agent's prompt disclosure. This result extends the classical revelation principle developed for static mechanism design problems (e.g., Dasgupta et al. 1979, Myerson 1979) to a dynamic setting with both moral hazard and costly state verification.

THEOREM 1 **(Optimality of Prompt Disclosure)**. *For any given policy* $\widehat{\mathcal{P}} := \left(\widehat{F}_t,\widehat{P}_t,\widehat{Q}_t\right)_{t\in[0,\infty)}$ *with the agent's best response strategy* $\widehat{\sigma}^\star$*, a policy* $\mathcal{P} := (F_t,P_t,Q_t)_{t\in[0,\infty)}$ *exists such that,*

1) *the fine upon detecting the issue through an audit is set to its maximum level, i.e.,* $F_t := F$*;*
2) *the agent always prefers disclosing the issue to evading or self-correcting, i.e.,*

$$P_t \leq r \wedge (h+U_t), \quad \textit{for all } t \geq 0, \tag{6}$$

where U_t *evolves according to*

$$U_t = (1-q_t^m)U_{t+} + q_t^m\left(\beta F + (1-\beta)U_{t+}^I\right), \quad \textit{for} \;\; q_t^m > 0, \tag{7}$$

$$\frac{dU_t}{dt} = \theta U_t - q_t^n\left[\beta F + (1-\beta)U_{t+}^I - U_t\right], \quad \textit{for} \;\; q_t^m = 0, \tag{8}$$

with U_{t+} *(resp.,* U_{t+}^I*) being the value of* U_t *right after time* t *in the absence (resp., presence) of an audit.*

3) the agent always prefers disclosing without delay, i.e., $C_a(\mathcal{P},T) \leq C_a(\mathcal{P},\sigma)$ *for all* σ*; or, equivalently,* P_t *evolves according to*

$$P_t \leq (1-q_t^m)P_{t+} + q_t^m F, \quad \text{for} \quad q_t^m > 0, \quad \text{and} \tag{9}$$

$$P_t \leq P_{t+}, \quad \text{or} \quad \frac{dP_t}{dt} \geq \theta P_t - q_t^n (F - P_t), \quad \text{for} \quad q_t^m = 0, \tag{10}$$

with P_{t+} *(resp.,* P_{t+}^I*) being the value of* P_t *right after time* t *in the absence (resp., presence) of an audit.*

4) the agent's total discounted expected cost remains the same, while the principal is not made worse off, i.e., $C_a(\mathcal{P},T) = C_a(\widehat{\mathcal{P}},\widehat{\sigma}^\star)$ *and* $C(\widehat{\mathcal{P}},\widehat{\sigma}^\star) \geq C(\mathcal{P},T) = \mathbb{E}\left[k\int_0^T e^{-\theta t}dN_t + e^{-\theta T}(r-P_T) \,\middle|\, \mathcal{P}\right]$.

To establish Theorem 1, we construct a new policy $\mathcal{P}$, under which the agent's payment upon disclosure P_t replicates the agent's minimum expected discounted cost under the agent's best response to policy $\widehat{\mathcal{P}}$. By doing so, the principal maintains the same expected discounted payment towards remediation and hence equivalent payoff to the agent. This further allows the principal to always remedy the adverse consequences of the issue without delay (i.e., avoid cost c) and uncover the issue through self-reporting (i.e., avoid future unnecessary auditing costs). The principal is thus better off replacing the payment scheme of any arbitrary policy with the one in Theorem 1.

In essence, Theorem 1 states that focusing on policies which always induce prompt disclosure is optimal. Under such a policy, penalty F_t never materializes and only serves as a threat to the agent. As a result, the principal maximizes the penality to the agent's limited liability F, as stated by the first point of the proposition. We thus refer to a policy $\mathcal{P}$ in the following as a pair $(P,Q) = (P_t,Q_t)_{t\in[0,\infty)}$, and take $F_t = F$ for all t.

This policy must also satisfy the *obedience constraint* (6), per the second point of the theorem. This constraint addresses the moral hazard (i.e., hidden action) incentive that emerges from our setting. The constraint requires that the agent always finds voluntary disclosure (at cost P_t) more economically attractive than either self-correction (at cost r), or evasion and potentially getting caught (at cost $h + U_t$). Therefore, obedience constraint (6), together with the limited liability F, imposes an upper bound for the disclosure payment $P_t \leq \bar{h}$. Furthermore, obedience constraint (6) becomes more stringent for a given Q_t when the evasion becomes relatively easy (i.e., lower value of h). In other words, the principal has to offer the agent higher disclosure incentives (i.e., lower the penalty P_t) so as to induce no evasive behavior. As a result, cost U_t never materializes on the equilibrium path, and again only serves as a threat to the agent. Theorem 1 further explicitly characterizes the dynamic evolution of threat utility U_t in (7) and (8), which are determined by the audit schedule Q_t. Note that whether the principal should induce a particular action from the agent is in general unclear for moral harzard problems. Theorem 1 shows, however, that it is indeed optimal for the principal to induce the agent not to evade in our setting.

Finally, *informational IC constraint* $C_a(\mathcal{P},T) \le C_a(\mathcal{P},\sigma)$ in the last point of the theorem addresses the adverse selection (i.e., hidden information) problem by inducing the agent to disclose the issue as soon as it occurs. Equations (9) and (10) in the proposition express this constraint in a recursive manner for each time instant given that the issue has occurred. They are derived by ensuring the agent's payment P_t (from immediately disclosing the adverse event that has occurred) is no higher than the payment of postponing the disclosure to the next moment. Note that the constraints regulating U_t, (7)-(8), share great similarity with those regulating P_t, (9)-(10). In fact, in the absence of an effective evasion capability (i.e., when $\beta = 1$), the evolution of U_t coincides with the (binding) trajectory of P_t. The presence of an evasion capability, however, requires P_t and U_t to diverge. (See Lemma A.1 and Remark A.1 of Appendix A for the feasible range of (P_t, U_t).)

Overall, Theorem 1 allows to reformulate the principal's problem (5) as

$$C^\star = \frac{\lambda}{\theta+\lambda} r + \min_{\mathcal{P}=(P_t,Q_t)_{t\in[0,\infty)}} \mathbb{E}\left[k\int_0^T e^{-\theta t} dN_t - e^{-\theta T} P_T \,\middle|\, \mathcal{P}\right], \quad \text{subject to (6)–(10).} \tag{11}$$

The principal's objective in (11) explicitly captures the fundamental tradeoff that the principal needs to make. Specifically, the principal can reduce her cost by increasing the agent's payment P_t, whereas larger payments require more frequent audits, resulting in higher audit costs $k\int_0^T e^{-\theta t} dN_t$. Note further that (11) does not depend on the impact of the adverse event, c. Indeed, per Theorem 1, the agent always immediately reports the event, which is then fixed, at optimality. And since the agent never evades either (per the obedience constraint (6)), the solution of (11) also holds when the agent's evasive effort aggravates the adverse event (by increasing its impact to $\bar{c} > c$, for instance).

5. Optimal Policy for Perfect Evasion

We first examine the case where the evasion technology is able to render the principal's audits completely ineffective, i.e., the detection probability $\beta = 0$. In this case, the agent has the strongest incentive to take the evasive action. This allows us to isolate the effect that the evasion effort h has on the principal's policy. When evasion is perfect, the principal is unable to threaten the agent once an evasive action is taken (i.e., $U_t \equiv 0$, see the proof of Theorem 2 of Appendix B). As a result, IC constraints (7)-(8) become irrelevant and the obedience constraint (6) reduces to $P_t \le h$ (since $h \le \bar{h}$ from (2)) and $U_t = 0$. The following result demonstrates that the agent's ability to evade audits induces the principal to use a random (as opposed to deterministic) inspection schedule.

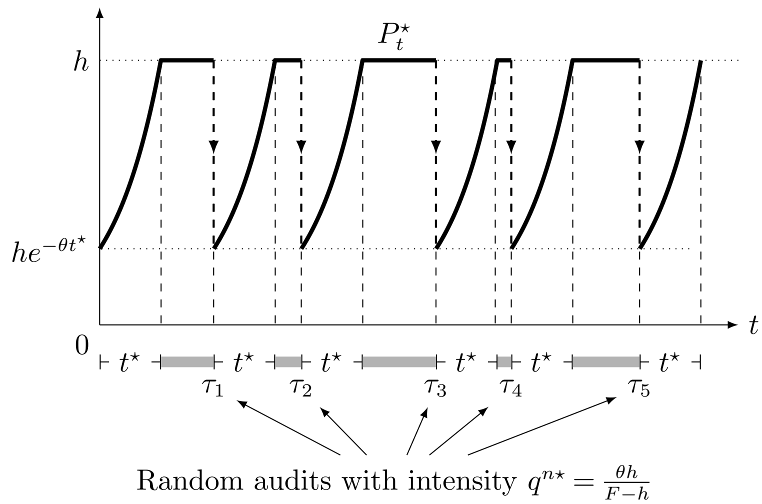


Figure 2 The optimal policy with $F = 10$, $k = 2$, $\lambda = 0.2$, $\theta = 1$, $h = 5$ and $\beta = 0$.

THEOREM 2. *If $\beta = 0$, then the principal's optimal policy $(P^\star, Q^\star)$ exhibits a cyclic structure marked by periodic* random *audits and persists as long as the issue has not been revealed by the agent. Specifically, let $t^\star$ be the unique solution to the following equation in t,*

$$\Gamma(t;h) := (\lambda + \theta)\left[F - (k+F)e^{-\lambda t}\right] - \lambda h\left[1 - e^{-(\lambda+\theta)t}\right] = 0. \tag{12}$$

Then, each cycle $i = 1, 2, \ldots$ starts with a deterministic period of length $t^\star$, immediately after the last audit at τ_{i-1} (with $\tau_0 = 0$), during which the principal conducts no audits (i.e., $q_t^{m\star} = q_t^{n\star} \equiv 0$ for $t \in (\tau_{i-1}, \tau_{i-1} + t^\star]$) and charges the agent a payment according to

$$P_t^\star = \Pi(t; t^\star, h) := he^{-\theta(\tau_{i-1} + t^\star - t)}, \quad \text{for } t \in (\tau_{i-1}, \tau_{i-1} + t^\star]. \tag{13}$$

Starting from $\tau_{i-1} + t^\star$, the principal conducts only intensive audits (i.e., $q_t^{m\star} \equiv 0$) at a finite constant rate while maintaining a constant payment level, respectively given by

$$q_t^{n\star} \equiv q^{n\star} := \frac{\theta h}{F - h}, \quad \text{and} \quad P_t^\star \equiv h, \quad \text{for } t \in (\tau_{i-1} + t^\star, \tau_i], \tag{14}$$

until the next audit takes place at time τ_i. Namely, conditional on τ_{i-1}, the time until the next audit $\tau_i - \tau_{i-1} - t^\star$ is an i.i.d. exponential random variable with rate $q^{n\star}$ given in (14).

Figure 2 illustrates a sample path of payment $P_t^\star$ under the optimal policy characterized by Theorem 2. As depicted by the figure, the optimal policy demonstrates a cyclic structure and alternates between *deterministic and increasing* monetary payments and *random* audits.

Specifically, the principal starts each cycle by first adjusting payment $P_t^\star$, which increases exponentially from the lower threshold $he^{-\theta t^\star}$ until it reaches the evasion cost h imposed by the obedience constraint (6). (Recall that $U_t \equiv 0$ if $\beta = 0$.) The increasing curve of $P_t^\star$ ensures that the agent is indifferent between disclosing the issue immediately (by paying $P_t^\star$ towards remediation) and delaying the disclosure to a later time, say $t+\Delta t$, which implies a cost of $(1-\theta\Delta t)P_{t+\Delta t}^\star$ due to discounting. Namely, the level of $P_t^\star$ is set such that $P_t^\star = (1-\theta\Delta t)P_{t+\Delta t}^\star$. Taking Δt to 0, we obtain $dP_t^\star/dt = \theta P_t^\star$,[5] which implies the optimal payment trajectory in (13). Further, as long as the increasing trajectory $P_t^\star$ remains below the evasion cost h, the agent has no incentive to take an evasive action. Hence, the monetary instrument provides sufficient incentive for the agent to promptly disclose the issue and the principal does not need to conduct any audit.

Once $P_t^\star$ reaches h (after $t^\star$ units of time since the last audit), the monetary incentives are exhausted. To discourage any evasion, the principal then resorts to audits, while maintaining $P_t^\star$ at the constant level h. The audit is actually *random* with a constant intensity rate $q^{n\star}$. This specific rate ensures that the time-discounting benefit of delaying the disclosure for Δt, which is $(\theta\Delta t)h + o(\Delta t)$, is exactly offset by the net loss of being caught and charged a fine, which is $(q_t^n\Delta t)(F-h)+o(\Delta t)$. That is, audit intensity q_t^n is set such that $(\theta\Delta t)h \approx (q_t^n\Delta t)(F-h)$, which yields the constant audit rate $q^{n\star}$ in (14). It is worth noting that this auditing intensity is time-independent, and is purely driven by the binding IC constraint (10). Notably, since rate $q^{n\star}$ induces voluntary disclosure *after* the issue has emerged, $q^{n\star}$ does not depend on λ, the rate at which the adverse event may occur. Interestingly, the audit cost k does not impact $q^{n\star}$ either, but only affects audits through their frequency, namely the time interval $t^\star$ per (12).

This random inspection structure is in sharp contrast to the *deterministic* audit policy in Wang et al. (2016) for the case where evasive actions are impossible (or equivalently when $h \geq F$). Here, the principal needs to run random audits to account for the agent's moral hazard incentive of evasion in our setting. When adverse selection is the only incentive issue, the agent's payment P_t upon disclosure is bounded by the limited liability, F. As a result, deterministic audits are optimal per Wang et al. (2016). In our setting, however, payment P_t needs to stay below $h \leq F$ in order to prevent the agent from evading audits, which induces audits to be random.

Note also that the principal periodically audits the agent at optimality, even though the agent is able to render these audits fully ineffective. This is because the optimal policy is precisely designed to prevent the agent from taking evasive actions (per the obedience constraint (6), with $U_t = 0$ when $\beta = 0$). As a result, audits and the penalty F serve as credible threats to enforce compliance.

Lastly, it is important to highlight the remarkable *simplicity* of the optimal policy. In essence, our policy motivates the agent to come clean before a random audit takes place. The principal runs random audits periodically, but offers a reduced penalty for self-reporting during a fixed amount of time before each audit. This penalty level gradually increases in a deterministic fashion due

[5] This corresponds to a binding informational IC constraint (10) at any point in time.

to time discounting. When the penalty reaches h, the time until an audit follows an exponential distribution.[6]

Overall, the optimal policy corresponds to a *dynamic cost sharing mechanism* with audits, where the agent is always responsible for a strictly positive portion of the remedial cost $P_t^\star \in (0, r]$ while the principal covers the remaining remedial cost $r - P_t^\star \in (0, r]$. Interestingly, the principal can in fact shift the *entire* auditing cost onto the agent, i.e., the agent's expected payment is larger than the principal's total expected auditing cost, as stated in the next corollary.

COROLLARY 1. *Under the optimal policy $(P^\star, Q^\star)$ specified in Theorem 2, the total expected discounted costs for the principal and agent are given by $C^\star = \frac{\lambda}{\lambda+\theta}\left(r - he^{-\theta t^\star}\right)$ and $C_a^\star = \frac{\lambda}{\lambda+\theta} he^{-\theta t^\star} + A^\star$, respectively, where $A^\star = \frac{\theta}{\lambda+\theta}\frac{khe^{-\theta t^\star}}{k+F-he^{-\theta t^\star}}$ is the principal's total auditing expense. Furthermore, we have $C^\star > 0$ and $C_a^\star > A^\star > 0$.*

6. Optimal Policy for Imperfect and Costly Evasions

We see in the last section that when evasion is perfect with $\beta = 0$, the agent has the strongest incentive to evade, and the obedience constraint (6) always binds at optimality (over some time intervals) per Theorem 2. This is actually true for any evasive cost $h < \bar{h}$, even though a high evasion cost h reduces the agent's incentive to evade and thus should relax the constraint. When the agent's evasive action is imperfect with $\beta > 0$ (and hence $U_t \geq 0$), however, the constraint $P_t \leq h + U_t$ may not always bind anymore. In this section we examine the case when the agent's incentive to evade is sufficiently weak for that constraint to be ignored. We find that this happens when the evasion cost is above a threshold, which we obtain in a closed form. As such, the moral hazard problem is mute and the optimal policy only binds the upper bound constraint $P_t \leq \bar{h}$.

THEOREM 3. *Let t° be the unique positive solution to equation $\Gamma(t; \bar{h}) = 0$ in t, in which function $\Gamma(t, \cdot)$ is defined in (12), and define*

$$\widehat{h}(\beta) := \frac{(1-\beta)\left[F - \bar{h}e^{-\theta t^\circ}\right]}{F - \bar{h}(1-\beta)e^{-\theta t^\circ}}\bar{h} \in [0, (1-\beta)\bar{h}], \quad \text{for } \beta \in [0,1]. \tag{15}$$

Then, for $\beta > 0$ and $h \geq \widehat{h}(\beta)$, the optimal control policy $(P^\star, Q^\star)$ exhibits a cyclic structure similar to Theorem 2. Specifically, each cycle $i = 1, 2, \ldots$ starts with a deterministic period of length t°, immediately after the last audit at τ_{i-1} (with $\tau_0 = 0$), during which $q_t^{m\star} = q_t^{n\star} \equiv 0$ for $t \in (\tau_{i-1}, \tau_{i-1} + t^\circ]$ and $P_t^\star = \Pi(t; t^\circ, \bar{h})$ for $t \in (\tau_{i-1}, \tau_{i-1} + t^\circ]$, in which $\Pi(t; \cdot, \cdot)$ is defined in (13).

Starting from $\tau_{i-1} + t^\circ$, the principal conducts audits in the following fashion:

[6] In practice, a random audit with a constant rate could be implemented in the following way. Say, the audit occurs with probability $x\%$ each day during the random audit phase. Then the policy is to audit the the agent when the first two digits after the decimal place of a commonly observable stock index's opening price is no larger than x.

- *If $r \geq F$, then a* deterministic *audit occurs at $\tau_i = \tau_{i-1} + t^\circ$ (i.e., $q^{m\star}_{\tau_i} = 1$, $q^{m\star}_t \equiv 0$ for $t \neq \tau_i$, and $q^{n\star}_t \equiv 0$ for all t).*
- *If $r < F$, then the principal conducts an* intensive *audit (i.e., $q^{m\star}_t \equiv 0$) at a finite constant rate while maintaining a constant payment level, given by*

$$q^{n\star}_t \equiv q^{n\circ} := \frac{\theta r}{F-r}, \quad \textit{and} \quad P^\star_t = r, \quad \textit{for } t \in (\tau_{i-1} + t^\circ, \tau_i], \tag{16}$$

respectively, until the next audit takes place at time τ_i.
The corresponding threat utility equals to

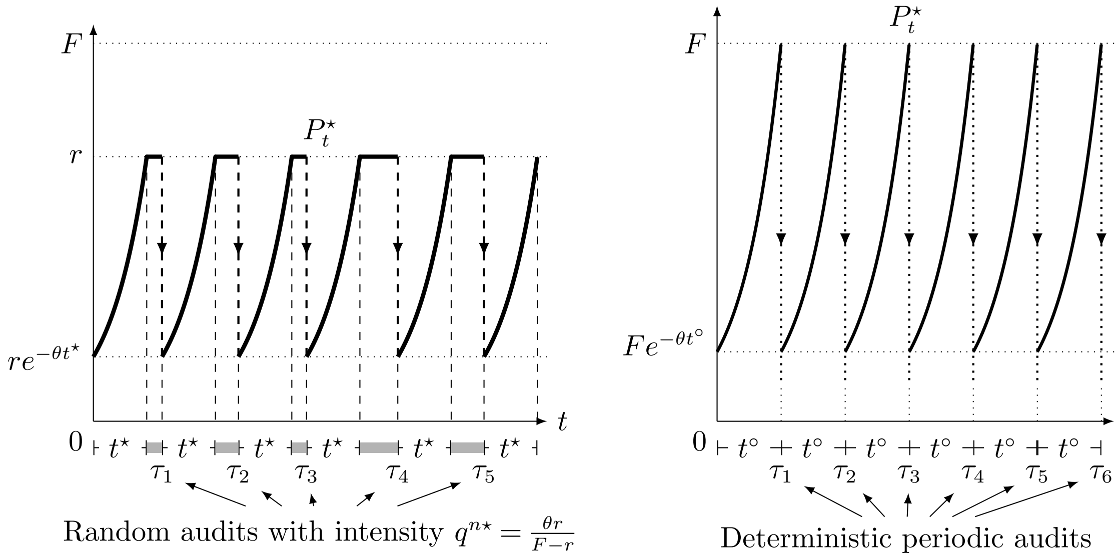
$$U^\star_t = \frac{\beta F}{F - (1-\beta)\bar{h}e^{-\theta t^\circ}} P^\star_t, \ \forall t \geq 0. \tag{17}$$

Based on Theorem 3, we can further obtain the optimal costs of the principal and that agent, which indicate that the agent bears the entire auditing costs.

Corollary 2. *Under the optimal policy (t°, F) specified in Theorem 3, the total expected discounted costs for the principal and agent are given by $C^\star = \frac{\lambda}{\lambda+\theta}\left(r - \bar{h}e^{-\theta t^\circ}\right)$ and $C^\star_a = \frac{\lambda}{\lambda+\theta}\bar{h}e^{-\theta t^\circ} + A^\star$, respectively, where $A^\star = \frac{\theta}{\lambda+\theta}\frac{k\bar{h}e^{-\theta t^\circ}}{k+F-\bar{h}e^{-\theta t^\circ}}$ is the principal's total auditing expense. Furthermore, we have $C^\star > 0$ and $C^\star_a > A^\star > 0$.*

In essence, Theorem 3 shows that a cyclic policy akin to the one identified in Theorem 2 remains optimal when evasions are imperfect ($\beta > 0$) and sufficiently costly ($h \geq \widehat{h}(\beta)$). More specifically, each cycle still features a deterministic no-audit period (with length t°) followed by an audit that resets the cycle. In particular, the audit is *random* with a constant rate if remedial cost r is lower than limited liability F (see Figure 3(a)). The random nature of the audit stems again from the binding obedience constraint (6). Specifically, Equation (16) indicates that the agent's payment P_t never goes beyond the remidial cost r, so as to mitigate the agent's self-correction incentive. However, when remedial costs more than the agent's limited liability (i.e., $r \geq F$), the entire policy becomes deterministic and periodic (see Figure 3(b)). In this case, the obedience constraint (6) is not binding under the optimal policy and the setting reduces to the one studied in Wang et al. (2016). Formally, we define the class of deterministic cyclic policies as follows.

Definition 1. A *Deterministic Cyclic Policy* $(\bar{t}, \bar{p})$, with periodicity $\bar{t} > 0$ and maximum payment $\bar{p} \in (0, F]$, is a policy (P, Q) such that a deterministic audit occurs at every time epoch $\tau_i = \bar{t} \times i$ (i.e., $q^m_{\tau_i} = 1$, $q^m_t = 0$ for $t \neq \tau_i$, and $q^{n\star}_t := 0$ for all t) for $i = 1, 2, \ldots$, as long as the agent does not reveal the issue. The payment between two consecutive audits follows the same trajectory $P_t = \bar{p}e^{-\theta(\tau_i - t)}$ for $t \in (\tau_{i-1}, \tau_i]$ and $i = 1, 2, \ldots$.



(a) $r = 7 < F$.

(b) $r = 12 > F$.

Figure 3 The optimal policy for $h \geq \widehat{h}(\beta)$, with $F = 10$, $k = 2$, $\lambda = 0.2$, and $\theta = 1$.

In particular, the optimal policy for $r \geq F$ in Theorem 3 is a deterministic cyclic policy (t°, F).

When the evasion cost h is sufficiently high $(h \geq \widehat{h}(\beta))$, the principal can ignore the agent's incentive to evade, and hence threat utility U_t. Problem (11) then reduces to a single-dimensional stochatic control on state P_t.

Figure 4 illustrates this point and depicts threshold $\widehat{h}(\beta)$ in the space of evasion capabilities (β, h) following (15). Threshold $\widehat{h}(\beta)$ is below the line $h = (1-\beta)\bar{h}$. Thus, when evasion is imperfect, there always exist some evasion capabilities (β, h) with $\widehat{h}(\beta) \leq h < \bar{h}$ that the principal can safely ignore. More generally, the optimal policy in Figure 4's shaded area is the same as descried in Theorem 3, regardless of evasion capabilities (β, h).

In contrast, threshold $\widehat{h}(\beta)$ converges to the evasion cost's upper bound $\bar{h}$ as evasion becomes more effective, i.e. β approaches zero. At the limit $\beta = 0$, the principal can never ignore the agent's evasion capability and associated obedience constraint (6), for *all* values of the evasion cost $h \in [0, \bar{h}]$. The optimal audit policy becomes random in this case per Theorem 2. In general, the principal needs to explicitly account for obedience constraint (6) in Figure 4's un-shaded area, for which $h < \widehat{h}(\beta)$ and $\beta > 0$. We explore these cases in the next section, where we leverage the fact that payment $P_t^\star$ for self-disclosure is proportional to threat utility $U_t^\star$ at optimality per Equation (17).

7. Policies for Imperfect and Inexpensive Evasion

We now explore situations where the agent has mixed incentives to evade. That is, the agent's evasive action is imperfect with $\beta > 0$, but also inexpensive with $h < \widehat{h}(\beta)$ (as defined in (15)),

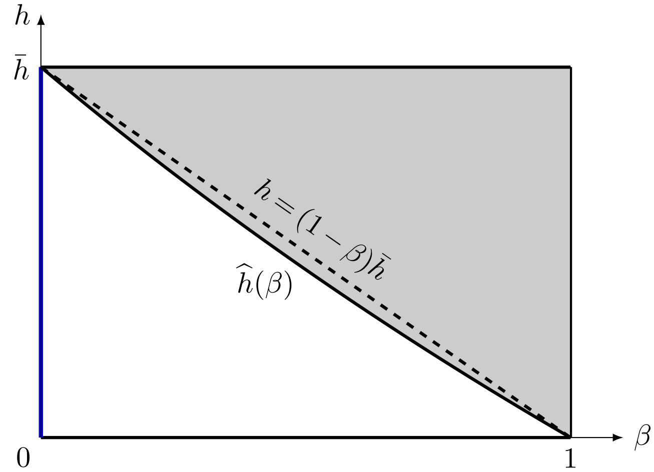


Figure 4 Shaded area represents parametric range of (β, h) with $h \geq \widehat{h}(\beta)$, for which Theorem 3 holds ($r = 7$, $F = 10, k = 2, \lambda = 0.2$ and $\theta = 1$).

corresponding to the un-shaded area of Figure 4. Recall that when evasion is either perfect ($\beta = 0$) or relatively costly ($h \geq \widehat{h}(\beta)$), the principal's problem (11) reduces to the stochastic control of a one-dimensional piecewise deterministic process (in P_t). This is because threat utility U_t reduces to zero when $\beta = 0$, or does not affect the agent's evasive action when $\beta > 0$ and $h \geq \widehat{h}(\beta)$. When $\beta > 0$ and $h < \widehat{h}(\beta)$, however, the problem (11) becomes a genuine *two*-dimensional control of piecewise deterministic process in (P_t, U_t) governed by the IC constraints (7)–(10) and a state constraint (6). Problems of this sort are known to be generally intractable not only analytically but also numerically (see, e.g., Chehrazi et al. 2019, and also Remark A.1 in Appendix A for more details). In addition, and perhaps more importantly, the optimal policy solving this problem may be too complex to be implementable in practice (see again Remark A.1 in Appendix A).

In the following, we restrict the search for efficient auditing policies within a large class of tractable policies. Note first that under all the optimal policies we have seen thus far, the threat utility U_t is always proportional to the disclosure incentive P_t. Motivated by this property, we define the class of *proportional* policies $\mathcal{P}_\gamma = (P_t, Q_t)_{t \in [0,\infty)}$, for some $\gamma \geq 0$, such that policy $\mathcal{P}_\gamma$ induces a threat utility U_t proportional to P_t, i.e. $U_t = \gamma P_t$ for all $t \geq 0$. For instance, a proportional policy is optimal when evasion is perfect or sufficiently costly ($\beta = 0$ or $h \geq \widehat{h}(\beta)$) with $\gamma = 0$ and $\gamma = \beta F / \left[F - (1-\beta)\bar{h} e^{-\theta t^\circ}\right]$, per Theorem 2 and Equation (17) of Theorem 3, respectively.

In other words, under a proportional policy, the principal rewards the agent for self-reporting an issue, by having the agent pay a fraction of the expected penalty of getting caught evading audits. Proportional policies do not need to be cyclic or deterministic; they may allow for random audits, a mixture of random and deterministic audits, random audits with different or time-varying rates, and different time interval lengths between audits. Even though the class of proportional policies remains fairly large, we show in the following the optimal proportional policy takes a cyclical structure as in the previous cases.

Finding the best proportional policy corresponds to solving the following optimization problem:

$$\widehat{C}^{\star} := \min_{\gamma\geq 0,(P_t,Q_t)_{t\in[0,\infty)}} \mathbb{E}\left[k\int_0^T e^{-\theta t}dN_t - e^{-\theta T}P_T \,\middle|\, \mathcal{P}\right], \text{ subject to (6)–(10) and } U_t=\gamma P_t,\ \forall t\geq 0. \tag{18}$$

In contrast to the principal's problem in (11), problem (18) involves an additional decision variable, γ, and a new *proportionality* constraint, $U_t=\gamma P_t$. This additional restriction renders the principal's problem more tractable both mathematically and numerically for $h<\widehat{h}(\beta)$. Indeed, the main result of this section shows that the infinite-dimensional optimization problem (18) can be solved through the following two-dimensional *static* optimization,

$$\begin{aligned} K^{\star} &:= \min_{\substack{\beta\leq\gamma\leq 1-(h/\bar{h})\\ 0\leq x\leq 1}} \left(\frac{1-\gamma}{h}\right)^{\rho} \frac{k+\beta\left[F/\gamma - hx/(1-\gamma)\right]}{\rho\left[(\beta F/h)(1-\gamma)/\gamma+(1-\beta)x\right]-(\rho-1)-x^{\rho}} \\ &\text{subject to} \qquad \frac{1-(\beta F/h)(1-\gamma)/\gamma}{1-\beta}\leq x\leq \frac{F(\gamma-\beta)(1-\gamma)}{h(1-\beta)\gamma}\wedge\frac{r(1-\gamma)}{h}, \end{aligned} \tag{19}$$

with $\rho:=(\theta+\lambda)/\theta$, where the constraint is mandated by the proportional policy's feasibility. (See Remark C.1 of Appendix C for additional explanations about this reframing of stochastic control problem (18) into static optimization problem (19).)

The following theorem shows that the optimal solution to (19) fully characterizes the optimal proportion and the optimal policy for problem (18).

THEOREM 4. *For $\beta>0$ and $h<\widehat{h}(\beta)$, a unique solution $(\gamma^{\star},x^{\star})$ to (19) exists, such that the optimal proportion solving (18) is equal to $\gamma^{\star}$. Further, for an initial period of length $t_0^{\star} = -(1/\theta)\ln\left[(1-\gamma^{\star})p_0^{\star}/h\right]$ with $p_0^{\star}=(\rho K^{\star})^{-\theta/\lambda}<h/(1-\gamma^{\star})$, the optimal policy $(P^{\star},Q^{\star})$ for (18) conducts no audits (i.e., $q_t^{m\star}=q_t^{n\star}\equiv 0$ for $t\in[0,t_0^{\star}]$) and charges the agent a payment according to*

$$P_t^{\star}=p_0^{\star}e^{\theta t}, \quad \text{for } t\in[0,t_0^{\star}]. \tag{20}$$

Starting from $t_0^{\star}$, the optimal policy $(P^{\star},Q^{\star})$ for (18) exhibits a cyclic structure marked by periodic random *audits and persists as long as the issue has not been revealed by the agent. Specifically, each cycle $i=1,2,\ldots$ starts with only intensive audits (i.e., $q_t^{m\star}\equiv 0$) at a finite constant rate while maintaining the constant payment level, respectively given by*

$$q_t^{n\star}\equiv q^{n\star} := \frac{\theta}{\beta\frac{F(1-\gamma^{\star})}{h\gamma^{\star}}+(1-\beta)x^{\star}-1}, \quad \text{and} \quad P_t^{\star}\equiv\frac{h}{1-\gamma^{\star}}, \quad \text{for } t\in(t_0^{\star},\tau_1]\bigcup_{i=1}^{\infty}(\tau_i+t^{\star},\tau_{i+1}], \tag{21}$$

where $t^{\star}=(-1/\theta)\ln x^{\star}$ and τ_i is the i-th audit. Immediately after the last audit at τ_i, the principal applies no audits (i.e., $q_t^{m\star}\equiv q_t^{n\star}:=0$) for a deterministic period of length $t^{\star}$ and charges the agent a payment according to

$$P_t^{\star}=\frac{h}{1-\gamma^{\star}}e^{-\theta(\tau_i+t^{\star}-t)}, \quad \text{for } t\in\bigcup_{i=1}^{\infty}(\tau_i,\tau_i+t^{\star}]. \tag{22}$$

Figure 5 illustrates the policy structure identified by Theorem 4 for a sample path of audits. This structure is similar to the prefect evasion case with $\beta = 0$ (see Theorem 2), except for the initial period of length $t_0^\star$. Specifically, obedience constraint (6) restricts the payment P_t to be upper bounded by $h/(1-\gamma^\star) \leq \bar{h}$. Between two consecutive intensive auditing episode, IC constraint (10) is binding, and therefore the payment P_t follows a deterministic exponential trajectory as in (20) and (22). Once the payment P_t reaches its upper bound $h/(1-\gamma^\star)$, the principal exhausts the monetary incentive, and the payment P_t remains constant at this level per (21). The principal then switches to random audits to incentivize the agent, with a constant audit rate also defined in (21).[7] In contrast to the perfect evasion case $\beta = 0$, however, IC constraint (10) may no longer bind in this case.[8]

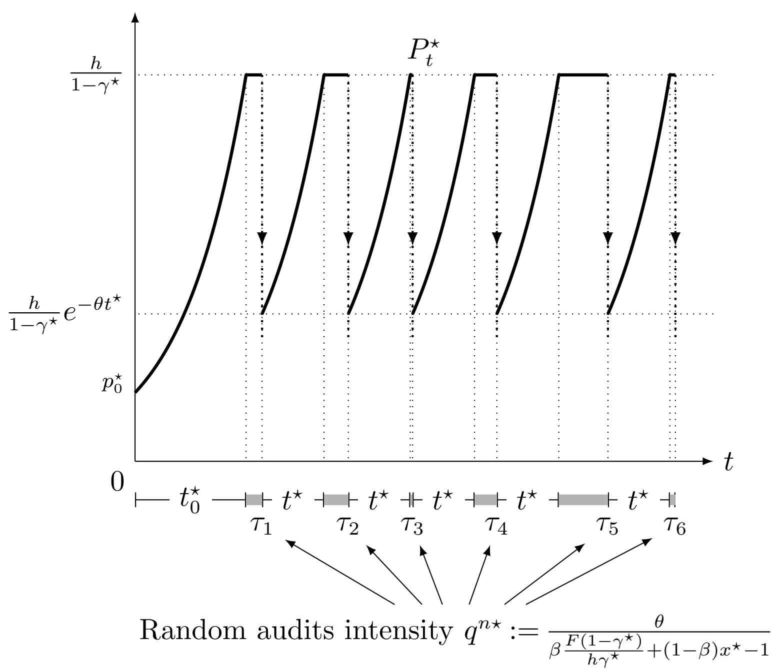


Figure 5 The optimal proportional policy with $F = 10$, $k = 2$, $\lambda = 0.2$, $\theta = 1$, $h = 1$ and $\beta = 0.6$.

Importantly, the optimal proportional policy in Theorem 4 always relies on random audits. The audits would be deterministic if the constant audit rate in (21) were infinite, which would happen if $x^\star$ were equal to its lower bound in (19). However, Theorem 4 states that this never occurs (see also Lemma C.3 in Appendix C).[9] This further implies that a deterministic cyclic policy cannot be optimal (over the whole set of feasible policies) when $h < \widehat{h}(\beta)$, as shown by the proposition below.

[7] Threat utility U_t is then also constant by ensuring $dU_t/dt = 0$ according to (8).

[8] IC constraint (10) is binding if and only if $x^\star$ binds its upper bound $\frac{F(\gamma^\star-\beta)(1-\gamma^\star)}{h(1-\beta)\gamma^\star}$ in (19).

[9] As commented in Section 3, only in the limiting case with $h = 0$, the auditing rate becomes infinite and hence cyclic deterministic audits (except for the initial period) is optimal.

PROPOSITION 1. *A deterministic cyclic policy is optimal if and only if* $r \geq F$ *and* $h \geq \widehat{h}(\beta)$.

Finally, the cyclic structure of the optimal policy in Theorem 4 yields a cost decomposition that is similar to the one characterized in Corollaries 1 and 2. (With a slight abuse of notation and for simplicity, we still use notations $A^\star$, $C^\star$ and $C_a^\star$ to denote the corresponding costs under the optimal proportional policy.)

COROLLARY 3. *Under the principal's optimal proportional policy* $(P^\star, Q^\star)$ *prescribed in Theorem 4, the total expected discounted costs for the principal and agent are given by* $C^\star = \frac{\lambda}{\lambda+\theta}(r - p_0^\star)$ *and* $C_a^\star = \frac{\lambda}{\lambda+\theta} p_0^\star + A^\star$*, respectively, where* $A^\star = \frac{\theta}{\lambda+\theta} \frac{kp_0^\star}{k+\beta[F/\gamma^\star - hx^\star/(1-\gamma^\star)]}$ *is the principal's total audit expense. Furthermore, we have* $C^\star > 0$ *and* $C_a^\star > A^\star > 0$.

8. The effect of Evasion Capability on Costs

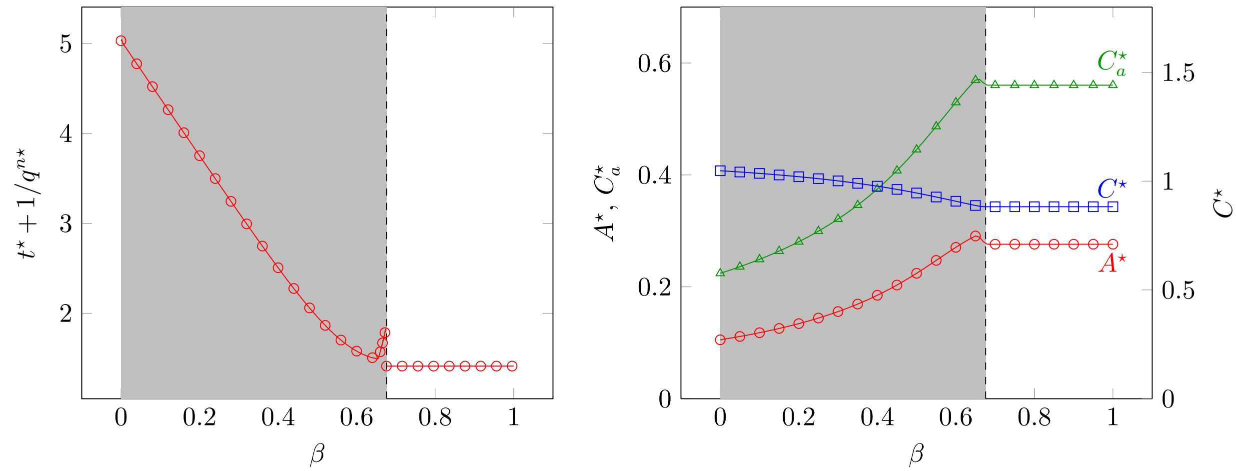
The previous analysis allows exploring the impact of the agent's evasion capacity on the principal's audit policy. To that end, we vary detection probability β and numerically evaluate the resulting audit frequency and the associated expected costs.

In our setup, the mean sojourn time between two consecutive audits is equal to $t^\star + 1/q^{n\star}$. Indeed, the auditing policies in Theorems 2, 3 and 4 alternate between a payment phase of fixed length $t^\star$ (or t°) and a random audit phase of average length $1/q^{n\star}$ (which is zero if the audit is deterministic). Thus a *lower* value of $t^\star + 1/q^{n\star}$ indicates *more frequent* audits.

Our results allows us to evaluate this sojourn time and the associated expected costs. In particular, we obtain the sojourn time from equations (12) and (14) of Theorem 2 when $\beta = 0$, and from t° and (16) of Theorem 3 when $h \geq \widehat{h}(\beta)$. When $\beta > 0$ and $h < \widehat{h}(\beta)$, we have $t^\star = (-1/\theta)\ln x^\star$, in which $x^\star$ is the optimal solution of (19), and $q^{n\star}$ as defined in (21). The expected costs are evaluated from Corollaries 2 and 3, depending on whether $h \geq \widehat{h}(\beta)$ or not.

Figure 6 depicts the principal's audit frequency and the corresponding expected costs as a function of detection probability β. Specifically, Figure 6(a) plots the mean sojourn time, while Figure 6(b) depicts the expected auditing cost $A^\star$, the principal's overall cost $C^\star$ and the agent's cost $C_a^\star$. Recall also that the agent bears the audit cost, such that $C_a^\star - A^\star > 0$ corresponds to the agent's expected contribution to the remedial cost.

Figure 6(a) demonstrates a non-monotone (and possibly discontinuous) relationship between detection probability β and audit frequency. Specifically, the principal audits first more and then less frequently as detection probability β increases (i.e., evasion becomes less effective) in the shaded region ($h < \widehat{h}(\beta)$). In the unshaded region (i.e., $h \geq \widehat{h}(\beta)$), the audit frequency remains constant per Theorem 3. As shown in Figure 6(b), auditing cost $A^\star$ exhibits a similar unimodal structure, albeit less pronounced. Furthermore, as evasion becomes less effective (i.e., β increases),



(a) Audit frequency. (b) Costs.

Figure 6 The effect of post-evasion detection probability β on the audit frequency and different cost components with $h = 2$, $r = F = 10$, $k = 2$, $\lambda = 0.2$ and $\theta = 1$. The shaded (resp., unshaded) parameter ranges correspond to $h < \widehat{h}(\beta)$ (resp., $h \geq \widehat{h}(\beta)$).

the principal can transfer a higher proportion of the remedial costs onto the agent. The principal's overall cost $C^\star$ decreases in probability β as a result.

To understand this unimodal structure, note that probability β has two countervailing effects. On one hand, as evasion becomes less and thus audits more effective, the principal can rely less on financial incentives and more on auditing to enforce compliance. In this case, the mean sojourn time decreases, and audit cost $A^\star$ increases. In addition, the principal decreases the financial incentives by transferring a higher proportion of the remedial cost to the agent, increasing the agent's cost but decreasing the principal's. On the other hand, because audits are more effective, the principal can audit less frequently to enforce compliance and thus reduce audit cost $A^\star$. The first effect dominates the second one when the evasive action is more effective, but the second effect dominates the first one when the action is more effective, yielding the overall unimodal impact of probability β we observe in Figure 6(b).

9. Extensions

In this section, we extend our base model and analysis in four directions to reflect additional considerations from practice. We will demonstrate that the results we obtained from previous sections still remain valid.

9.1. Post-evasion penalty

In our base model (Section 3), we assume that the principal charges the agent the same penalty F_t upon detection by an audit, irrespective of whether the agent has taken an evasive action or not. In the following, we allow the principal to use different penalties for the two types of violation. Specifically, we assume that when an audit uncovers an adverse event (which happens with probability β), it also reveals whether the agent has exerted evasive efforts. Notations $\overline{F}_t$ and F_t denote then the penalties upon detecting an advert event with or with evasion, respectively. When the agent voluntarily discloses the issue, the principal continues to charge the agent with payement P_t. All three penalties are bounded by maximum fine F.

With this extension, the principal's *policy*, $\mathcal{P} := \left(F_t, \overline{F}_t, P_t, Q_t\right)_{t\in[0,\infty)}$, now specifies three payments charged to the agent (depending on the discovery channel and the presence of evasion) and the audit schedule Q_t. In response to such a policy, the agent's strategy σ is again to choose whether and when to self-report the issue, or to conceal it through the evasive action. Notably, *threat utility* U_t (which is the agent's expected discounted cost from time t onwards after taking an evasive action), is now determined by $\left(\overline{F}_t, Q_t\right)$ (see (D.1) of Appendix D for a formal definition). The sequence of events remains the same as in our base model (Figure 1).

The following theorem establishes a key result for this extension. Similar to Theorem 1, we can again restrict our search for the optimal policy among all policies that induce the agent to self-report the issue without any delay or evasion. Consequently, only the payment P_t will be induced and all the other two payments $\left(F_t, \overline{F}_t\right)$ act as *off-equilibrium* threats.

Theorem 5 **(Optimality of Prompt Disclosure: Extended)**. *For any given policy* $\widehat{\mathcal{P}} := \left(\widehat{F}_t, \widehat{\overline{F}}_t, \widehat{P}_t, \widehat{Q}_t\right)_{t\in[0,\infty)}$*, there always exists a policy* $\mathcal{P} := \left(F_t, \overline{F}_t, P_t, Q_t\right)_{t\in[0,\infty)}$*, under which*

1) the fine upon detecting the issue or the evasion through an audit is set to its maximum level, i.e., $F_t = \overline{F}_t := F$*;*

2) the agent always prefers disclosure of the issue to evasion or self-correction, i.e., (6) holds with U_t *still evolving according to (7) and (8);*

3) the agent always prefers to disclose without delay, so that (9) and (10) hold;

4) the agent's total discounted expected cost remains the same, while the principal is not made worse off, compared to under policy $\widehat{\mathcal{P}}$*.*

Theorem 5 shows that to induce the agent's timely disclosure (i.e., the equilibrium outcome), the principal should simply maximize the threat (i.e., the off-equilibrium outcome) by setting the fine upon detection to the agent's limited liability $F_t = \overline{F}_t := F$ just as in our base model, regardless of whether an evasion is revealed. As a result, the threat to the agent U_t still follows the same evolution as in (7) and (8). As such, just like our base model, Theorem 5 essentially allows us to again reduce any policy $\left(F_t, \overline{F}_t, P_t, Q_t\right)$ simply to (P_t, Q_t) which satisfies (6), (9) and (10). Therefore, the principal's problem remains the same as (11) and all the results in Sections 5-8 still apply. In other words, the principal does not benefit from penalizing the non-disclosure of an issue differently when the agent also took evasive actions. In this sense, our base model is without loss of generality.

9.2. Third-party discovery

In our base model (Section 3), the principal has two information channels to uncover the adverse event, namely the agent's voluntary disclosure and the audit's detection. Other channels may exist, for instance, when a non-governmental organization or an independent news media uncovers the adverse event. In this section, we extend our model to incorporate such a setting. More specifically, we assume that the issue can be revealed by a third party with a constant probability rate μ after its occurrence regardless of whether an evasion has been taken. (This means that a not-yet-uncovered issue will come to the principal's awareness after an exponentially distributed period of time.) We focus on the case such that $\mu < \bar{\mu} := \min\{\lambda, \theta r/(F-r)^+\}$, i.e. the third-party discovery rate is not too high.[10] When μ takes sufficiently high values such that $\mu \geq \bar{\mu}$, the incentive problem discussed in this paper is arguably less relevant. In this case, the principal relies more on third parties to uncover the problem, rather than its own audit policy.[11]

The following proposition shows that the optimal policy in this extended setting can still be identified within our current framework.

PROPOSITION 2. *Let $(P_t^\star, Q_t^\star)_{t\in[0,\infty)}$ be the solution to (11) where the discount rate θ is replaced by $\theta+\mu$, the limited liability F by $\frac{\theta}{\theta+\mu}F$, the remedial cost r by $r - \frac{\mu}{\theta+\mu}F > 0$, the hazard rate λ by $\lambda - \mu > 0$, and the auditing cost k by $k(\lambda-\mu)/\lambda$. Then, the optimal policy $\widetilde{\mathcal{P}}^\star := \left(\widetilde{F}_t^\star, \widetilde{P}_t^\star, \widetilde{Q}_t^\star\right)_{t\in[0,\infty)}$ with an exogenous discovery rate μ is given by*

$$\widetilde{F}_t^\star = F, \quad \widetilde{P}_t^\star = P_t^\star + \frac{\mu}{\theta+\mu}F, \quad \textit{and} \quad \widetilde{Q}_t^\star = Q_t^\star. \tag{23}$$

In essence, the exogenous discovery channel plays two roles. First, it acts as a costless random audit (with constant rate μ and perfect detection probability), which helps the principal to reduce the agent's disclosure benefit and inflate the penalty $P_t^\star$ by $\frac{\mu}{\theta+\mu}F$ (i.e., the expected discounted penalty due to the exogenous discovery). Second, it acts to speed up the discounting as an exogenous discovery would immediately terminates the strategic interaction between the principal and the agent. Thus, the discount rate θ is inflated to $\theta+\mu$.

Proposition 2 immediately implies that the optimal policies we obtained for perfect evasions (Theorem 2) and for imperfect but sufficiently costly evasions (Theorem 3) can be re-parameterized as the optimal policies in the presence of exogenous discovery channel. For imperfect and inexpensive evasions (Section 7), we consider the class of *proportional* policies in the form $U_t = \gamma P_t$ for

[10] In particular, we have $\bar{\mu} = \lambda$ if $F \leq r$.

[11] Technically, it can be optimal for the principal to induce self-correction if $\mu \geq \frac{\theta r}{(F-r)^+}$. For $\lambda \leq \mu < \frac{\theta r}{(F-r)^+}$, the principal's problem can be reformulated as (11) in our base model, albeit with different discount rates for the principal and agent. Dynamic contract design problems with different discount rates between the principal and the agent involves more complex control, and is beyond the scope of this paper (see, e.g., Cao et al. 2023).

some constant γ. Under the re-parameterization identified in Proposition 2, we can generalize it to and optimize within the class of *affine* policies in the form $\widetilde{U}_t - \frac{\mu}{\theta+\mu}F = \gamma\left(\widetilde{P}_t - \frac{\mu}{\theta+\mu}F\right)$ in the presence of exogenous discovery channel.

9.3. Social welfare

In our current setting, the principal's objective is to minimize her total discounted cost in (4). This objective is reasonable for many settings when the principal is a self-interested party such as a private enterprise (e.g., Walmart). Yet, when the principal is a regulatory agency such as EPA, she may also care about the cost incurred by the agent in (3) and aims to minimize the total *social cost* (i.e., the principal is a social welfare maximizer). In this case, we follow the mainstream literature on regulation economics (e.g., Baron and Myerson 1982, Laffont and Tirole 1993), public economics (e.g., Dahlby 2008), and environmental regulations (e.g., Boyer and Laffont 1999, Lyon and Maxwell 2003, Wang et al. 2016) to assume that any cost incurred by the principal is $\alpha > 0$ times more expensive than that of the agent, where the fact α corresponds to the deadweight loss of applying public funds, and captures economic frictions created by regulation (e.g., by raising distortionary taxes).[12] As a result, the principal's problem (5) can be revised as

$$\min_{\mathcal{P}} (1+\alpha)C(\mathcal{P},\sigma) + C_a(\mathcal{P},\sigma), \quad \text{subject to} \quad C_a(\mathcal{P},\sigma) \le C_a(\mathcal{P},\sigma') \text{ for all } \sigma'. \tag{24}$$

PROPOSITION 3. *The solution to (24) is the same as that to (11) with auditing cost k replaced by $(1+1/\alpha)k$.*

Proposition 3 shows that the socially optimal policy can essentially be identified by re-parameterizing the principal's problem as the one in our base model. This is because the agent's problem and hence the IC constraint in (24) remains unchanged. As a result, we can still focus on the class of policies inducing the agent's prompt disclosure according to Theorem 1. In the principal's objective function, the principal's auditing cost is amplified by the factor α and the monetary transfer P_t is not completely canceled due to the deadweight loss.

9.4. Imperfectly informed agent

In our base model, the agent is assumed to be perfectly *informed* about the adverse issue once it occurs. However, it is plausible that the agent is genuinely unaware of the occurrence of the issue. Assume that the agent can observe the adverse event's occurrence only with a probability $\delta \in (0,1)$, and cannot find it with probability $1-\delta$. In the later case, the agent cannot disclose the issue even

[12] A reasonable estimate for α is significantly positive in the magnitude of 0.3 for the U.S. economy (see, for example, Ballard et al. 1985, Jones et al. 1990) for empirical estimations of α.

if it has occurred. However, we assume that an audit can still uncover the issue and whether it was observable to the agent. Let $\underline{F}$ be the penalty that the principal levies upon audit detection on the agent who did not observe the issue.[13] As in our base model, it is optimal for the principal to lever both disclosure penalty P_t and audits Q_t to induce the agent's prompt disclosure without evasion or self-correction.

In this alternative setting, the principal faces a trade-off concerning audits, which need to detect unobservable issues to the agent, while properly incentivizing the agent to disclose them when they are observable. Therefore, the principal's problem becomes

$$\begin{aligned}\min_{\mathcal{P}=(P_t,Q_t)_{t\in[0,\infty)}} \quad & \delta\left\{\frac{\lambda}{\theta+\lambda}r+\mathbb{E}\left[k\int_0^T e^{-\theta t}dN_t-e^{-\theta T}P_T\,\middle|\,\mathcal{P}\right]\right\}\\ & +(1-\delta)\mathbb{E}\left[k\int_0^T e^{-\theta t}dN_t+c\int_T^{\tau(T)}e^{-\theta t}dt+e^{-\theta\tau(T)}(k+r-\underline{F})\,\middle|\,\mathcal{P}\right] \qquad (25)\\ \text{subject to} \quad & (6)\text{–}(10).\end{aligned}$$

PROPOSITION 4. *If $\underline{F}=k+r-c/\theta$, then the solution to (25) is the same as that to (11) with auditing cost k replaced by k/δ.*

The sufficient condition $\underline{F}=k+r-c/\theta$ in Proposition 4 essentially charges the uninformed agent for the cost of detecting and repairing the issue but deducts the cost of harm to the principal caused by the issue due to its delayed detection. This condition acts to render the effects that the uninformed agent inflicts on the principal independent of the delay of detection $\tau(T)-T$. Otherwise, the principal's problem would be of a fundamentally different nature and needs a separate analytical treatment that we leave for future research.[14]

10. Conclusion

This paper studies the impact of audit evasion capabilities on the efficient audit and remedial strategies. We represent the evasion capability as a costly effort that reduces the audits' detection probability. To evaluate the impact of this capacity on auditing schedules, we allow very general classes of control policies, instead of restricting to a few specific structures. In particular, we do not assume a priori whether the policy is deterministic or random.

The presence of this evasion capability gives rise to a moral hazard problem in which the agent may self-repair or opt to exert effort aimed at avoiding detection. Further, the adverse issue's

[13] If the principal is not able to distinguish between whether the issue is observable or not to the agent, the agent can always claim to be uninformed even upon audit detection, effectively lowering the informed agent's limited liability to $\min\{F,\underline{F}\}$.

[14] If $\underline{F}\neq k+r-c/\theta$, the objective function in (25) involves evaluating $\mathbb{E}\left[e^{-\theta(\tau(T)-T)}\,\middle|\,T,\mathcal{P}\right]$ as shown in the proof of Proposition 4, where $\mathbb{P}[\tau(T)-T>t\,|\,T,\mathcal{P}]=\mathbb{P}[N_{t+T}-N_T=0\,|\,T,\mathcal{P}]=e^{-\int_T^{t+T}q_s^m ds}\prod_{s=T}^{t+T}(1-q_s^n)$. Hence, the objective function in (25) can no longer be expressed as an expectation of the integral with respect to dN_t.

occurrence is the agent's private information. Even if the agent does not actively evade audits, he may nonetheless decide not to disclose or remedy the problem immediately, which would cause damage to the principal. This gives rise to an adverse selection problem. And because the time at which the issue occurs is random, the problem is dynamic. As such, audits act as a threat and deter the agent from both taking evasive actions and delaying the disclosure of a violation.

Taken together, this situation corresponds to a dynamic principal-agent problem with costly state verification, adverse selection and moral hazard. We reformulate this problem as the stochastic optimal control of a Piecewise Deterministic Process. The analysis of this dynamic stochastic control problem yields two important new managerial insights.

First, the presence of an evasion capability may require the principal to run audits randomly. This contrasts with the deterministic audit schedules that are optimal when the agent cannot hide the issue from audits (but may still not disclose the issue voluntarily) (Wang et al. 2016). More specifically, the principal should randomly audit the agent, unless the agent's evasion capacity is not very effective and the agent cannot afford to self-correct the issue. In this later case, the principal should follow pre-determined audit schedules. In this sense, our findings provide a novel rationale for why audits are sometimes random in practice. Technically, the key driver for random audits in our set-up is the upper limit that the moral hazard problem imposes on the agent's contribution toward the remedial costs.

Second, as we increase the audit's probability of detection, the principal should audit the agent first more and then less frequently. This means, in particular, that an improvement in the agent's evasion capability can actually decrease the principal's audit costs (but always increases the principal's total cost).

Overall, our analysis yields a policy that is easy to understand and implement: the policy runs a series of random audits, but always motivates the agent to come clean. After each audit, the principal first offers a penalty reduction, which is discounted over time according to basic accounting principles. After a fixed amount of time, the penalty reduction stops changing and stays constant until the next inspection, which takes a simple exponentially distributed random time to occur.

Importantly, this policy outperforms any implementable audit schedules (including non-exponential inspection times, combinations of pre-scheduled audits with random inspections, etc.). In addition, this structure continues to hold when 1) different levels of penalties can be inflicted depending on whether or not the violation is accompanied with evasive actions, 2) a third party can independently uncover the violation, 3) the principal maximizes social welfare, 4) the agent may not be able to observe the issue's occurrence, 5) the penalty associated with a violation can take any finite value, 6) the agent's evasive actions can aggravate the environmental impact, 7) the cost of effort is either a lump sum or a flow overtime.

Our model can potentially be extended in a few other directions. In particular, our set-up could be generalized to account for n different effort levels, such that higher evasive effort reduces audits' efficacy. Our present model corresponds to a case where $n = 2$, with effort costs being h and r, respectively. We suspect that conditions exist for $n > 2$ such that the agent either evades at the highest intensity or not at all, in which case our results should hold. If not, the threat utility becomes multi-dimensional, which requires different and novel analytical approaches.

Another potential direction is to consider additional sources of adverse selection in our model. For instance, cost h could be the agent's private information. Alternatively, the agent may privately know upfront whether or not the agent will be able to observe the event's occurrence (similar to Baliga and Ely 2016). Accounting for these extensions require introducing agents of different types, which, following the Revelation Principle, requires the principal to offer a *menu* of dynamic contracts. The design of these contracts, in turn, requires representing the agent's dynamic optimal responses to the contract of each type. Overall, this yields a highly non-trivial problem,[15] which we leave for future research.

From a technical perspective, although the optimal control of general PDP is notoriously difficult, we solve this problem in closed form for a given class of PDP (Theorems 2 and 3). When the problem becomes intractable (as in Section 7), we optimize over a subset of policies and again solve the problem in closed form (Theorem 4). We accomplish this result by reducing the stochastic dynamic optimization problem into a deterministic one. Note also that the subset of policies we consider is quite general and focuses on policies that are implementable in practice.

Besides the problem of evading detection that we address in this paper, the optimal control of Piecewise Deterministic Markov Processes provides a fruitful framework to address other types of issues related to auditing. For example, in different situations, the agent does not exert effort to evade audits, but rather directly influences the likelihood of an adverse issue occurring. This can be modeled as the agent's effort level determining λ. Variations of our analytical framework could help study this and other settings related to audit and remedial strategies. The rise of sustainability and corporate social responsibility concerns is conferring increasing importance on these questions.

References

Abreu D, Pearce D, Staccheti E (1990) Toward a theory of discounted repeated games with imperfect monitoring. *Econometrica* 58(5):1041–1063.

[15] We can consider an example with pure dynamic moral hazard, arguably simpler than our setting (with both static moral hazard and dynamic adverse selection), studied in Sun and Tian (2018). The working paper Tian et al. (2023) attempts to generalize Sun and Tian (2018) to include an upfront static adverse selection such that the principal does not know the agent's effort cost. The results in Tian et al. (2023) are much more complex than those in Sun and Tian (2018). In fact, the optimal menu itself does not seem to possess tractable, let alone implementable, structures. So Tian et al. (2023) resorts to restricting contract spaces, and approximating the optimal solution by upper and lower bounding its performance.

Babich V, Tang C (2012) Managing opportunistic supplier adulteration: Deferred payments, inspection, and combined mechanisms. *M&SOM* 14(2):301–314.

Bakshi N, Gans N (2010) Securing the containerized supply chain–analysis of government incentives for private investment. *Management Sci.* 56(2):219–233.

Baliga S, Ely JC (2016) Torture and the commitment problem. *Rev. Econ. Stud.* 83(4):1406–1439.

Ballard C, Shoven J, Whalley J (1985) General equilibrium computations of the marginal welfare costs of taxes in the United State. *Amer. Econ. Rev.* 75(1):128–138.

Barlow RE, Proschan F (1996) *Mathematical theory of reliability* (Philadelphia: SIAM).

Baron D, Myerson RB (1982) Regulating a monopolist with unknown costs. *Econometrica* 50(4):911–930.

Becker GS (1968) Crime and punishment: An economic approach. *J. Polit. Economy* 72(2):169–217.

Bensoussan A, Lions JL (1982) *Impulse Control and Quasivariational Inequalities* (Bordas), translation of *Contrôle Impulsionnel et Inéquations Quasi-variationnelles.*

Biais B, Mariotti T, Rochet JC, Villeneuve S (2010) Large risks, limited liability, and dynamic moral hazrd. *Econometrica* 78(1):73–118.

Board S, Meyer-Ter-Vehn M (2013) Reputation for quality. *Econometrica* 81:2381–2462.

Boyer M, Laffont JJ (1999) Toward a political theory of the emergence of environmental incentive regulation. *RAND J. Econ.* 30(1):137–157.

Cao P, Sun P, Tian F (2023) Punish underperformance with suspension – optimal dynamic contracts in the presence of switching cost. *Management Sci.* Forthcoming.

Carreyrou J (2018) *Bad Blood: Secrets and Lies in a Silicon Valley Startup* (New York: Alfred A. Knopf).

Chehrazi N, Glynn PW, Weber TA (2019) Dynamic credit-collections optimization. *Management Sci.* 65(6):2737–2769.

Chen J, Ai A, Dawande D (2020) Supplier centrality and auditing priority in socially responsible supply chains. *M&SOM* 22(6):1199–1214.

Cho SH, Fang X, Tayur S (2015) Combating strategic counterfeiters in licit and illicit supply chains. *M&SOM* 17(3):273–289.

Clifford S, Greenhouse S (2013) Fast and flawed inspections of factories abroad. *New York Times* URL http://www.nytimes.com/2013/09/02/business/global/superficial-visits-and-trickery-undermine-foreign-factory-inspections.html, accessed on October 24, 2016.

Dahlby D (2008) *The Marginal Cost of Public Funds* (Cambridge, MA: MIT Press).

Dasgupta P, Hammond P, Maskin E (1979) The implementation of social choice rules. *Rev. Econ. Stud.* 46(2):185–216.

Davis MHA, Dempster MAH, Sethi SP, Vermes D (1987) Optimal capacity expansion under uncertainty. *Adv. Appl. Prob.* 19(1):156–76.

EPA (2000) *Incentives for Self-Policing: Discovery, Disclosure, Correction and Prevention of Violations.* U.S. Environmental Protection Agency, URL http://www.epa.gov/compliance/incentives/auditing/auditpolicy.html, accessed on February 27, 2014.

Ewing J (2016) VW presentation in '06 showed how to foil emissions tests. *New York Times* URL http://www.nytimes.com/2016/04/27/business/international/vw-presentation-in-06-showed-how-to-foil-emissions-tests.html?_r=0, accessed on October 24, 2016.

Hadler T (2013) Reward supplier honesty, not performance. *Network for Business Sustainability* URL https://nbs.net/reward-supplier-honesty-not-performance/, accessed on June 8, 2023.

Harrington W (1988) Enforcement leverage when penalties are restricted. *J. Public Econ.* 37(1):29–53.

Hwang I, Radhakrishnan S, Su L (2006) Vendor certification and appraisal: Implications for supplier quality. *Management Sci.* 52(10):1472–1482.

Jones LP, Tandon P, Vogelsang I (1990) *Selling Public Enterprise* (Cambridge, MA: MIT Press).

Kim SH (2015) Time to come clean? Disclosure and inspection policies for green production. *Oper. Res.* 63(1):1–20.

Kim Y, Xu Y (2023) Operational risk management: Optimal inspection policy. *Management Sci.* Forthcoming.

Lacker JM, Weinberg JA (1989) Optimal contracts under costly state falsification. *J. Polit. Econ.* 97(6):1345–1363.

Laffont JJ, Tirole J (1993) *A Theory of Incentives in Procurement and Regulation* (Cambridge, MA: MIT Press).

Levi R, Singhvi S, Zheng Y (2019) Economically motivated adulteration in farming supply chains. *Management Sci.* 66(1):209–226.

Li H, Zhang H, Fine CH (2013) Dynamic business share allocation in a supply chain with competing suppliers. *Oper. Res.* 61(2):280–297.

Ljungqvist L, Sargent TJ (2004) *Recursive Macroeconomic Theory* (Cambridge, MA: MIT Press).

Lyon TP, Maxwell JW (2003) Self-regulation, taxation and public voluntary environmental agreements. *J. Public Econ.* 87(7-8):1453–1486.

Malik AS (1990) Avoidance, screening and optimum enforcement. *Rand J. Econ.* 21(3):341–353.

Mookherjee D, Png I (1989) Optimal auditing, insurance, and redistribution. *Q. J. Econ.* 104(2):399–415.

Myerson R (1979) Incentive compatibility and the bargaining problem. *Econometrica* 47(1):61–73.

Myerson RB (2012) A model of moral-hazard credit cycles. *J. Polit. Economy* 120(5):847–878.

Parmigiani G (1993) Scheduling inspections in reliability. Basu AP, ed., *Advances in Reliability*, 303–319 (North Holland).

Plambeck E, Taylor T (2016) Supplier evasion of a buyer's audit: Implications for motivating supplier social and environmental responsibility. *M&SOM* 18(2):184–197.

Polinsky AM, Shavell S (2000) The economic theory of public enforcement of law. *J. Econ. Lit.* 38(1):45–76.

Ravikumar B, Zhang Y (2012) Optimal auditing insurance in a dynamic model of tax compliance. *Theor. Econ.* 7(2):241–282.

Sannikov Y (2008) A continuous-time version of the principal-agent problem. *Rev. Econ. Stud.* 75(3):957–984.

Spear S, Srivastava S (1987) On repeated moral hazard with discounting. *Rev. Econ. Stud.* 54(4):599–617.

Sun P, Tian F (2018) Optimal contract to induce continued effort. *Management Sci.* 64(9):4193–4217.

Tian F, Zhang F, Sun P, Duenyas I (2023) Dynamic moral hazard with adverse selection – probation, sign-on bonus and delayed payment, working paper, Duke University.

Townsend RM (1979) Optimal contracts and competitive markets with costly state verification. *J. Econ. Theory* 21(2):265–293.

Townsend RM (1988) Information constrained insurance: The revelation principle extended. *J. Monet. Econ.* 21(2-3):411–450.

Varas F, Marinovics I, Skrzypacz A (2020) Random inspections and periodic reviews: Optimal dynamic monitoring. *Rev. Econ. Stud.* 87:2893–2937.

Wang S, Sun P, de Véricourt F (2016) Inducing environmental disclosures: A dynamic mechanism design approach. *Oper. Res.* 64(2):371–389.

Appendix A: Definitions in Section 3 and Proofs in Section 4

We first formally define the audit policy by adopting Definition A.1 from Wang et al. (2016).

Definition A.1 (Audit Policy). Let dt and δ_t denote the usual Lebesgue measure and Dirac measure on time horizon $t \in [0,\infty)$, respectively. We call $\{q_t^n \in [0,\infty) : t \geq 0\}$ and $\{q_t^m \in [0,1] : t \geq 0\}$ an *intensity audit policy* and an *impulsive audit policy*, respectively, if

1. the process q_t^n and q_t^m are $\mathscr{F}_t$-predictable, where $\mathscr{F}_t$ is the filtration generated by N_t;
2. the measure $\mu(dt) := q_t^n dt + q_t^m \delta_t$ satisfies

$$\int_0^t \mu(ds) < \infty, \quad t \geq 0; \text{ and} \tag{A.1}$$

3. the measure $\mu(dt)$ consists of an $\mathscr{F}_t$-predictable compensator (e.g., Brémaud 1981, Lipster and Shiryaev 2010) for the counting process N_t, i.e.,

$$\mathbb{E}\left[\int_0^\infty X_t dN_t\right] = \mathbb{E}\left[\int_0^\infty X_t \mu(dt)\right] \tag{A.2}$$

for any bounded $\mathscr{F}_t$-predictable process X_t. □

Let random variable $Y_t \in \{0,1\}$ denote the result of an audit conducted at time t, with $Y_t = 0$ if an issue is detected and $Y_t = 1$ otherwise. Thus, the random process $Z_t = \Pi_{\tau \in \mathcal{I}_t} Y_\tau \in \{0,1\}$ denotes whether the agent survive the principal's audits by time t. Thus, Z_t starts with value 1 representing no detection by an audit up to time t and jumps down to value 0 once an audit detects an issue at time t. Furthermore, let the ternary process $H_t \in \{0, \pm 1\}$ denote whether the agent has taken any action: $H_t = 0$ before the agent makes any action (i.e., for all $t \leq \sigma(T)$); H_t enters the absorbing state 1 once the agent takes an evasive action (and the auditing accuracy reduces to β); H_t enters the absorbing state -1 once the agent conducts self-correction (and the auditing accuracy reduces to 0 and $Z_s := 1$ for all $s \geq t$). Using these notation, we have

$$\mathbb{P}[Y_t = 0 \mid T > t] = 0, \; \mathbb{P}[Y_t = 0 \mid T \leq t, Z_t = 1, H_t = 0] = 1, \text{ and } \mathbb{P}[Y_t = 0 \mid T \leq t, Z_t = 1, H_t = 1] = \beta. \tag{A.3}$$

In particular, having taken an evasive action at time t (i.e., $H_t = 1$), the agent's expected discounted cost onwards under policy $\mathcal{P} := (F_t, P_t, Q_t)_{t \in [0,\infty)}$ can be written as

$$U_t := \mathbb{E}\left[-\int_t^\infty e^{-\theta(\zeta - t)} F_\zeta dZ_\zeta \,\middle|\, T \leq t, Z_t = 1, H_t = 1, \mathcal{I}_t, \mathcal{P}\right]. \tag{A.4}$$

Correspondingly, the principal's expected total cost after the agent takes an evasive action from t onwards can be similarly computed as

$$V_t := \mathbb{E}\left[\int_t^\infty e^{-\theta(\zeta - t)} \left\{Z_\zeta (cd\zeta + kdN_\zeta) - (r - F_\zeta) dZ_\zeta\right\} \,\middle|\, T \leq t, Z_t = 1, H_t = 1, \mathcal{I}_t, \mathcal{P}\right]. \tag{A.5}$$

If a self-correction is conducted by time t, the principal will only incur auditing cost afterwards indefinitely (because no detection will ever occur), resulting an expected total cost of

$$W_t := \mathbb{E}\left[\int_t^\infty e^{-\theta(\zeta - t)} k dN_\zeta \,\middle|\, T \leq t, Z_t = 1, H_t = -1, \mathcal{I}_t, \mathcal{P}\right]. \tag{A.6}$$

Proof of Theorem 1. Denote $\widehat{U}_t$ and $\widehat{V}_t$ (resp., $\widehat{W}_t$) as the agent's and the principal's expected discounted cost from time t onwards under the policy $\widehat{\mathcal{P}} := \left(\widehat{F}_t, \widehat{P}_t, \widehat{Q}_t\right)_{t\in[0,\infty)}$ after the agent takes evasive action (resp., self-correction) at time t. Also, denote the agent's corresponding best response strategy as $\widehat{\sigma}^\star(\cdot)$, the optimal stopping time to take action (i.e., disclosure, evasion, or self-correction). According to (A.4), (A.5) and (A.6),

$$\widehat{U}_t := \mathbb{E}\left[-\int_t^\infty e^{-\theta(\zeta-t)}\widehat{F}_\zeta dZ_\zeta \,\middle|\, T\le t, Z_t=1, H_t=1, \mathcal{I}_t, \widehat{\mathcal{P}}\right], \tag{A.7}$$

$$\widehat{V}_t := \mathbb{E}\left[\int_t^\infty e^{-\theta(\zeta-t)}\left\{Z_\zeta\left(cd\zeta + kdN_\zeta\right) - (r-\widehat{F}_\zeta)dZ_\zeta\right\} \,\middle|\, T\le t, Z_t=1, H_t=1, \mathcal{I}_t, \widehat{\mathcal{P}}\right], \quad \text{and} \tag{A.8}$$

$$\widehat{W}_t := \mathbb{E}\left[\int_t^\infty e^{-\theta(\zeta-t)}kdN_\zeta \,\middle|\, T\le t, Z_t=1, H_t=-1, \mathcal{I}_t, \widehat{\mathcal{P}}\right]. \tag{A.9}$$

Now we construct an alternative policy $\mathcal{P} := (F_t, P_t, Q_t)_{t\in[0,\infty)}$ by letting $F_t := \widehat{F}_t$, $Q_t := \widehat{Q}_t$, and,

$$\begin{aligned} P_t :=& \min_{\widehat{\sigma}\ge t}\mathbb{E}\left[e^{-\theta(\widehat{\sigma}-t)}\min\left\{\widehat{P}_{\widehat{\sigma}}, h+\widehat{U}_{\widehat{\sigma}}, r\right\}Z_{\widehat{\sigma}} - \int_t^{\widehat{\sigma}} e^{-\theta(\zeta-t)}\widehat{F}_\zeta dZ_\zeta \,\middle|\, T\le t, Z_t=1, H_t=0, \mathcal{I}_t, \widehat{\mathcal{P}}\right],\\ =& \mathbb{E}\left[e^{-\theta(\widehat{\sigma}^\star(t)-t)}\min\left\{\widehat{P}_{\widehat{\sigma}^\star(t)}, h+\widehat{U}_{\widehat{\sigma}^\star(t)}, r\right\}Z_{\widehat{\sigma}^\star(t)} - \int_t^{\widehat{\sigma}^\star(t)} e^{-\theta(\zeta-t)}\widehat{F}_\zeta dZ_\zeta \,\middle|\, T\le t, Z_t=1, H_t=0, \mathcal{I}_t, \widehat{\mathcal{P}}\right], \quad \forall \mathcal{I}_t, \end{aligned} \tag{A.10}$$

namely P_t is the agent's minimum expected discounted cost from t onwards under policy $\widehat{\mathcal{P}}$, given that the issue has emerged ($T\le t$), no disclosure nor detection has occurred ($Z_t=1$), and the agent has not yet taken any evasive action ($H_t=0$). Under this specification of $\mathcal{P}$, we immediately have $U_t=\widehat{U}_t$ by (A.4) and (A.7), $V_t=\widehat{V}_t$ by (A.5) and (A.8), and $W_t=\widehat{W}_t$ by (A.6) and (A.9), for all $\mathcal{I}_t$.

Now we demonstrate that the above-defined policy $\mathcal{P}$ satisfies the following properties.

Property 1: $\mathcal{P}$ is well defined (i.e., $P_t\le F$) and, in particular, $P_t\le\min\{h+U_t, r\}$ for all t, suggesting that the agent always weakly prefers disclosure to evasion and self-correction, namely (6). Indeed, it is obvious that F_t and Q_t is, by construction, well defined. By the definition of P_t in (A.10), we immediately note that, since $\widehat{P}_t\le F$ and $\widehat{F}_t\le F$,

$$P_t \le F\mathbb{E}\left[e^{-\theta(\widehat{\sigma}^\star(t)-t)}Z_{\widehat{\sigma}} - \int_t^{\widehat{\sigma}^\star(t)} e^{-\theta(\zeta-t)}dZ_\zeta \,\middle|\, T\le t, Z_t=1, H_t=0, \mathcal{I}_t, \widehat{\mathcal{P}}\right] \le F.$$

The property that $P_t\le\min\{h+U_t, r\}$ follows from the optimality of $\widehat{\sigma}^\star$ in (A.10): $P_t\le\min\left\{\widehat{P}_t, h+\widehat{U}_t, r\right\}\le \min\{h+U_t, r\}$.

Property 2: Prompt disclosure is the agent's best response to $\mathcal{P}$ (in the sense of weakly dominant strategy). Indeed, we have, for all $s\ge t$,

$$\begin{aligned} P_t =& \mathbb{E}\left[e^{-\theta(\widehat{\sigma}^\star(t)-t)}\min\left\{\widehat{P}_{\widehat{\sigma}^\star(t)}, h+\widehat{U}_{\widehat{\sigma}^\star(t)}, r\right\}Z_{\widehat{\sigma}^\star(t)} - \int_t^{\widehat{\sigma}^\star(t)} e^{-\theta(\zeta-t)}\widehat{F}_\zeta dZ_\zeta \,\middle|\, T\le t, Z_t=1, H_t=0, \mathcal{I}_t, \widehat{\mathcal{P}}\right]\\ \le& \mathbb{E}\left[e^{-\theta(\widehat{\sigma}^\star(s)-t)}\min\left\{\widehat{P}_{\widehat{\sigma}^\star(s)}, h+\widehat{U}_{\widehat{\sigma}^\star(s)}, r\right\}Z_{\widehat{\sigma}^\star(s)} - \int_t^{\widehat{\sigma}^\star(s)} e^{-\theta(\zeta-t)}\widehat{F}_\zeta dZ_\zeta \,\middle|\, T\le t, Z_t=1, H_t=0, \mathcal{I}_t, \widehat{\mathcal{P}}\right]\\ =& \mathbb{E}\left[e^{-\theta(s-t)}Z_sP_s - \int_t^s e^{-\theta(\zeta-t)}\widehat{F}_\zeta dZ_\zeta \,\middle|\, T\le t, Z_t=1, H_t=0, \mathcal{I}_t, \widehat{\mathcal{P}}\right]\\ =& \mathbb{E}\left[e^{-\theta(s-t)}Z_sP_s - \int_t^s e^{-\theta(\zeta-t)}F_\zeta dZ_\zeta \,\middle|\, T\le t, Z_t=1, H_t=0, \mathcal{I}_t, \mathcal{P}\right], \end{aligned} \tag{A.11}$$

where the first equality follows from the construction of P_t in (A.10), the first inequality follows from the optimality of $\widehat{\sigma}^\star$ in (A.10), the second equality follows by splitting the time interval $[t,\widehat{\sigma}^\star(s)]$ into $[t,s]$ and $[s,\widehat{\sigma}^\star(s)]$ and the construction of P_s in (A.10), and the last equality follows from the construction that $F_t=\widehat{F}_t$. The right-hand side of (A.11) is nothing but the agent's total expected discounted cost of delaying the disclosure to any stopping time $s\geq t$ under $\mathcal{P}$ (by Property 1, there is no incentive to evade or self-correct at any point in time under $\mathcal{P}$). As such, the agent always prefers to disclose without delay. By taking unconditional expectation on both sides of (A.11) immediately yields $C_a(\mathcal{P},T)\leq C_a(\mathcal{P},\sigma)$ for all σ.

Property 3: The principal is not worse off under $\mathcal{P}$ than under $\widehat{\mathcal{P}}$. We first note that since $c\geq\theta r$ and $k\geq 0$, (A.8) implies that

$$\begin{aligned}\widehat{V}_t=&\mathbb{E}\left[\int_t^\infty e^{-\theta(\zeta-t)}\Big\{Z_\zeta\left(cd\zeta+kdN_\zeta\right)-(r-\widehat{F}_\zeta)dZ_\zeta\Big\}\,\Big|\,T\leq t,Z_t=1,H_t=1,\mathcal{I}_t,\widehat{\mathcal{P}}\right]\\ \geq& r\mathbb{E}\left[\theta\int_t^\infty e^{-\theta(\zeta-t)}Z_\zeta d\zeta-\int_t^\infty e^{-\theta(\zeta-t)}dZ_\zeta\,\Big|\,T\leq t,Z_t=1,H_t=1,\mathcal{I}_t,\widehat{\mathcal{P}}\right]\\ &+\mathbb{E}\left[\int_t^\infty e^{-\theta(\zeta-t)}\widehat{F}_\zeta dZ_\zeta\,\Big|\,T\leq t,Z_t=1,H_t=1,\mathcal{I}_t,\widehat{\mathcal{P}}\right]\\ =&r-\widehat{U}_t,\end{aligned}\tag{A.12}$$

where the last equality follows from the definition (A.4) and the direct calculation:

$$e^{-\theta(t'-t)}Z_{t'}-1=\int_t^{t'}e^{-\theta(\zeta-t)}dZ_\zeta-\theta\int_t^{t'}e^{-\theta(\zeta-t)}Z_\zeta d\zeta,\quad\text{for } Z_t=1,\ \text{and then let } t'\to\infty.\tag{A.13}$$

Then, by (4), the principal's expected cost under policy $\widehat{\mathcal{P}}$ is given by

$$\begin{aligned}C\left(\widehat{\mathcal{P}},\widehat{\sigma}^\star\right)=&\mathbb{E}\Bigg[k\int_0^T e^{-\theta t}dN_t+c\int_T^{\widehat{\sigma}^\star(T)\wedge\widehat{\tau}(T)}e^{-\theta t}dt+\mathbb{1}_{\{\widehat{\sigma}^\star(T)>\widehat{\tau}(T)\}}e^{-\theta\widehat{\tau}(T)}\left(k+r-\widehat{F}_{\widehat{\tau}(T)}\right)\\ &+\mathbb{1}_{\{\widehat{\sigma}^\star(T)\leq\widehat{\tau}(T)\}}e^{-\theta\widehat{\sigma}^\star(T)}\Big\{\mathbb{1}_{\{\widehat{P}_{\widehat{\sigma}^\star(T)}\leq\min\{h+\widehat{U}_{\widehat{\sigma}^\star(T)},r\}\}}\left(r-\widehat{P}_{\widehat{\sigma}^\star(T)}\right)\\ &\quad+\mathbb{1}_{\{r<\widehat{P}_{\widehat{\sigma}^\star(T)},r\leq h+\widehat{U}_{\widehat{\sigma}^\star(T)}\}}W_{\widehat{\sigma}^\star(T)}+\mathbb{1}_{\{h+\widehat{U}_{\widehat{\sigma}^\star(T)}<\min\{r,\widehat{P}_{\widehat{\sigma}^\star(T)}\}\}}V_{\widehat{\sigma}^\star(T)}\Big\}\,\Big|\,\widehat{\mathcal{P}},\widehat{\sigma}^\star\Bigg].\\ \geq&\mathbb{E}\Bigg[k\int_0^T e^{-\theta t}dN_t+c\int_T^{\widehat{\sigma}^\star(T)\wedge\widehat{\tau}(T)}e^{-\theta t}dt+\mathbb{1}_{\{\widehat{\sigma}^\star(T)>\widehat{\tau}(T)\}}e^{-\theta\widehat{\tau}(T)}\left(k+r-\widehat{F}_{\widehat{\tau}(T)}\right)\\ &+\mathbb{1}_{\{\widehat{\sigma}^\star(T)\leq\widehat{\tau}(T)\}}e^{-\theta\widehat{\sigma}^\star(T)}\Big\{\mathbb{1}_{\{\widehat{P}_{\widehat{\sigma}^\star(T)}\leq\min\{h+\widehat{U}_{\widehat{\sigma}^\star(T)},r\}\}}\left(r-\widehat{P}_{\widehat{\sigma}^\star(T)}\right)\\ &\quad+\mathbb{1}_{\{r<\widehat{P}_{\widehat{\sigma}^\star(T)},r\leq h+\widehat{U}_{\widehat{\sigma}^\star(T)}\}}(r-r)+\mathbb{1}_{\{h+\widehat{U}_{\widehat{\sigma}^\star(T)}<\min\{r,\widehat{P}_{\widehat{\sigma}^\star(T)}\}\}}\left(r-\widehat{U}_{\widehat{\sigma}^\star(T)}\right)\Big\}\,\Big|\,\widehat{\mathcal{P}},\widehat{\sigma}^\star\Bigg]\\ \geq&\mathbb{E}\Bigg[k\int_0^T e^{-\theta t}dN_t+r\left(e^{-\theta T}-e^{-\theta\widehat{\sigma}^\star(T)\wedge\widehat{\tau}(T)}\right)+\mathbb{1}_{\{\widehat{\sigma}^\star(T)>\widehat{\tau}(T)\}}e^{-\theta\widehat{\tau}(T)}\left(r-\widehat{F}_{\widehat{\tau}(T)}\right)\\ &+\mathbb{1}_{\{\widehat{\sigma}^\star(T)\leq\widehat{\tau}(T)\}}e^{-\theta\widehat{\sigma}^\star(T)}\Big\{\mathbb{1}_{\{\widehat{P}_{\widehat{\sigma}^\star(T)}\leq\min\{h+\widehat{U}_{\widehat{\sigma}^\star(T)},r\}\}}\left(r-\widehat{P}_{\widehat{\sigma}^\star(T)}\right)\\ &\quad+\mathbb{1}_{\{r<\widehat{P}_{\widehat{\sigma}^\star(T)},r\leq h+\widehat{U}_{\widehat{\sigma}^\star(T)}\}}(r-r)+\mathbb{1}_{\{h+\widehat{U}_{\widehat{\sigma}^\star(T)}<\min\{r,\widehat{P}_{\widehat{\sigma}^\star(T)}\}\}}\left(r-\widehat{U}_{\widehat{\sigma}^\star(T)}\right)\Big\}\,\Big|\,\widehat{\mathcal{P}},\widehat{\sigma}^\star\Bigg]\\ =&\mathbb{E}\Bigg[k\int_0^T e^{-\theta t}dN_t+e^{-\theta T}\Big\{r-\mathbb{1}_{\{\widehat{\sigma}^\star(T)>\widehat{\tau}(T)\}}e^{-\theta(\widehat{\tau}(T)-T)}\widehat{F}_{\widehat{\tau}(T)}\\ &-\mathbb{1}_{\{\widehat{\sigma}^\star(T)\leq\widehat{\tau}(T)\}}e^{-\theta(\widehat{\sigma}^\star(T)-T)}\Big\{\mathbb{1}_{\{\widehat{P}_{\widehat{\sigma}^\star(T)}\leq\min\{h+\widehat{U}_{\widehat{\sigma}^\star(T)},r\}\}}\widehat{P}_{\widehat{\sigma}^\star(T)}+\mathbb{1}_{\{r<\widehat{P}_{\widehat{\sigma}^\star(T)},r\leq h+\widehat{U}_{\widehat{\sigma}^\star(T)}\}}r\\ &\quad+\mathbb{1}_{\{h+\widehat{U}_{\widehat{\sigma}^\star(T)}<\min\{r,\widehat{P}_{\widehat{\sigma}^\star(T)}\}\}}\widehat{U}_{\widehat{\sigma}^\star(T)}\Big\}\Big\}\,\Big|\,\widehat{\mathcal{P}},\widehat{\sigma}^\star\Bigg]\\ \geq&\mathbb{E}\Bigg[k\int_0^T e^{-\theta t}dN_t+e^{-\theta T}\Big\{r-\Big[\mathbb{1}_{\{\widehat{\sigma}^\star(T)>\widehat{\tau}(T)\}}e^{-\theta(\widehat{\tau}(T)-T)}\widehat{F}_{\widehat{\tau}(T)}\end{aligned}$$

$$+\mathbb{1}_{\{\widehat{\sigma}^\star(T)\le\widehat{\tau}(T)\}}e^{-\theta(\widehat{\sigma}^\star(T)-T)}\min\left\{\widehat{P}_{\widehat{\sigma}^\star(T)},h+\widehat{U}_{\widehat{\sigma}^\star(T)},r\right\}\Big]\Big\}\,\Big|\,\widehat{\mathcal{P}},\widehat{\sigma}^\star\Big]$$

$$=\mathbb{E}\left[k\int_0^T e^{-\theta t}dN_t+e^{-\theta T}\left(r-P_T\right)\,\middle|\,\mathcal{P}\right]=C(\mathcal{P},T), \tag{A.14}$$

where the first inequality follows from (A.12) and the fact that $W_{\widehat{\sigma}^\star(T)}\ge r-r=0$, the second inequality follows from the fact that $k\ge 0$ and $c/\theta\ge r$, the second last equality follows from the construction of $\mathcal{P}$ (particularly P_t in (A.10)), and the last equality follows from (4) and by recognizing that the agent always prefers to disclose the issue under $\mathcal{P}$ once it occurs at T by Property 1 and 2 above.

Property 4: It is optimal for the principal to set $F_t:=F$ for all $t\ge 0$, which immediately yields the recursive representation of (A.4) and (A.11) in (7), (8), (9) and (10) by following a similar derivation as in Lemma 1 of Wang et al. (2016). Indeed, we note that the variable F_t is absent from the principal's expected discounted cost (A.14). Therefore, it is optimal for the principal to relax the constraints (A.11) and (6) to the extent that is allowed. The limited liability constraint $F_t\le F$ hence suggests the optimality of setting $F_t:=F$.

Property 5: Policy $\mathcal{P}$ and policy $\widehat{\mathcal{P}}$ are payoff-equivalent to the agent, i.e., $C_a(\mathcal{P},T)=C_a(\widehat{\mathcal{P}},\widehat{\sigma}^\star)$. This is because, by construction in (A.10), P_t is the agent's minimum expected discounted cost from t onwards under policy $\widehat{\mathcal{P}}$ by following $\widehat{\sigma}^\star$ as the response. On the other hand, by Properties 1 and 2, the agent will always promptly disclose at time T under policy $\mathcal{P}$ and hence incur the same expected cost P_t, leading to the conclusion. □

Lemma A.1. *The optimal policy must satisfy* $0\le\beta P_t\le U_t\le P_t\le\bar{h}$.

Proof of Lemma A.1. To show $P_t\ge 0$ in the optimum, we first note that $P_t:=0$ and $U_t:=0$ for all $t\ge 0$ always satisfy (6)–(10) with $q_t^m=q_t^n=0$, which results in no auditing cost. Therefore, any policy with $P_t<0$ is dominated by the policy with $P_t=U_t=0$.

To see $\beta P_t\le U_t$, we first note that by definition (A.4), $U_t\in[0,F]$. Denote $D_t=U_t-\beta P_t$ and then, by (7)-(10), we have

$$\begin{aligned}&(1-q_t^m)(D_{t+}-D_t)\le -q_t^m\left[(1-\beta)U_{t+}^I-D_t\right], \quad \text{for } q_t^m>0, \quad \text{and}\\ &D_{t+}\le D_t, \quad \text{or} \quad \frac{dD_t}{dt}\le(\theta+q_t^n)D_t-q_t^n(1-\beta)U_{t+}^I, \quad \text{for } q_t^m=0.\end{aligned}$$

Thus, if $D_{t_0}<0$ for some t_0, then D_t will be decreasing in $t\ge t_0$ and $D_t\to-\infty$ as $t\to\infty$, which must imply that $P_t\to\infty$, leading to a contradiction.

Finally, to see that $U_t\le P_t$, it suffice to argue that a policy with $P_t:=U_t$ for all $t\ge 0$ is incentive feasible, because it dominates any policy with $P_t<U_t$ (as the principal would like to elicit a payment P_t as high as possible). Indeed, it is obvious that $P_t:=U_t$ satisfies (6); and (7)-(8) imply

$$\begin{aligned}U_t&=(1-q_t^m)U_{t+}+q_t^m\left(\beta F+(1-\beta)U_{t+}^I\right)\le(1-q_t^m)U_{t+}+q_t^mF, \quad \text{for} \quad q_t^m>0, \quad \text{and}\\ \frac{dU_t}{dt}&=\theta U_t-q_t^n\left[\beta F+(1-\beta)U_{t+}^I-U_t\right]\ge\theta U_t-q_t^n\left[F-U_t\right], \quad \text{for} \quad q_t^m=0.\end{aligned}$$

Thus, $P_t:=U_t$ also satisfies (9)-(10). Finally, $P_t\le\min\{r,F\}$ follows from the limited liability constraint and (6). □

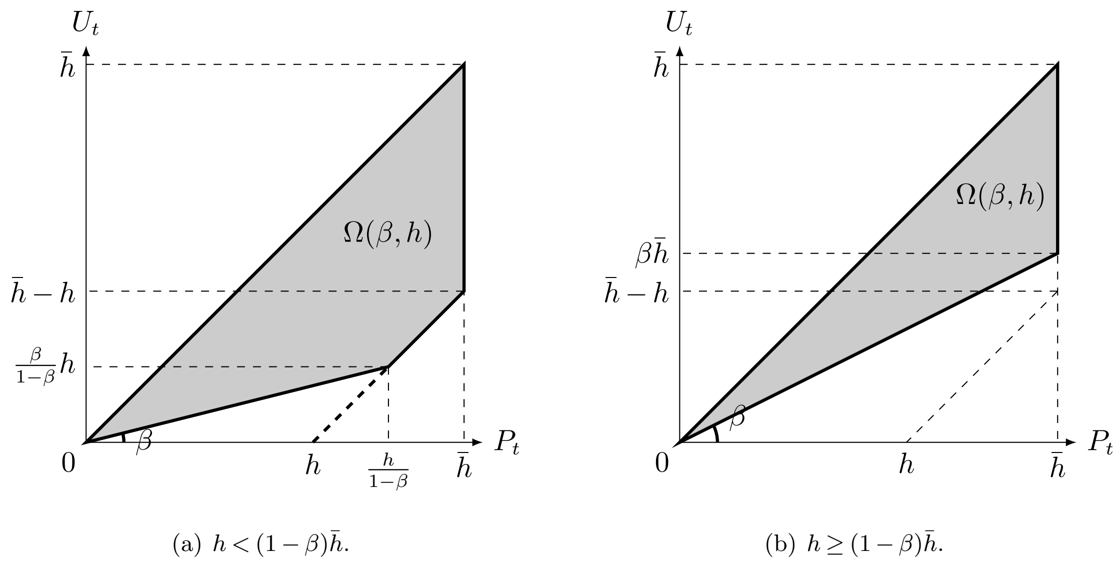


Figure A.1 Feasible range of (P_t,U_t) (shaded area).

REMARK A.1. Intuitively, because evasion reduces the audit effectiveness, the threat that the principal will be able to impose on the evading agent, namely U_t, cannot exceed the payment she is able to charge from the agent without any evasion, namely P_t. While an audit after an evasion can only detect the issue with probability β per each audit, repeated audits allow the principle to impose a threat U_t higher than βP_t.

Together with (6), Lemma A.1 allows us to narrow down the feasible region of (P_t,U_t) to $\Omega(h,\beta):=\left\{(p,u): 0\leq\beta p\leq u\leq p\leq\bar{h},\ p\leq h+u\right\}$, which is illustrated in Figure A.1. As will become evident later, the boundary of $\Omega(h,\beta)$ critically determines the binding constraints in the optimal policy, and hence play an important role in shaping the optimal policy. In particular, as can be seen from Figure 1(b), the obedience constraint (6) will never be active when either evasion is too costly or less effective so that $h\geq(1-\beta)\bar{h}$, suggesting the irrelevance of the moral hazard issue due to the agent's evasion. Indeed, Theorem 3 characterizes the *exact* condition $h\geq\widehat{h}(\beta)$, under which the principal's optimal policy does not bind the obedience constraint (6). For $h<\widehat{h}(\beta)$, however, (P_t,U_t) can move in a plethora of trajectories in the feasible region of Figure 1(a), making the characterization of the optimal policy extremely challenging. □

Appendix B: Proofs in Sections 5 and 6

LEMMA B.1 **(Verification of Optimality)**. *For a constant $B\leq F$, the policy $\mathcal{P}:=(P_t,Q_t)_{t\in[0,\infty)}$ solves*

$$\frac{\lambda}{\theta+\lambda}r+\min_{\mathcal{P}=(P_t,Q_t)_{t\in[0,\infty)}}\mathbb{E}\left[k\int_0^T e^{-\theta t}dN_t-e^{-\theta T}P_T\,\middle|\,\mathcal{P}\right],\quad \text{subject to } P_t\leq B,\ (9)\ \text{and}\ (10),\tag{B.1}$$

if the principal's cost-to-go function under $\mathcal{P}$,

$$C(\mathcal{I}_t,t):=\frac{\lambda}{\theta+\lambda}r+\mathbb{E}\left[k\int_t^T e^{-\theta(\zeta-t)}dN_\zeta-e^{-\theta(T-t)}P_T\,\middle|\,\mathcal{I}_t,t\leq T\right],\tag{B.2}$$

satisfies the following properties:

Property 1: $C(\mathcal{I}_t,t)$ depends on $(\mathcal{I}_t,t)$ only through P_t and can hence be denoted as $C(\mathcal{I}_t,t)=C(P_t)$. That is, $C(\mathcal{I}_t,t)=C\left(\widehat{\mathcal{I}}_{\hat{t}},\hat{t}\right)$ for any $(\mathcal{I}_t,t)$ and $\left(\widehat{\mathcal{I}}_{\hat{t}},\hat{t}\right)$ such that $P_t(\mathcal{I}_t)=P_{\hat{t}}(\widehat{\mathcal{I}}_{\hat{t}})$.

Property 2: $C(p)$ is bounded, non-decreasing, and continuously differentiable in $p \in [0, B]$.

Property 3: $C(p)$ satisfies

$$\lambda(r-p) - (\theta+\lambda)C(p) + \mathcal{N}C(p) \geq 0 \quad \text{and} \quad \mathcal{M}C(p) - C(p) \geq 0, \quad \text{for } p \in [0, B], \tag{B.3}$$

where the functional operators $\mathcal{N}$ and $\mathcal{M}$ are defined as

$$\mathcal{N}C(p) := \min_{\substack{p_+^I \leq B \\ q^n \geq 0, z \geq 0}} q^n \left[k + C\left(p_+^I\right) - C(p)\right] + \{\theta p - q^n [F-p] + z\} \frac{dC(p)}{dp}, \quad \text{and} \tag{B.4}$$

$$\begin{aligned} \mathcal{M}C(p) := \min_{\substack{p_+^I \leq B \\ q^m \in [0,1], z \geq 0}} \quad & q^m \left(k + C\left(p_+^I\right)\right) + (1-q^m) C(p_+) \\ \text{subject to} \quad & p = (1-q^m)p_+ + q^m F - z. \end{aligned} \tag{B.5}$$

Proof of Lemma B.5. By item (3) of Theorem 1, P_t is a controlled piecewise deterministic process and hence Markovian (Davis 1993) with $\left(P_{t+}^I, Q_t, z_t\right)$ as the control variables, which uniquely determine the evolution of P_t according to (9) and (10). Therefore, for any $\mathcal{I}_{t_0}$ and $\widehat{\mathcal{I}}_{t_0}$ that yields the same $P_{t_0} = p$, $(P_t)_{t \geq t_0}$ will follow the same trajectory under the same control $\left(P_{t+}^I, Q_t, z_t\right)_{t \geq t_0}$. We further note that the principal's *optimal* cost-to-go function (in the current value) starting from any $(\mathcal{I}_{t_0}, t_0)$ (assuming $t_0 \leq T$) can be rewritten as

$$\begin{aligned} \widehat{C}^\star(\mathcal{I}_{t_0}, t_0) := \frac{\lambda}{\lambda+\theta} r + \min_{\left(P_{t+}^I, Q_t, z_t\right)_{t \geq t_0}} & e^{(\lambda+\theta)t_0} \mathbb{E}\left[\int_{t_0}^{\infty} e^{-(\lambda+\theta)t} \left(kq_t^n - \lambda P_t\right) dt + \sum_{t \geq t_0, q_t^m > 0} e^{-(\lambda+\theta)t} k q_t^m \,\middle|\, \mathcal{I}_{t_0}\right] \\ & \text{subject to } P_t \leq B, \text{ (9) and (10)}, \end{aligned}$$

where we use the property of T being exponential distribution and the definition of audits. As such, the objective function above depends on $\mathcal{I}_t$ only through P_t and Q_t.

Therefore, for any $\mathcal{I}_{t_0}$ and $\widehat{\mathcal{I}}_{t_0}$ that yields the same $P_{t_0} = p$, $\widehat{C}^\star(\mathcal{I}_{t_0}, t_0) = \widehat{C}^\star(\widehat{\mathcal{I}}_{t_0}, t_0) =: \widetilde{C}^\star(p, t_0)$ is given by

$$\begin{aligned} \widetilde{C}^\star(p, t_0) := \frac{\lambda}{\lambda+\theta} r + \min_{\left(P_{t+}^I, Q_t, z_t\right)_{t \geq t_0}} & e^{(\lambda+\theta)t_0} \mathbb{E}\left[\int_{t_0}^{\infty} e^{-(\lambda+\theta)t} \left(kq_t^n - \lambda P_t\right) dt + \sum_{t \geq t_0, q_t^m > 0} e^{-(\lambda+\theta)t} k q_t^m \,\middle|\, P_{t_0} = p\right] \\ \text{subject to } P_t \leq B, \text{ (9) and (10)}. \end{aligned} \tag{B.6}$$

That is, (P_t, t) can be used as the (payoff-relevant) state variables for the principal's problem.

By a time shifting, we can further rewrite (B.6) as follows

$$\begin{aligned} \widetilde{C}^\star(p, t_0) = \frac{\lambda}{\lambda+\theta} r + & \min_{\left(P_{(t+t_0)+}^I, Q_{t+t_0}, z_{t+t_0}\right)_{t \geq 0}} \mathbb{E}\left[\int_0^{\infty} e^{-(\lambda+\theta)t} \left(kq_{t+t_0}^n - \lambda P_{t+t_0}\right) dt + \sum_{t \geq 0, q_{t+t_0}^m > 0} e^{-(\lambda+\theta)t} k q_{t+t_0}^m \,\middle|\, P_{t_0} = p\right] \\ & \text{subject to } P_{t+t_0} \leq B; \quad P_{t+t_0} = (1-q_{t+t_0}^m) P_{(t+t_0)+} + q_{t+t_0}^m F - z_{t+t_0}, \text{ if } q_{t+t_0}^m > 0; \text{and} \\ & \qquad P_{t+t_0} = P_{(t+t_0)+} - z_{t+t_0}, \quad \text{or} \quad \frac{dP_{t+t_0}}{dt} = \theta P_{t+t_0} - q_{t+t_0}^n [F - P_{t+t_0}] + z_{t+t_0}, \text{ if } q_{t+t_0}^m = 0. \end{aligned}$$

By the virtue of the Markovian property of P_t, we immediately see that $\widetilde{C}^\star(p, t_0) = \widetilde{C}^\star(p, 0) = C^\star(p)$, where

$$C^\star(p) := \frac{\lambda}{\lambda+\theta} r + \min_{\left(P_{t+}^I, Q_t, Z_t\right)_{t \geq 0}} \mathbb{E}\left[\int_0^{\infty} e^{-(\lambda+\theta)t} \left(kq_t^n - \lambda P_t\right) dt + \sum_{t \geq 0, q_t^m > 0} e^{-(\lambda+\theta)t} k q_t^m \,\middle|\, P_0 = p\right]$$

$$\text{subject to } P_t \le B, \quad P_t = (1-q_t^m)P_{t+} + q_t^m F - z_t, \text{ if } q_t^m > 0, \text{and} \qquad (B.7)$$
$$P_t = P_{t+} - z_t, \quad \text{or} \quad \frac{dP_t}{dt} = \theta P_t - q_t^n\left[F - P_t\right] + z_t, \text{ if } q_t^m = 0.$$

That is, the optimal cost-to-go function is in fact time-homogeneous.

Now suppose that the principal's current-value cost-to-go function $C(\mathcal{I}_t, t)$ satisfies the three properties listed in the lemma; in particular, $C(\mathcal{I}_t, t) = C(P_t)$. Thus, the optimality of $C^\star(p)$ immediately implies that $C(p) \ge C^\star(p)$ for all $p \le B$. We now demonstrate that $C(p) \le C^\star(p)$ also holds for all $p \le B$, which immediately implies that the policy $\mathcal{P}$ is optimal for the principal.

Let $\left(\widetilde{P}^I_{t+}, \widetilde{Q}_t, \widetilde{z}_t\right)_{t\ge 0}$ be an arbitrary admissible policy, under which the transfer trajectory $\widetilde{P}_t$ is given by

$$\left.\begin{array}{ll}\widetilde{P}_t = (1-\tilde{q}_t^m)\widetilde{P}_{t+} + \tilde{q}_t^m F - \tilde{z}_t, & \text{if} \quad t \in \widetilde{\Gamma} \\ \frac{d\widetilde{P}_t}{dt} = \theta \widetilde{P}_t - \tilde{q}_t^n\left[F - \widetilde{P}_t\right] + \tilde{z}_t, & \text{if} \quad t \notin \widetilde{\Gamma}\end{array}\right\} \quad \forall t \notin \mathcal{I}_{[0,\infty)}, \quad \text{and } \widetilde{P}_t \text{ is reset to } \widetilde{P}^I_{t+}, \text{ for } t \in \mathcal{I}_{[0,\infty)}, \qquad (B.8)$$

where $\widetilde{\Gamma} := \{t \ge 0 : \tilde{q}_t^m > 0 \text{ or } P_{t+} > P_t\} = \{\nu_1, \nu_2, \cdots\}$ with $\nu_0 = 0$. Therefore, by Davis (1993, Theorem 31.3 and 31.9), we have[16]

$$\begin{aligned}
&\mathbb{E}^p\left[e^{-(\lambda+\theta)\nu_j} C\left(\widetilde{P}_{\nu_j}\right) - e^{-(\lambda+\theta)\nu_{j-1}} C\left(\widetilde{P}_{\nu_{j-1}+}\right)\right] \\
=&\mathbb{E}^p\left[\int_{\nu_{j-1}}^{\nu_j} e^{-(\lambda+\theta)t}\left\{\theta\widetilde{P}_t - \tilde{q}_t^n\left[F - \widetilde{P}_t\right] + \tilde{z}_t^n\right\}\frac{dC\left(\widetilde{P}_t\right)}{dt}dt\right] \\
&+\mathbb{E}^p\left[\int_{\nu_{j-1}}^{\nu_j} e^{-(\lambda+\theta)t}\left\{\tilde{q}_t^n\left[C\left(\widetilde{P}^I_{t+}\right) - C\left(\widetilde{P}_t\right)\right] - (\lambda+\theta)C\left(\widetilde{P}_t\right)\right\}dt\right] \\
\ge&\mathbb{E}^p\left[\int_{\nu_{j-1}}^{\nu_j} e^{-(\lambda+\theta)t}\left(-k\tilde{q}_t^n + \mathcal{N}C(\widetilde{P}_t) - (\lambda+\theta)C\left(\widetilde{P}_t\right)\right)dt\right] \\
\ge& -\mathbb{E}^p\left[\int_{\nu_{j-1}}^{\nu_j} e^{-(\lambda+\theta)t}\left(\lambda(r - \widetilde{P}_t) + k\tilde{q}_t^n\right)dt\right],
\end{aligned} \qquad (B.9)$$

where the first inequality follows from the definition of operator $\mathcal{N}$ in(B.4) and the second inequality follows from the first inequality of(B.3).

We then observe that, between two consecutive intervention time epochs,

$$\begin{aligned}
&\mathbb{E}^p\left[e^{-(\lambda+\theta)\nu_j}\left(C\left(\widetilde{P}_{\nu_j+}\right) - C\left(\widetilde{P}_{\nu_j}\right)\right)\right] \\
=&\mathbb{E}^p\left[\mathbb{E}^p\left[e^{-(\lambda+\theta)\nu_j}\left(C\left(\widetilde{P}_{\nu_j+}\right) - C\left(\widetilde{P}_{\nu_j}\right)\right) \,\middle|\, \widetilde{P}_{\nu_j}\right]\right] \\
=&\mathbb{E}^p\left[\mathbb{E}^p\left[e^{-(\lambda+\theta)\nu_j}\left(\tilde{q}^m_{\nu_j} C\left(\widetilde{P}^I_{\nu_j+}\right) + (1-\tilde{q}^m_{\nu_j})C\left(\widetilde{P}_{\nu_j+}\right) - C\left(\widetilde{P}_{\nu_j}\right)\right) \,\middle|\, \widetilde{P}_{\nu_j}\right]\right] \\
\ge&\mathbb{E}^p\left[\mathbb{E}^p\left[e^{-(\lambda+\theta)\nu_j}\left(-k\tilde{q}^m_{\nu_j} + \mathcal{M}C(\widetilde{P}_{\nu_j}) - C(\widetilde{P}_{\nu_j})\right) \,\middle|\, \widetilde{P}_{\nu_j}\right]\right] \\
\ge& -\mathbb{E}^p\left[e^{-(\lambda+\theta)\nu_j} k\tilde{q}^m_{\nu_j}\right],
\end{aligned} \qquad (B.10)$$

where the first equality follows from the tower rule of expectation operator, the second one from the fact that $\widetilde{P}_{\nu_j}$ is rest to $\widetilde{P}^I_{\nu_j+}$ with probability $\tilde{q}^m_{\nu_j}$ (in which case an audit takes place) and to $\widetilde{P}_{\nu_j+}$ with probability $1 - \tilde{q}^m_{\nu_j}$ (in which case no audit takes place) subject to the first constraint in(B.8), the first inequality from the definition of operator $\mathcal{M}$ in(B.5) and the second inequality from the second inequality in(B.3).

[16] We denote $\mathbb{E}^p[\cdot] := \mathbb{E}\left[\cdot \,\middle|\, \widetilde{P}_0 = p\right]$.

For any $p \le B$ and $n = 1, 2, \ldots$, we can then make the following decomposition:

$$\begin{aligned} & C(p) - \mathbb{E}^p\left[e^{-(\lambda+\theta)\nu_n} C\left(P_{\nu_n}\right)\right] \\ =& \sum_{j=1}^{n} \mathbb{E}^p\left[e^{-(\lambda+\theta)\nu_{j-1}} C\left(\widetilde{P}_{\nu_{j-1}+}\right) - e^{-(\lambda+\theta)\nu_j} C\left(\widetilde{P}_{\nu_j}\right)\right] + \sum_{j=0}^{n-1} \mathbb{E}^p\left[e^{-(\lambda+\theta)\nu_j}\left(C\left(\widetilde{P}_{\nu_j}\right) - C\left(\widetilde{P}_{\nu_j+}\right)\right)\right], \\ \le& \mathbb{E}^p\left[\int_0^{\nu_n} e^{-(\lambda+\theta)t}\left(\lambda(r-\widetilde{P}_t) + k\tilde{q}_t^n\right) dt + \sum_{j=0}^{n-1} e^{-(\lambda+\theta)\nu_j} k\tilde{q}_{\nu_j}^m\right], \end{aligned} \tag{B.11}$$

where the last inequality follows from(B.9) and (B.10).

Since $\lim_{n\to\infty} \nu_n = \infty$ with probability one by the admissibility of policy $\left(\widetilde{P}_{t+}^I, \widetilde{Q}_t, \widetilde{z}_t\right)_{t\ge0}$ and $C(\cdot)$ is bounded, letting n go to infinity in (B.11) yields

$$C(p) \le \frac{\lambda}{\lambda+\theta} r + \mathbb{E}^p\left[\int_0^{\infty} e^{-(\lambda+\theta)t}\left(k\tilde{q}_t^n - \lambda\widetilde{P}_t\right) dt + \sum_{j=0}^{\infty} e^{-(\lambda+\theta)\nu_j} k\tilde{q}_{\nu_j}^m\right],$$

which, by (B.7), implies $C(p) \le C^\star(p)$ for $p \le B$ due to the arbitrariness of $\left(\widetilde{P}_{t+}^I, \widetilde{Q}_t, \widetilde{z}_t\right)_{t\ge0}$. □

LEMMA B.2. *There exists a unique solution $t^\star > 0$ to (12), which is increasing in $h \in [0, \bar{h}]$.*

Proof. Let $f(t) := \theta F + \lambda(F-h) + \lambda h e^{-(\lambda+\theta)t} - (\lambda+\theta)(k+F)e^{-\lambda t}$. Then, (12) is equivalent to $f(t^\star) = 0$. The existence and uniqueness of $t^\star$ thus follow from the straightforward verification that $f(0) = -(\lambda+\theta)k < 0$, $f(\infty) = \theta F + \lambda(F-h) > 0$, and $f(t)$ is increasing:

$$f'(t) = \lambda(\lambda+\theta)e^{-\lambda t}\left(k + F - he^{-\theta t}\right) \ge \lambda(\lambda+\theta)e^{-\lambda t}(k + F - h) > 0.$$

To see that $t^\star$ is increasing in h, it suffices to note that $f(t)$ is decreasing in h for any given t. □

LEMMA B.3. *The policies prescribed in Theorems 2 and 3 both satisfy (9) and (10), and furthermore, satisfy $P_t^\star \le h$ and $P_t^\star \le \bar{h}$, respectively.*

Proof. Under the policy prescribed in Theorem 2, it is straightforward to verify:

- For any $t \in (\tau_{i-1}, \tau_{i-1} + t^\star]$ and i, (13) implies that $P_t^\star \le h$ and

$$\frac{dP_t^\star}{dt} = \theta h e^{\theta(t-\tau_i-t^\star)} = \theta P_t^\star,$$

which is essentially the binding constraint (10) by noticing $q_t^{n\star} := 0$ during $(\tau_{i-1}, \tau_{i-1} + t^\star]$.

- For any $t \in (\tau_{i-1} + t^\star, \tau_i]$ and i, (14) and $P_t^\star := h$ imply that

$$\frac{dP_t^\star}{dt} = 0 = \theta h - \frac{\theta h}{F-h}(F-h) = \theta P^\star - q^{n\star}(F - P_t^\star),$$

which is again essentially the binding constraint (10).

Under the policy prescribed in Theorem 3, it is also straightforward to verify:

- For any $t \in (\tau_{i-1}, \tau_{i-1} + t^\circ]$ and i, (**??**) implies that $P_t^\star \le \bar{h}$ and

$$\frac{dP_t^\star}{dt} = \theta\bar{h}e^{-\theta(\tau_i - t)} = \theta P_t^\star,$$

which is essentially the binding constraint (10) as $q_t^{n\star} := 0$. In particular, if $r \ge F$, we have $\tau_i = \tau_{i-1} + t^\circ$ and $P_{\tau_i}^\star = F$. Thus, (9) holds with equality at those impulsive audit epochs τ_i.

- If $r < F$, for any $t \in (\tau_{i-1} + t^\circ, \tau_i]$ and i, (16) and $P_t^\star := r$ imply that

$$\frac{dP_t^\star}{dt} = 0 = \theta r - \frac{\theta r}{F - r}(F - r) = \theta P^\star - q^{n\star}(F - P_t^\star),$$

which is again essentially the binding constraint (10).

Lemma B.4. *The current-value cost-to-go functions under the policies prescribed in Theorems 2 and 3 depend on the past history $(\mathcal{I}_t, t)$ only through $P_t^\star$ and are given by a bounded, strictly convex increasing, and continuously differentiable function*

$$C(p) = \frac{\lambda}{\lambda+\theta} r + \kappa^\star p^{\frac{\lambda+\theta}{\theta}} - p, \text{ for } p \in [\underline{p}^\star, h], \text{ with } \kappa^\star = \frac{1}{h^{\frac{\lambda+\theta}{\theta}}} \frac{k + F - he^{-\theta t^\star}}{\frac{\lambda+\theta}{\theta}\frac{F-h}{h} + 1 - e^{-(\lambda+\theta)t^\star}} \text{ and } \underline{p}^\star = he^{-\theta t^\star}, \text{ and} \tag{B.12}$$

$$C(p) = \frac{\lambda}{\lambda+\theta} r + \kappa^\star p^{\frac{\lambda+\theta}{\theta}} - p, \text{ for } p \in [\underline{p}^\star, \bar{h}], \text{ with } \kappa^\star = \frac{1}{\bar{h}^{\frac{\lambda+\theta}{\theta}}} \frac{k + F - \bar{h}e^{-\theta t^\circ}}{\frac{\lambda+\theta}{\theta}\frac{F-\bar{h}}{\bar{h}} + 1 - e^{-(\lambda+\theta)t^\circ}} \text{ and } \underline{p}^\star = \bar{h}e^{-\theta t^\circ}, \text{ respectively.} \tag{B.13}$$

In particular, both functions satisfy

$$C(\underline{p}^\star) = \frac{\lambda}{\lambda+\theta}(r - \underline{p}^\star) \quad \text{and} \quad \frac{dC(\underline{p}^\star)}{dp} = \frac{\lambda+\theta}{\theta}\kappa^\star (\underline{p}^\star)^{\frac{\lambda}{\theta}} - 1 = 0. \tag{B.14}$$

Proof. It is clear that the policies in Theorems 2 and 3 are prescribed purely as a function of $P_t^\star$, and hence its current-value cost-to-go function depends on $(\mathcal{I}_t, t)$ only through $P_t^\star$. Let $B = h$ and $B = F$ under policies prescribed in Theorems 2 and 3, respectively. Then, (13) and $P_t^\star = Fe^{-\theta(\tau_i - t)}$ imply that $P_t^\star$ evolves deterministically according to $P_t^\star = pe^{\theta t}$ staring from any $P_0^\star = p$ before reaching the threshold B, which takes $\tau(p) := \frac{1}{\theta}\ln\frac{B}{p}$ amount of time, i.e., $P_{\tau(p)}^\star = B$. No audit ($q_t^{m\star} = q_t^{n\star} := 0$) is conducted between $[0, \tau(p))$. Therefore, we compute the cost-to-go function as follows:

$$\begin{aligned}
C(p) =& \mathbb{E}\left[\int_0^\infty e^{-(\lambda+\theta)t}\left(kq_t^{n\star} + \lambda(r - P_t^\star)\right)dt + \sum_{t \geq 0, q_t^{m\star} > 0} e^{-(\lambda+\theta)t} kq_t^{m\star} \,\middle|\, P_0^\star = p\right] \\
=& \int_0^{\tau(p)} \lambda e^{-(\lambda+\theta)t}\left[r - pe^{\theta t}\right]dt + e^{-(\lambda+\theta)\tau(p)} C(B) \\
=& \frac{\lambda}{\lambda+\theta} r\left(1 - e^{-(\lambda+\theta)\tau(p)}\right) - p\left(1 - e^{-\lambda\tau(p)}\right) + e^{-(\lambda+\theta)\tau(p)} C(B) \\
=& \frac{\lambda r}{\lambda+\theta} + \left[C(B) - \frac{\lambda r}{\lambda+\theta} + B\right]\left(\frac{p}{B}\right)^{\frac{\lambda+\theta}{\theta}} - p.
\end{aligned} \tag{B.15}$$

Furthermore, direct calculation reveals

$$\frac{dC(p)}{dp} = \frac{1}{B}\frac{\lambda+\theta}{\theta}\left[C(B) - \frac{\lambda r}{\lambda+\theta} + B\right]\left(\frac{p}{B}\right)^{\lambda/\theta} - 1, \tag{B.16}$$

which is increasing in p. Therefore, $C(p)$ is strictly convex in p. It is also straightforward to verify

$$C(p) = -\frac{\theta}{\lambda+\theta} p \frac{dC(p)}{dp} + \frac{\lambda}{\lambda+\theta}(r - p) = \frac{\lambda}{\lambda+\theta}(r - p). \tag{B.17}$$

- Under the policy prescribed in Theorem 2, $B = h$ and an intensity audit with constant rate prescribed in (14) is used while maintaining $P_t^\star := h$, which suggests that

$$C(B) = C(h) = \int_0^\infty e^{-(\theta+\lambda+q^{n\star})t}\left\{\lambda(r - h) + q^{n\star}\left[k + C(\underline{p}^\star)\right]\right\}dt$$

$$=\frac{1}{\frac{\lambda+\theta}{q^{n\star}}+1}\left[\frac{\lambda}{q^{n\star}}(r-h)+k+C(he^{-\theta t^\star})\right]$$
$$=\frac{1}{\frac{\lambda+\theta}{\theta}\frac{F-h}{h}+1}\left\{\frac{\lambda}{\theta}\frac{F-h}{h}(r-h)+k+\frac{\lambda r}{\lambda+\theta}+\left[C(h)-\frac{\lambda r}{\lambda+\theta}+h\right]e^{-(\lambda+\theta)t^\star}-he^{-\theta t^\star}\right\},$$

where the first equality follows from the fact that $P_t^\star$ is reset to $\underline{p}^\star$ right after an audit and we use (B.15) to obtain the last equality. From the above equality, we can solve for

$$C(h)-\frac{\lambda r}{\lambda+\theta}+h=\frac{k+F-he^{-\theta t^\star}}{\frac{\lambda+\theta}{\theta}\frac{F-h}{h}+1-e^{-(\lambda+\theta)t^\star}},$$

which renders (B.15) to (B.12). Now substituting $\underline{p}^\star=he^{-\theta t^\star}$ into (B.16) yields

$$\frac{dC(\underline{p}^\star)}{dp}=\frac{dC(he^{-\theta t^\star})}{dp}=\frac{(\lambda+\theta)\left(k+F-he^{-\theta t^\star}\right)}{(\lambda+\theta)(F-h)+\theta h\left(1-e^{-(\lambda+\theta)t^\star}\right)}e^{-\lambda t^\star}-1$$
$$=\frac{\lambda h\left(1-e^{-(\lambda+\theta)t^\star}\right)-(\lambda+\theta)\left[F-(k+F)e^{-\lambda t^\star}\right]}{(\lambda+\theta)(F-h)+\theta h\left(1-e^{-(\lambda+\theta)t^\star}\right)}=0, \tag{B.18}$$

where the last equality follows from the fact that $t^\star$ is the solution to (12). Thus, we obtain the second equation in (B.14), which, together with the convexity of $C(p)$, suggests that $C(p)$ is strictly increasing in $p\in[\underline{p}^\star,h]$. By (B.17), this also leads to the first equation of (B.14).

- Under the policy prescribed in Theorem 3, $B=\bar{h}$. If $r<F$, then $B=r$ and the proof follows the same argument as in the case of Theorem 2 above, where we replace h with r and $t^\star$ with t° given by (**??**). Otherwise ($r\geq F$), an impulsive audit is conducted at $P_t^\star=F$, suggesting that

$$C(F)=k+C(\underline{p}^\star)=k+C\left(Fe^{-\theta t^\circ}\right)=k+\frac{\lambda r}{\lambda+\theta}+\left(C(F)-\frac{\lambda r}{\lambda+\theta}+F\right)e^{-(\lambda+\theta)t^\circ}-Fe^{-\theta t^\circ},$$

where the first equality follows from the fact that $P_t^\star$ is reset to $\underline{p}^\star$ right after an audit and we use (B.15) to obtain the last equality. From the above equality, we can solve for

$$C(F)-\frac{\lambda r}{\lambda+\theta}+F=\frac{k+F-Fe^{-\theta t^\circ}}{1-e^{-(\lambda+\theta)t^\circ}},$$

which renders (B.15) to (B.13). Now substituting $\underline{p}^\star=Fe^{-\theta t^\circ}$ into (B.16) yields

$$\frac{dC(\underline{p}^\star)}{dp}=\frac{dC(Fe^{-\theta t^\circ})}{dp}=\frac{(\lambda+\theta)\left(k+F-Fe^{-\theta t^\circ}\right)}{\theta F\left(1-e^{-(\lambda+\theta)t^\circ}\right)}e^{-\lambda t^\circ}-1$$
$$=\frac{\lambda F\left(1-e^{-(\lambda+\theta)t^\circ}\right)-(\lambda+\theta)\left[F-(k+F)e^{-\lambda t^\circ}\right]}{\theta F\left(1-e^{-(\lambda+\theta)t^\circ}\right)}=0, \tag{B.19}$$

where the last equality follows from the fact that t° is the solution to (**??**). Thus, we obtain the second equation in (B.14), which, together with the convexity of $C(p)$, suggests that $C(p)$ is strictly increasing in $p\in[\underline{p}^\circ,F]$. By (B.17), this also leads to the first equation of (B.14). ☐

Lemma B.5. *We extend the cost-to-go functions $C(p)$ in (B.12) and (B.13) by defining $C(p):=C\left(\underline{p}^\star\right)$ for $p\in\left[0,\underline{p}^\star\right]$. Then, the extended $C(p)$ satisfies (B.3).*

Proof of Lemma B.5. The extended cost-to-go functions $C(p)$ can be written as

$$C(p)=\begin{cases}\frac{\lambda}{\lambda+\theta}r+\kappa^\star p^{\lambda/\theta+1}-p, & \text{for} \quad p\in\left[\underline{p}^\star,B\right],\\ C\left(\underline{p}^\star\right), & \text{for} \quad p\in\left[0,\underline{p}^\star\right],\end{cases} \tag{B.20}$$

where $B=h$ and $B=\bar{h}$ under policies prescribed in Theorems 2 and 3, respectively, and $C\left(\underline{p}^\star\right)$ is given by (B.14).

We first show $\lambda p-(\theta+\lambda)C(p)+\mathcal{N}C(p)\geq 0$, where the functional operator $\mathcal{N}$ is defined in (B.4).

- For $p \le \underline{p}^\star$, $C(p)$ is a constant $C\left(\underline{p}^\star\right)$, which is the minimal value of $C(p)$ with $\frac{d}{dp}C(\underline{p}^\star)=0$. Hence, by definition (B.4),

$$\lambda(r-p)-(\lambda+\theta)C(p)+\mathcal{N}C(p) = \lambda(r-p)-(\lambda+\theta)C\left(\underline{p}^\star\right)+\min_{p_+^I\le B, q^n\ge 0} q^n\left[k+C(p_+^I)-C\left(\underline{p}^\star\right)\right]$$
$$= \lambda(r-p)-(\lambda+\theta)C\left(\underline{p}^\star\right)+\min_{q^n\ge 0} kq^n=\lambda\left(\underline{p}^\star-p\right)\ge 0,$$

where the last equality follows from $C\left(\underline{p}^\star\right)=\frac{\lambda}{\lambda+\theta}(r-\underline{p}^\star)$ according to (B.14).

- For $p\in\left(\underline{p}^\star, B\right]$, we have

$$\begin{aligned}
&\lambda(r-p)-(\lambda+\theta)C(p)+\mathcal{N}C(p)\\
=&\lambda(r-p)-(\lambda+\theta)C(p)+\theta p\frac{dC(p)}{dp}+\min_{p_+^I\le B, q^n\ge 0} q^n\left[k+C\left(p_+^I\right)-C(p)+(p-F)\frac{dC(p)}{dp}\right]\\
=&\lambda(r-p)-(\lambda+\theta)C(p)+\theta p\frac{dC(p)}{dp}+\min_{q^n\ge 0} q^n\left[k+C(\underline{p}^\star)-C(p)+(p-F)\frac{dC(p)}{dp}\right],
\end{aligned}\tag{B.21}$$

where the first equality follows from the definition (B.4) (with $z=0$ therein because $\frac{dC(p)}{dp}\ge 0$ by Lemma B.4), the second equality follows from the fact that $C(p)$ reaches its minimum value of $C\left(\underline{p}^\star\right)$ at $\underline{p}^\star$ by Lemma B.4.

For $p\in\left(\underline{p}^\star, B\right]$, since $C\left(\underline{p}^\star\right)=\frac{\lambda}{\lambda+\theta}(r-\underline{p}^\star)$ by (B.14), direct calculation reveals

$$\begin{aligned}
&k+C\left(\underline{p}^\star\right)-C(p)+(p-F)\frac{dC(p)}{dp}\\
=&k+\frac{\lambda}{\lambda+\theta}\left(r-\underline{p}^\star\right)-\frac{\lambda}{\lambda+\theta}r-\kappa^\star p^{\lambda/\theta+1}+p+(p-F)\left[\frac{(\lambda+\theta)}{\theta}\kappa^\star p^{\lambda/\theta}-1\right]\\
=&k+F-\frac{\lambda}{\lambda+\theta}\underline{p}^\star-\frac{\kappa^\star}{\theta}\,p^{\lambda/\theta}\left[(\lambda+\theta)F-\lambda p\right]\\
\ge&k+F-\frac{\lambda}{\lambda+\theta}\underline{p}^\star-\frac{\kappa^\star}{\theta}B^{\lambda/\theta}\left[(\lambda+\theta)F-\lambda B\right],
\end{aligned}\tag{B.22}$$

where the last inequality follows by letting $p=B$ because the function $p^{\lambda/\theta}\left[(\lambda+\theta)F-\lambda p\right]$ is decreasing in p:

$$\frac{d}{dp}\left\{p^{\lambda/\theta}\left[(\lambda+\theta)F-\lambda p\right]\right\}=\frac{\lambda(\lambda+\theta)}{\theta}p^{\lambda/\theta-1}(F-p)\ge 0 \quad \text{for } p\le B\le F.$$

—In the case of (B.12), we have $B=h$ and hence

$$k+F-\frac{\lambda}{\lambda+\theta}\underline{p}^\star-\frac{\kappa^\star}{\theta}B^{\lambda/\theta}\left[(\lambda+\theta)F-\lambda B\right]\tag{B.23}$$
$$=k+F-\frac{\lambda}{\lambda+\theta}he^{-\theta t^\star}-\frac{k+F-he^{-\theta t^\star}}{\frac{\lambda+\theta}{\theta}\frac{F-h}{h}+1-e^{-(\lambda+\theta)t^\star}}\left[\frac{\lambda+\theta}{\theta}\frac{F}{h}-\frac{\lambda}{\theta}\right]$$
$$=\frac{1}{\lambda+\theta}\frac{e^{-\theta t^\star}}{\frac{\lambda+\theta}{\theta}\frac{F-h}{h}+1-e^{-(\lambda+\theta)t^\star}}\left[(\lambda+\theta)\left[F-(k+F)e^{-\lambda t^\star}\right]-\lambda h\left(1-e^{-(\lambda+\theta)t^\star}\right)\right]=0,\tag{B.24}$$

where the last equality follows from the fact that $t^\star$ is the solution to (12).

—In the case of (B.13), we have $B=\bar{h}$ and hence

$$k+F-\frac{\lambda}{\lambda+\theta}\underline{p}^\star-\frac{\kappa^\star}{\theta}B^{\lambda/\theta}\left[(\lambda+\theta)F-\lambda B\right]\tag{B.25}$$
$$=k+F-\frac{\lambda}{\lambda+\theta}\bar{h}e^{-\theta t^\circ}-\frac{k+F-\bar{h}e^{-\theta t^\circ}}{\frac{\lambda+\theta}{\theta}\frac{F-\bar{h}}{\bar{h}}-e^{-(\lambda+\theta)t^\circ}}\left[\frac{\lambda+\theta}{\theta}\frac{F}{\bar{h}}-\frac{\lambda}{\theta}\right]$$
$$=\frac{1}{\lambda+\theta}\frac{e^{-\theta t^\circ}}{\frac{\lambda+\theta}{\theta}\frac{F-\bar{h}}{\bar{h}}-e^{-(\lambda+\theta)t^\circ}}\left[(\lambda+\theta)\left[F-(k+F)e^{-\lambda t^\circ}\right]-\lambda\bar{h}\left(1-e^{-(\lambda+\theta)t^\circ}\right)\right]=0,\tag{B.26}$$

where the last equality follows from the fact that t° is the solution to (**??**).

Combining (B.21) with (B.24) and (B.26), we must have $\min_{q^n\geq 0} q^n\left[k+C(\underline{p}^\star)-C(p)+(p-F)\frac{dC(p)}{dp}\right]=0$, reducing (B.21) to

$$\begin{aligned}\lambda(r-p)-(\lambda+\theta)C(p)+\mathcal{N}C(p)&=\lambda(r-p)-(\lambda+\theta)C(p)+\theta p\frac{dC(p)}{dp}\\ &=\lambda(r-p)-(\lambda+\theta)\left[\frac{\lambda}{\lambda+\theta}r+\kappa^\star p^{\lambda/\theta+1}-p\right]+\theta p\left[\frac{(\lambda+\theta)}{\theta}\kappa^\star p^{\lambda/\theta}-1\right]=0.\end{aligned}$$

That is, $\lambda(r-p)-(\theta+\lambda)C(p)+\mathcal{N}C(p)\geq 0$ holds with equality for $p\in[\underline{p}^\star,B]$.

We then show $\mathcal{M}C(p)-C(p)\geq 0$. The functional operator $\mathcal{M}$ is defined by(B.5) and can be rewritten as

$$\mathcal{M}C(p):=\min_{q^m\in[0,1]}\Upsilon(q^m\mid p):=q^m\left(k+C\left(\underline{p}^\star\right)\right)+(1-q^m)C\left(\max\left\{\frac{p-q^mF}{1-q^m},\underline{p}^\star\right\}\right),\tag{B.27}$$

where we use the fact that $C(p)$ is increasing in $p\geq\underline{p}^\star$ and reaches its minimal value of $C\left(\underline{p}^\star\right)$ at $p=\underline{p}^\star$ by Lemma B.4.

- For $p\leq\underline{p}^\star$, the minimality of $\underline{p}^\star$ suggests that

$$\begin{aligned}C(p):=C\left(\underline{p}^\star\right)\leq&q^m\left(k+C\left(\underline{p}^\star\right)\right)+(1-q^m)C\left(\underline{p}^\star\right)\\ \leq&q^m\left(k+C\left(\underline{p}^\star\right)\right)+(1-q^m)C\left(\max\left\{\frac{p-q^mF}{1-q^m},\underline{p}^\star\right\}\right),\ \forall q^m\in[0,1].\end{aligned}$$

Therefore, by definition(B.27), we must have $\mathcal{M}C(p)-C(p)\geq 0$.

- We now consider $p\in\left(\underline{p}^\star,B\right]$. If $\frac{p-q^mF}{1-q^m}\leq\underline{p}^\star$, or, equivalently, $q^m\geq(p-\underline{p}^\star)/(F-\underline{p}^\star)$, then $\Upsilon(q^m\mid p)$ reduces to

$$\Upsilon(q^m\mid p)=kq^m+C\left(\underline{p}^\star\right),$$

which is obviously increasing in q^m. Thus, we can restrict the search for the minimizer of $\Upsilon(q^m\mid p)$ within $q^m\leq(p-\underline{p}^\star)/(F-\underline{p}^\star)$, or equivalently, $\hat{p}:=\frac{p-q^mF}{1-q^m}\geq\underline{p}^\star$, in which case

$$\Upsilon(q^m\mid p)=q^m\left[k+C\left(\underline{p}^\star\right)\right]+(1-q^m)C\left(\frac{p-q^mF}{1-q^m}\right),$$

which is also increasing in q^m because its derivative with respect to q^m can be calculated as

$$\frac{d}{dq^m}\Upsilon(q^m\mid p)=k+C\left(\underline{p}^\star\right)-C(\hat{p})+(\hat{p}-F)\frac{dC(\hat{p})}{dp}\geq 0,$$

where the last inequality follows from the same argument as in (B.21), (B.24) and (B.26). Altogether, we have shown that $\Upsilon(q^m\mid p)$ is monotonically increasing in q^m, and hence, by (B.27), $\mathcal{M}C(p)=\Upsilon(0\mid p)=C(p)$, implying that $\mathcal{M}C(p)-C(p)\geq 0$ holds with equality for $p\in\left(\underline{p}^\star,B\right]$. □

Proof of Theorem 2. By definition in A.3, if $\beta=0$, then we have $\mathbb{P}[Z_\zeta=1\mid T\leq t,Z_t=1,H_t=1]=1$ for all $\zeta\geq t$. Therefore, it follows from the definition of U_t in (A.4) that $U_t:=0$ for all $t\geq 0$. As a result, the constraint (6) reduces to $P_t\leq h$ and hence the principal's problem (11) reduces to (B.1) with $B=h$. Then, Lemmas B.4 and B.5 immediately imply the optimality of the policy prescribed in Theorem 2. □

Proof of Corollary 1. The principal's cost $C^\star$ immediately follows from the first equation in (B.14) with $\underline{p}^\star=he^{-\theta t^\star}$. To compute the agent's cost, we denote the agent's cost-to-go function as $C_a(p):=\mathbb{E}\left[e^{-\theta(T-t)}P_T^\star\,\middle|\,P_t^\star=p,t<T\right]$. Then, letting $r=k=0$ in (B.12) yields $-C_a(p)$, namely

$$C_a(p)=p-\frac{1}{h^{\frac{\lambda+\theta}{\theta}}}\frac{F-he^{-\theta t^\star}}{\frac{\lambda+\theta}{\theta}\frac{F-h}{h}+1-e^{-(\lambda+\theta)t^\star}}p^{\frac{\lambda+\theta}{\theta}}.\tag{B.28}$$

Thus, we can easily obtain the agent's cost as follows:

$$\begin{aligned} C_a^\star = C_a(\underline{p}^\star) =& \underline{p}^\star - \frac{1}{h^{\frac{\lambda+\theta}{\theta}}} \frac{F - he^{-\theta t^\star}}{\frac{\lambda+\theta}{\theta}\frac{F-h}{h} + 1 - e^{-(\lambda+\theta)t^\star}} \left(\underline{p}^\star\right)^{\frac{\lambda+\theta}{\theta}} \\ =& \left[1 - \frac{\theta}{\lambda+\theta}\frac{F - he^{-\theta t^\star}}{k + F - he^{-\theta t^\star}}\right]\underline{p}^\star = \frac{\lambda}{\lambda+\theta}he^{-\theta t^\star} + \frac{\theta}{\lambda+\theta}\frac{khe^{-\theta t^\star}}{k+F-he^{-\theta t^\star}}, \end{aligned}$$

where the second equality follows from the second equation in (B.14) and the third equality follows from $\underline{p}^\star = he^{-\theta t^\star}$. According to (A.14), we then have

$$\frac{\lambda}{\lambda+\theta}(r - \underline{p}^\star) = C^\star = \frac{\lambda}{\theta+\lambda}r + \underbrace{\mathbb{E}\left[k\int_0^T e^{-\theta t} dN_t \,\middle|\, Q^\star\right]}_{A^\star} - \underbrace{\mathbb{E}\left[e^{-\theta T}P_T^\star\right]}_{C_a^\star}$$

which immediately implies that $A^\star = C_a^\star - \frac{\lambda}{\lambda+\theta}he^{-\theta t^\star} = \frac{\theta}{\lambda+\theta}\frac{khe^{-\theta t^\star}}{k+F-he^{-\theta t^\star}}$. □

Proof of Theorem 3. We first note that (B.1) with $B = \bar{h}$ is a relaxed problem of (11) with constraints (6)–(8) ignored, whose solution is given by the policy prescribed in Theorem 3, as implied by Lemmas B.4 and B.5.

We now complete the proof of Theorem 3 by showing that this policy automatically satisfies the ignored constraints (6)–(8) for $h \geq \widehat{h}(\beta)$ with $\widehat{h}(\beta)$ given by (15). To that end, we construct below a cyclical process $U_t^\star$ according to (7) and (8) under the policy prescribed in Theorem 3. Denote $\underline{u}^\star := U_0^\star = U_{\tau_i+}^\star$ for all i. For any $t \in (\tau_{i-1}, \tau_{i-1} + t^\circ)$, because $q_t^{n\star} = q_t^{m\star} = 0$, we have $U_t^\star = \underline{u}^\star e^{\theta(t-\tau_{i-1})}$ (because $\frac{dU_t^\star}{dt} = \theta U_t^\star$ according to (8)). This implies that $U_{\tau_i}^\star = \underline{u}^\star e^{\theta t^\circ}$.

- If $r \geq F$, then $\tau_i = \tau_{i-1} + t^\circ$ (i.e., $q_{\tau_{i-1}+t^\circ}^{m\star} = 1$) and (7) implies that

$$\underline{u}^\star e^{\theta t^\circ} = U_{\tau_i}^\star = \beta F + (1-\beta)\underline{u}^\star. \tag{B.29}$$

- If $r < F$, then for $t \in [\tau_{i-1} + t^\circ, \tau_i]$, we have $q_t^{m\star} = 0$, $q_t^{n\star} = \frac{\theta r}{F-r}$ and $P_t^\star = r$ by (16). Thus, (8) implies that

$$\begin{aligned} \frac{dU_t^\star}{dt} =& \theta U_t^\star - \frac{\theta r}{F-r}\left[\beta F + (1-\beta)\underline{u}^\star - U_t^\star\right] \\ =& \frac{\theta}{F-r}\left\{FU_t^\star - r\left[\beta F + (1-\beta)\underline{u}^\star\right]\right\}, \end{aligned}$$

which implies that $U_t^\star$ would be unbounded for $t \in [\tau_{i-1} + t^\circ, \tau_i]$ if $\frac{dU_t^\star}{dt} \gtrless 0$ (i.e., $U_t^\star \gtrless r/F\left[\beta F + (1-\beta)\underline{u}^\star\right]$). Thus, we must have $\frac{dU_t^\star}{dt} = 0$ for $t \in [\tau_{i-1} + t^\circ, \tau_i]$, implying

$$U_t^\star := U_{\tau_{i-1}+t^\circ}^\star = \underline{u}^\star e^{\theta t^\circ} = r/F\left[\beta F + (1-\beta)\underline{u}^\star\right], \text{ for } t \in [\tau_{i-1} + t^\circ, \tau_i]. \tag{B.30}$$

Combining (B.29) and (B.30) yields

$$\underline{u}^\star e^{\theta t^\circ} = \frac{\bar{h}}{F}\left[\beta F + (1-\beta)\underline{u}^\star\right] \quad \Leftrightarrow \quad \underline{u}^\star = \frac{\beta F \bar{h}}{Fe^{\theta t^\circ} - (1-\beta)\bar{h}}. \tag{B.31}$$

Thus, for $t \in (\tau_{i-1}, \tau_{i-1} + t^\circ]$, (**??**), together with (B.31), implies that

$$\begin{aligned} P_t^\star - U_t^\star =& \left[\bar{h} - \underline{u}^\star e^{\theta t^\circ}\right] e^{-\theta(\tau_{i-1}+t^\circ - t)} \\ \leq& \left[\bar{h} - \underline{u}^\star e^{\theta t^\circ}\right] = \frac{(1-\beta)\left[F - \bar{h}e^{-\theta t^\circ}\right]}{F - \bar{h}(1-\beta)e^{-\theta t^\circ}}\bar{h} = \widehat{h}(\beta), \end{aligned}$$

which is bounded from above by h if $h \geq \widehat{h}(\beta)$ with $\widehat{h}(\beta)$ given by (15). Therefore, the ignored constraint (6) is indeed automatically satisfied. Finally, we note that

$$\frac{U_t^\star}{P_t^\star} = \frac{\underline{u}^\star e^{\theta(t-\tau_{i-1})}}{\bar{h}e^{-\theta(\tau_i - t)}} = \frac{\beta F}{F-(1-\beta)\bar{h}e^{-\theta t^\circ}}. \quad \square$$

Proof of Corollary 2. The derivation of the principal's cost $C^\star$, auditing cost $A^\star$ and the agent's cost $C_a^\star$ follows a similar argument as in the proof of Corollary 1 by replacing $\underline{p}^\star = he^{-\theta t^\star}$ with $\underline{p}^\star = \bar{h}e^{-\theta t^\circ}$. $\square$

Appendix C: Proofs in Section 7

We first outline the proof strategy in the following remark.

Remark C.1. First, we establish Lemma C.1, which allows us to convert (18) to the following problem

$$\frac{\lambda}{\theta+\lambda}r + \min_{\gamma\geq 0,(P_t,Q_t)_{t\in[0,\infty)}} \mathbb{E}\left[k\int_0^T e^{-\theta t}dN_t - e^{-\theta T}P_T \,\middle|\, \mathcal{P}\right], \text{ subject to (C.3)–(C.8)}, \tag{C.1}$$

whose solution is denoted as $\{\gamma^\star, P_t^\star, Q_t^\star\}$. Then, $(P_t^\star, Q_t^\star)$ together with $U_t^\star := \gamma^\star P_t^\star$ satisfy (6)–(10), thus obtaining the complete solution to (18). To that end, we solve (C.1) using the following two steps.

1. For any given γ satisfying (C.3), we first solve the following one-dimensional stochastic control in P_t using the verification approach:

$$c_\gamma^\star := \frac{\lambda}{\theta+\lambda}r + \min_{(P_t,Q_t)_{t\in[0,\infty)}} \mathbb{E}\left[k\int_0^T e^{-\theta t}dN_t - e^{-\theta T}P_T \,\middle|\, \mathcal{P}\right], \text{ subject to (C.4)–(C.8)}. \tag{C.2}$$

As characterized by Lemma C.4, the optimal policy for (C.2) exhibits a cyclical structure, which allows us to explicitly compute the principal's cost-to-go function. All key policy parameters enter the cost-to-go function through the objective function in (C.18). Thus, the stochastic control problem is turned into a one-dimensional static optimization problem (C.18) in Lemma C.3, in which the decision variable x corresponds to an exponential transformation of the deterministic phase of each cycle t, i.e., $x = e^{-\theta t}$.

2. We then optimize $c_\gamma^\star$ over γ satisfying (C.3) to identify the optimal $\gamma^\star$. The two-dimensional static optimization problem (19) essentially combines the one-dimensional static optimization problem (C.18) from the previous step and the optimization of $c_\gamma^\star$ over γ in this step. The optimal policy solving (C.2) for $\gamma^\star$ is the solution to (C.1). $\square$

Lemma C.1. *Under $U_t = \gamma P_t$ for all $t \geq 0$, constraints (6)–(10) imply*

$$\beta \leq \gamma \leq 1 - h/\bar{h}, \tag{C.3}$$

$$0 \leq P_t \leq \frac{h}{1-\gamma}, \tag{C.4}$$

$$\frac{1}{1-\beta}\left(\frac{h}{1-\gamma} - \frac{\beta}{\gamma}F\right)^+ \leq P_{t+}^I \leq \min\left\{\frac{h}{1-\gamma}, \frac{1-\beta/\gamma}{1-\beta}F, r\right\}. \tag{C.5}$$

$$P_t = (1-q_t^m)P_{t+} + q_t^m F - z_t, \quad \text{for } q_t^m > 0, \tag{C.6}$$

$$\frac{dP_t}{dt} = \theta P_t - q_t^n (F - P_t) + z_t, \quad \text{for } q_t^m = 0, \quad \text{and} \tag{C.7}$$

$$z_t = \begin{cases} q_t^m\left[(1-\beta/\gamma)F - (1-\beta)P_{t+}^I\right], & \text{for } q_t^m > 0, \\ q_t^n\left[(1-\beta/\gamma)F - (1-\beta)P_{t+}^I\right], & \text{for } q_t^m = 0. \end{cases} \tag{C.8}$$

Conversely, for any γ, P_t and Q_t satisfying (C.3)–(C.8), the pair (P_t, U_t) with $U_t := \gamma P_t$, together with Q_t, for all $t \geq 0$ satisfies (6)–(10).

Proof of Lemma C.1. By Lemma A.1 (and its proof), we must have $\gamma \geq \beta$, and further by (6), we have $0 \leq P_t - U_t \leq h$, which immediately implies (C.4) given that $U_t = \gamma P_t$. Since $P_t \leq \bar{h}$, we can restrict to $h/(1-\gamma) \leq \bar{h}$, namely $\gamma \leq 1 - h/\bar{h}$. Altogether, we have (C.3).

Under $U_t = \gamma P_t$, P_t must be continuous between audits because so is U_t by (7)-(8). Further, (9) and (10), which can be written as (C.6) and (C.7), imply

$$U_t = (1-q_t^m)U_{t+} + q_t^m \gamma F - \gamma z_t, \quad \text{for } q_t^m > 0, \quad \text{and} \tag{C.9}$$

$$\frac{dU_t}{dt} = \theta U_t - q_t^n(\gamma F - U_t) + \gamma z_t, \quad \text{for } q_t^m = 0, \tag{C.10}$$

where $z_t \geq 0$ is the slack variable. By contrasting (C.9)-(C.10) with (7)-(8), we must have

$$\gamma z_t = q_t^m\left[(\gamma-\beta)F - (1-\beta)U_{t+}^I\right] = q_t^m\left[(\gamma-\beta)F - (1-\beta)\gamma P_{t+}^I\right] \geq 0, \text{ for } q_t^m > 0, \text{ and} \tag{C.11}$$

$$\gamma z_t = q_t^n\left[(\gamma-\beta)F - (1-\beta)U_{t+}^I\right] = q_t^n\left[(\gamma-\beta)F - (1-\beta)\gamma P_{t+}^I\right] \geq 0, \text{ for } q_t^m = 0, \tag{C.12}$$

which immediately yield (C.8) and, together with (C.4), yield the upper bound on P_{t+}^I in (C.5). To show the lower bound on P_{t+}^I in (C.5), we note (C.4) requires (7)-(8) at $P_t = \frac{h}{1-\gamma}$ to satisfy

$$\frac{h}{1-\gamma} = (1-q_t^m)P_{t+} + q_t^m F - z_t \leq (1-q_t^m)\frac{h}{1-\gamma} + q_t^m F - z_t \text{ for } q_t^m > 0, \quad \text{and} \tag{C.13}$$

$$\left.\frac{dP_t}{dt}\right|_{P_t = \frac{h}{1-\gamma}} = \theta\frac{h}{1-\gamma} - q_t^n\left(F - \frac{h}{1-\gamma}\right) + z_t \leq 0 \text{ for } q_t^m = 0 \text{ and } q_t^n > 0. \tag{C.14}$$

Substituting (C.11) and (C.12) respectively into (C.13) and (C.14) yields

$$q_t^m\left[\frac{\beta}{\gamma}F + (1-\beta)P_{t+}^I - \frac{h}{1-\gamma}\right] \geq 0, \quad \text{and} \quad q_t^n\left[\frac{\beta}{\gamma}F + (1-\beta)P_{t+}^I - \frac{h}{1-\gamma}\right] \geq 0,$$

which gives the lower bound on P_{t+}^I in (C.5).

Conversely, given P_t satisfying (C.4)–(C.7) and nonnegative slack variable z_t specified by (C.8), it is straightforward to verify that P_t satisfies (9)–(10) and $U_t = \gamma P_t$ satisfies (7) and (8). Furthermore, if γ satisfies (C.3), then (P_t, U_t) also satisfies (6). □

By contrasting problem (C.2) with problem (B.1), we modify Lemma B.5 to obtain the following:

LEMMA C.2 **(Verification of Optimality)**. *The policy $\mathcal{P} := (P_t, Q_t)_{t\in[0,\infty)}$ solves (C.2) if the principal's cost-to-go function under $\mathcal{P}$, namely $C(\mathcal{I}_t, t)$ defined in (B.2), satisfies the following properties:*

Property 1: $C(\mathcal{I}_t, t)$ depends on $(\mathcal{I}_t, t)$ only through P_t and can hence be denoted as $C(\mathcal{I}_t, t) = C(P_t)$. That is, $C(\mathcal{I}_t, t) = C\left(\widehat{\mathcal{I}}_{\hat{t}}, \hat{t}\right)$ for any $(\mathcal{I}_t, t)$ and $\left(\widehat{\mathcal{I}}_{\hat{t}}, \hat{t}\right)$ such that $P_t(\mathcal{I}_t) = P_{\hat{t}}(\widehat{\mathcal{I}}_{\hat{t}})$.

Property 2: $C(p)$ is bounded, non-decreasing, and continuously differentiable in $p \in \left[0, \frac{h}{1-\gamma}\right]$.

Property 3: $C(p)$ satisfies

$$\lambda(r-p) - (\theta+\lambda)C(p) + \mathcal{N}C(p) \geq 0 \quad \text{and} \quad \mathcal{M}C(p) - C(p) \geq 0, \quad \text{for } p \in \left[0, \tfrac{h}{1-\gamma}\right], \tag{C.15}$$

where the functional operators $\mathcal{N}$ and $\mathcal{M}$ are defined as

$$\begin{aligned} \mathcal{N}C(p) := \min_{p_+^I, q^n, z \geq 0} \quad & q^n\left[k + C\left(p_+^I\right) - C(p)\right] + \{\theta p - q^n[F-p] + z\}\tfrac{dC(p)}{dp} \\ \text{subject to} \quad & \tfrac{1}{1-\beta}\left(\tfrac{h}{1-\gamma} - \tfrac{\beta}{\gamma}F\right)^+ \leq p_+^I \leq \min\left\{\tfrac{h}{1-\gamma}\tfrac{1-\beta/\gamma}{1-\beta}F, r\right\} \\ & z = q^n\left[(1-\beta/\gamma)F - (1-\beta)p_+^I\right], \end{aligned} \tag{C.16}$$

and

$$\begin{aligned} \mathcal{M}C(p) := \min_{p_+, p_+^I, q^m, z \geq 0} \quad & q^m \left(k + C\left(p_+^I\right)\right) + (1-q^m) C(p_+) \\ \text{subject to} \quad & \frac{1}{1-\beta}\left(\frac{h}{1-\gamma} - \frac{\beta}{\gamma} F\right)^+ \leq p_+^I \leq \min\left\{\frac{h}{1-\gamma}\frac{1-\beta/\gamma}{1-\beta}F, r\right\} \\ & z = q^m\left[(1-\beta/\gamma)F - (1-\beta)p_+^I\right] \\ & p = (1-q^m)p_+ + q^m F - z. \end{aligned} \tag{C.17}$$

Proof of Lemma C.2. The proof follows the same argument as that of Lemma B.5 by replacing the bound B with $\frac{h}{1-\gamma}$ and the constraint $p_+^I \leq B$ with the constraint implied by (C.5) and (C.8). □

LEMMA C.3. *For any given γ satisfying (C.3), the solution to the following static constrained optimization problem, denoted as x_γ, exists and is unique:*

$$K_\gamma := \min_{\frac{1}{1-\beta}\left[1-\beta\frac{F(1-\gamma)}{h\gamma}\right]^+ \leq x \leq 1 \wedge \frac{F(\gamma-\beta)(1-\gamma)}{h(1-\beta)\gamma} \wedge \frac{r(1-\gamma)}{h}} \left(\frac{1-\gamma}{h}\right)^{\frac{\theta+\lambda}{\theta}} \frac{k + \beta\left[\frac{F}{\gamma} - \frac{h}{1-\gamma}x\right]}{\frac{\theta+\lambda}{\theta}\frac{1-\gamma}{h}\left[\beta\frac{F}{\gamma} + (1-\beta)\frac{h}{1-\gamma}x\right] - \frac{\lambda}{\theta} - x^{\frac{\theta+\lambda}{\theta}}}, \tag{C.18}$$

with $x_\gamma \in (0,1)$ and $K_\gamma > \frac{\theta}{\theta+\lambda}\left(\frac{1-\gamma}{h}\right)^{\frac{\lambda}{\theta}}$. Furthermore, let $\gamma^\star = \arg\min_{\beta \leq \gamma \leq 1 - h/\bar{h}} K_\gamma$. Then, $(\gamma^\star, x_{\gamma^\star})$ is the solution to (19) with $K^\star = K_{\gamma^\star}$ and, in particular, $\frac{1}{1-\beta}\left[1 - \beta\frac{F(1-\gamma^\star)}{h\gamma^\star}\right]^+ < x_{\gamma^\star} < 1$.

Proof of Lemma C.3. Direct calculation reveals that the sign of the derivative of the objective function in (C.18) with respect to x is given by

$$\begin{aligned} & \operatorname{sign}\left\{\frac{\partial}{\partial x}\left(\frac{1-\gamma}{h}\right)^{\frac{\theta+\lambda}{\theta}} \frac{k + \beta\left[\frac{F}{\gamma} - \frac{h}{1-\gamma}x\right]}{\frac{\theta+\lambda}{\theta}\frac{1-\gamma}{h}\left[\beta\frac{F}{\gamma} + (1-\beta)\frac{h}{1-\gamma}x\right] - \frac{\lambda}{\theta} - x^{\frac{\theta+\lambda}{\theta}}}\right\} \\ = & -\frac{\beta h}{1-\gamma}\left\{\frac{\theta+\lambda}{\theta}\frac{1-\gamma}{h}\left[\beta\frac{F}{\gamma} + (1-\beta)\frac{h}{1-\gamma}x\right] - \frac{\lambda}{\theta} - x^{\frac{\theta+\lambda}{\theta}}\right\} - \frac{\theta+\lambda}{\theta}\left\{k + \beta\left[\frac{F}{\gamma} - \frac{h}{1-\gamma}x\right]\right\}\left\{1 - \beta - x^{\frac{\lambda}{\theta}}\right\} \\ = & \frac{\theta+\lambda}{\theta}\left\{\beta\left(\frac{\lambda}{\lambda+\theta}\frac{h}{1-\gamma} - \frac{F}{\gamma}\right) - (1-\beta)k + \left(k + \beta\frac{F}{\gamma}\right)x^{\frac{\lambda}{\theta}} - \frac{\lambda}{\lambda+\theta}\beta\frac{h}{1-\gamma}x^{\frac{\lambda+\theta}{\theta}}\right\} := \Xi_\gamma(x). \end{aligned}$$

We then note that function $\Xi_\gamma(x)$ satisfies the following properties:

- $\Xi_\gamma'(x) = \lambda/\theta x^{\lambda/\theta - 1}\left(k + \beta F/\gamma - \beta h x/(1-\gamma)\right) > 0$ for $0 \leq x \leq 1 \wedge \frac{F(\gamma-\beta)(1-\gamma)}{h(1-\beta)\gamma} \wedge \frac{r(1-\gamma)}{h}$;
- $\Xi_\gamma(0) = \beta\left(\frac{\lambda}{\lambda+\theta}\frac{h}{1-\gamma} - F/\gamma\right) - (1-\beta)k < 0$ for γ satisfying (C.3);
- and $\Xi_\gamma(1) = \beta k > 0$.

Thus, the objective function in (C.18) is strictly quasi-convex in x with positive derivative at $x = 0$. Thus, solution $x_\gamma \in (0,1)$ must exist and is unique, which immediately implies that $x_{\gamma^\star} \in (0,1)$.

To show $K_\gamma > \frac{\theta}{\theta+\lambda}\left(\frac{1-\gamma}{h}\right)^{\frac{\lambda}{\theta}}$, we note that it suffices to show

$$\frac{k + \beta\left[\frac{F}{\gamma} - \frac{h}{1-\gamma}x\right]}{\left[\beta\frac{F}{\gamma} + (1-\beta)\frac{h}{1-\gamma}x\right] - \frac{\lambda}{\theta+\lambda}\frac{h}{1-\gamma} - \frac{\theta}{\theta+\lambda}\frac{h}{1-\gamma}x^{\frac{\theta+\lambda}{\theta}}} > 1 \quad \Leftrightarrow \quad \Gamma(x) := k + \frac{\lambda}{\theta+\lambda}\frac{h}{1-\gamma} + \frac{\theta}{\theta+\lambda}\frac{h}{1-\gamma}x^{\frac{\theta+\lambda}{\theta}} - \frac{h}{1-\gamma}x > 0,$$

which holds for all $x \in [0,1]$, because function $\Gamma(x)$ is non-increasing in $x \in [0,1]$ (as $\Gamma'(x) = \frac{h}{1-\gamma}\left(x^{\frac{\lambda}{\theta}} - 1\right) \leq 0$) with $\Gamma(1) = k > 0$.

It is straightforward to see that $(\gamma^\star, x_{\gamma^\star})$ is the solution to (19), which can be solved via a two-stage optimization by first optimizing over x given γ as in (C.18) and then optimizing over γ. To show $x_{\gamma^\star} >$

$\frac{1}{1-\beta}\left[1-\beta\frac{F(1-\gamma^\star)}{h\gamma^\star}\right]$, we take the variable transformation, $\bar{p}:=\frac{h}{1-\gamma}$ and $\underline{p}:=\frac{h}{1-\gamma}x$, under which the objective function in (C.18) can be rewritten as

$$\left(\frac{1-\gamma}{h}\right)^{\frac{\theta+\lambda}{\theta}}\frac{k+\beta\left[\frac{F}{\gamma}-\frac{h}{1-\gamma}x\right]}{\frac{\theta+\lambda}{\theta}\frac{1-\gamma}{h}\left[\beta\frac{F}{\gamma}+(1-\beta)\frac{h}{1-\gamma}x\right]-\frac{\lambda}{\theta}-x^{\frac{\theta+\lambda}{\theta}}}=\frac{k+\phi(\underline{p},\bar{p})-\underline{p}}{\frac{\theta+\lambda}{\theta}\phi(\underline{p},\bar{p})\bar{p}^{\frac{\lambda}{\theta}}-\frac{\lambda}{\theta}\bar{p}^{\frac{\lambda+\theta}{\theta}}-\underline{p}^{\frac{\lambda+\theta}{\theta}}}:=\kappa(\underline{p},\bar{p}), \quad \text{(C.19)}$$

with $\phi(\underline{p},\bar{p}):=\beta\frac{F}{1-h/\bar{p}}+(1-\beta)\underline{p}$, and the constraint $x\geq\frac{1}{1-\beta}\left[1-\beta\frac{F(1-\gamma)}{h\gamma}\right]$ can be rewritten as

$$\bar{p}\leq\phi(\underline{p},\bar{p}) \quad \text{(C.20)}$$

with "=" holding at the same time. We further note that (C.20) is equivalent to

$$\underline{p}\geq\frac{1}{1-\beta}\left[\bar{p}-\beta\frac{F}{1-h/\bar{p}}\right],$$

whose right-hand side is monotonically increasing in $\bar{p}$. Hence, there exists a monotonically increasing function $\psi(\cdot)$ such that (C.20) is equivalent to $\bar{p}\leq\psi(\underline{p})$.

If $x_{\gamma^\star}=\frac{1}{1-\beta}\left[1-\beta\frac{F(1-\gamma^\star)}{h\gamma^\star}\right]$, then $\bar{p}^\star:=\frac{h}{1-\gamma^\star}$ and $\underline{p}^\star:=\frac{h}{1-\gamma^\star}x^\star$ must satisfy $\bar{p}^\star=\phi(\underline{p}^\star,\bar{p}^\star)$, or equivalently $\bar{p}^\star=\psi(\underline{p}^\star)$. Then, direct calculation reveals

$$\text{sign}\left\{\frac{\partial\kappa(\underline{p}^\star,\bar{p}^\star)}{\partial\bar{p}}\right\}=\text{sign}\left\{\underbrace{\frac{\partial\phi(\underline{p}^\star,\bar{p}^\star)}{\partial\bar{p}}}_{<0}\underbrace{\left[1-\frac{\theta+\lambda}{\theta}\underbrace{\kappa(\underline{p}^\star,\bar{p}^\star)}_{=K_{\gamma^\star}}(\bar{p}^\star)^{\frac{\lambda}{\theta}}\right]}_{<0}\right\}>0,$$

where we have shown above that $K_{\gamma^\star}>\frac{\theta}{\theta+\lambda}\left(\frac{1-\gamma^\star}{h}\right)^{\frac{\lambda}{\theta}}$. That is, there exists $\bar{p}<\bar{p}^\star=\psi(\underline{p}^\star)$ such that $\kappa(\underline{p},\bar{p}^\star)<\kappa(\underline{p}^\star,\bar{p}^\star)=K_{\gamma^\star}$, contradicting to the optimality of $K_{\gamma^\star}$. ☐

LEMMA C.4. *For any given γ satisfying (C.3), let x_γ be the solution to (C.18) given in Lemma C.3. Then, the optimal policy that solves (C.2) is given as follows: for an initial period of length $t_\gamma^\circ=-\frac{1}{\theta}\ln\frac{(1-\gamma)p_\gamma}{h}$ with $p_\gamma=\left(\frac{\theta+\lambda}{\theta}K_\gamma\right)^{-\theta/\lambda}<\frac{h}{1-\gamma}$, the principal applies no audits (i.e., $q_t^{m\star}=q_t^{n\star}:=0$ for $t\in[0,t_\gamma^\circ]$) and charges the agent a payment according to*

$$P_t^\star=p_\gamma e^{\theta t}, \quad \textit{for } t\in[0,t_\gamma^\circ]. \quad \text{(C.21)}$$

Starting from t_γ°, the principal applies intensive audits at constant rate while maintaining the constant payment level, respectively given by

$$q_t^{n\star}:=\frac{\theta}{\beta\frac{F(1-\gamma)}{h\gamma}+(1-\beta)x_\gamma-1}, \quad \textit{and} \quad P_t^\star:=\frac{h}{1-\gamma}, \quad \textit{for } t\in(t_\gamma^\circ,\tau_1]\bigcup_{i=1}^{\infty}(\tau_i+t_\gamma,\tau_{i+1}], \quad \text{(C.22)}$$

where $t_\gamma=-\frac{1}{\theta}\ln x_\gamma$ and τ_i is the i-th audit. (If $q_t^{n\star}$ in (C.22) becomes infinity, then the intensive audit is equivalent to an impulsive audit with probability $q_t^{m\star}=1$.) The principal applies no audits (i.e., $q_t^{m\star}=q_t^{n\star}:=0$) for $t\in(\tau_i,\tau_i+t_\gamma]$ and charges the agent a payment according to

$$P_t^\star=\frac{h}{1-\gamma}e^{-\theta(\tau_i+t_\gamma-t)}, \quad \textit{for } t\in\bigcup_{i=1}^{\infty}(\tau_i,\tau_i+t_\gamma]. \quad \text{(C.23)}$$

Finally, the principal's optimal total expected discounted cost in (C.2) is given by $c_\gamma^\star:=\frac{\lambda}{\lambda+\theta}(r-p_\gamma)$, out of which she spends $a_\gamma^\star=\frac{\theta}{\lambda+\theta}\frac{kp_\gamma}{k+\beta\left[\frac{F}{\gamma}-\frac{h}{1-\gamma}x_\gamma\right]}$ on audits. The corresponding agent's total expected discounted cost is given by $c_{a\gamma}^\star=\frac{\lambda}{\lambda+\theta}p_\gamma+a_\gamma^\star$.

Proof of Lemma C.4. It is clear that the policy in the lemma are prescribed purely as a function of $P_t^\star$, and hence its current-value cost-to-go function depends on $(\mathcal{I}_t, t)$ only through $P_t^\star$. Then, (C.21) and (C.23) imply that staring from any $P_0^\star = p \in \left[0, \frac{h}{1-\gamma}\right]$, $P_t^\star$ evolves deterministically according to $P_t^\star = pe^{\theta t}$ before reaching the threshold $\frac{h}{1-\gamma}$, which takes $\tau(p) := \frac{1}{\theta}\ln\frac{h}{p(1-\gamma)}$ amount of time, i.e., $P_{\tau(p)}^\star = \frac{h}{1-\gamma}$. No audit ($q_t^{m\star} = q_t^{n\star} := 0$) is conducted between $[0, \tau(p))$. Therefore, we compute the cost-to-go function as follows:

$$
\begin{aligned}
C(p) =& \mathbb{E}\left[\int_0^\infty e^{-(\lambda+\theta)t}\left(kq_t^{n\star} + \lambda(r - P_t^\star)\right)dt + \sum_{t\geq 0, q_t^{m\star}>0} e^{-(\lambda+\theta)t}kq_t^{m\star} \,\middle|\, P_0^\star = p\right] \\
=& \int_0^{\tau(p)} \lambda e^{-(\lambda+\theta)t}\left[r - pe^{\theta t}\right]dt + e^{-(\lambda+\theta)\tau(p)}C\left(\frac{h}{1-\gamma}\right) \\
=& \frac{\lambda}{\lambda+\theta}r\left(1 - e^{-(\lambda+\theta)\tau(p)}\right) - p\left(1 - e^{-\lambda\tau(p)}\right) + e^{-(\lambda+\theta)\tau(p)}C\left(\frac{h}{1-\gamma}\right) \\
=& \frac{\lambda r}{\lambda+\theta} + \left[C\left(\frac{h}{1-\gamma}\right) - \frac{\lambda r}{\lambda+\theta} + \frac{h}{1-\gamma}\right]\left[\frac{p(1-\gamma)}{h}\right]^{\frac{\lambda+\theta}{\theta}} - p. \qquad \text{(C.24)}
\end{aligned}
$$

Under the policy prescribed in the lemma, once $P_t^\star$ reaches $\frac{h}{1-\gamma}$, an intensity audit with constant rate prescribed in (C.22) is used while maintaining $P_t^\star := \frac{h}{1-\gamma}$, which suggests that

$$
\begin{aligned}
C\left(\frac{h}{1-\gamma}\right) =& \int_0^\infty e^{-(\theta+\lambda+q_t^{n\star})t}\left\{\lambda\left(r - \frac{h}{1-\gamma}\right) + q_t^{n\star}\left[k + C(\underline{p}^\star)\right]\right\}dt \\
=& \frac{1}{\lambda+\theta+\frac{\theta}{\beta\frac{F(1-\gamma)}{h\gamma}+(1-\beta)x_\gamma-1}}\left\{\lambda\left(r - \frac{h}{1-\gamma}\right) + \frac{\theta}{\beta\frac{F(1-\gamma)}{h\gamma}+(1-\beta)x_\gamma-1}\left[k + C\left(\frac{h}{1-\gamma}e^{-\theta t_\gamma}\right)\right]\right\} \\
=& \frac{1}{\frac{\lambda+\theta}{\theta}\left[\beta\frac{F(1-\gamma)}{h\gamma}+(1-\beta)x_\gamma-1\right]+1}\left\{\frac{\lambda}{\theta}\left[\beta\frac{F(1-\gamma)}{h\gamma}+(1-\beta)x_\gamma-1\right]\left(r - \frac{h}{1-\gamma}\right) + k + C\left(\frac{h}{1-\gamma}e^{-\theta t_\gamma}\right)\right\}. \\
& \qquad \text{(C.25)}
\end{aligned}
$$

Note that if $q_t^{n\star} := \infty$, or equivalently $\beta\frac{F(1-\gamma)}{h\gamma} + (1-\beta)x_\gamma - 1 = 0$, then (C.25) also holds and can be verified to coincide with the cost-to-go function for an impulsive audit with probability $q_t^{m\star} = 1$.

Meanwhile, (C.24) implies

$$
C\left(\frac{h}{1-\gamma}e^{-\theta t_\gamma}\right) = \frac{\lambda r}{\lambda+\theta} + \left[C\left(\frac{h}{1-\gamma}\right) - \frac{\lambda r}{\lambda+\theta} + \frac{h}{1-\gamma}\right]x_\gamma^{\frac{\lambda+\theta}{\theta}} - \frac{h}{1-\gamma}x_\gamma, \qquad \text{(C.26)}
$$

where we use $t_\gamma = -\frac{1}{\theta}\ln x_\gamma$. Combining (C.25) and (C.26) yields

$$
C\left(\frac{h}{1-\gamma}\right) - \frac{\lambda r}{\lambda+\theta} + \frac{h}{1-\gamma} = \frac{k + \beta\left[\frac{F}{\gamma} - \frac{h}{1-\gamma}x_\gamma\right]}{\frac{\theta+\lambda}{\theta}\frac{1-\gamma}{h}\left[\beta\frac{F}{\gamma} + (1-\beta)\frac{h}{1-\gamma}x_\gamma\right] - \frac{\lambda}{\theta} - x_\gamma^{\frac{\theta+\lambda}{\theta}}}. \qquad \text{(C.27)}
$$

Substituting (C.27) into (C.24) yields

$$
C(p) = \frac{\lambda r}{\lambda+\theta} + K_\gamma p^{\frac{\lambda+\theta}{\theta}} - p, \quad \text{for } p \in \left[0, \frac{h}{1-\gamma}\right], \qquad \text{(C.28)}
$$

where K_γ is defined in (C.18) and it is straightforward to verify that $p_\gamma = \left(\frac{\theta+\lambda}{\theta}K_\gamma\right)^{-\theta/\lambda} < \frac{h}{1-\gamma}$ because $K_\gamma > \frac{\theta}{\theta+\lambda}\left(\frac{1-\gamma}{h}\right)^{\frac{\lambda}{\theta}}$ by Lemma C.3, and that p_γ satisfies

$$
\frac{dC(p_\gamma)}{dp} = \frac{\lambda+\theta}{\theta}K_\gamma p_\gamma^{\frac{\lambda}{\theta}} - 1 = 0 \quad \Rightarrow \quad C(p_\gamma) = \frac{\lambda}{\lambda+\theta}(r - p_\gamma). \qquad \text{(C.29)}
$$

It is straightforward to verify that $C(p)$ is bounded, non-decreasing, convex, and continuously differentiable function on $\left[0, \frac{h}{1-\gamma}\right]$. Therefore, by Lemma C.2, it remains to show that $C(p)$ satisfies (C.15) to establish the optimality of the policy specified in this lemma.

We first show $\lambda(r-p)-(\lambda+\theta)C(p)+\mathcal{N}C(p)\geq 0$ *for* $p\in\left[0,\frac{h}{1-\gamma}\right]$. By definition (C.16) (after eliminating slack variable z via its constraint therein), we have

$$\mathcal{N}C(p) = \theta p\frac{dC(p)}{dp} + \min_{\substack{\frac{1}{1-\beta}\left(\frac{h}{1-\gamma}-\frac{\beta}{\gamma}F\right)^+ \leq p_+^I \leq \min\left\{\frac{h}{1-\gamma},\frac{\gamma-\beta}{1-\beta}F\right\}\\ q^n\geq 0}} q^n\left[k+C\left(p_+^I\right)-C(p)-\left(\beta/\gamma F+(1-\beta)p_+^I-p\right)\frac{dC(p)}{dp}\right], \tag{C.30}$$

where we note that the coefficient of q^n satisfies

$$\frac{\partial}{\partial p}\left\{k+C\left(p_+^I\right)-C(p)-\left(\beta/\gamma F+(1-\beta)p_+^I-p\right)\frac{dC(p)}{dp}\right\} = -\left(\beta/\gamma F+(1-\beta)p_+^I-p\right)\frac{d^2C(p)}{dp^2}\leq 0,$$

for all $p\leq\frac{h}{1-\gamma}$, because the lower bound on $p_+^I\geq\frac{1}{1-\beta}\left(\frac{h}{1-\gamma}-\frac{\beta}{\gamma}F\right)^+$ and the convexity of $C(p)$. Therefore, the coefficient of q^n must be nonnegative:

$$\begin{aligned}
&k+C\left(p_+^I\right)-C(p)-\left(\beta/\gamma F+(1-\beta)p_+^I-p\right)\frac{dC(p)}{dp}\\
\text{(with “=” at } p=\tfrac{h}{1-\gamma}\text{)}\quad &\geq k+C\left(p_+^I\right)-C\left(\frac{h}{1-\gamma}\right)-\left(\beta/\gamma F+(1-\beta)p_+^I-\frac{h}{1-\gamma}\right)\frac{d}{dp}C\left(\frac{h}{1-\gamma}\right)\\
\text{(by (C.28))}\quad &= k+\beta\left(F/\gamma-p_+^I\right)-K_\gamma\left[\frac{\lambda+\theta}{\theta}\left(\beta\frac{F}{\gamma}+(1-\beta)p_+^I\right)\left(\frac{h}{1-\gamma}\right)^{\frac{\lambda}{\theta}}-\frac{\lambda}{\theta}\left(\frac{h}{1-\gamma}\right)^{\frac{\lambda+\theta}{\theta}}-\left(p_+^I\right)^{\frac{\lambda+\theta}{\theta}}\right]\\
&=\left(\frac{1-\gamma}{h}\right)^{\frac{\theta+\lambda}{\theta}}\frac{k+\beta\left[\frac{F}{\gamma}-\frac{h}{1-\gamma}x\right]}{\frac{\theta+\lambda}{\theta}\frac{1-\gamma}{h}\left[\beta\frac{F}{\gamma}+(1-\beta)\frac{h}{1-\gamma}x\right]-\frac{\lambda}{\theta}-x^{\frac{\theta+\lambda}{\theta}}}-K_\gamma\geq 0,
\end{aligned}\tag{C.31}$$

where $x=\frac{(1-\gamma)p_+^I}{h}\in\left[\frac{1}{1-\beta}\left[1-\beta\frac{F(1-\gamma)}{h\gamma}\right]^+, 1\wedge\frac{F(\gamma-\beta)(1-\gamma)}{h(1-\beta)\gamma}\right]$, and the nonnegativity in (C.31) follows from (C.18) with the “=” holding at $x=x_\gamma$ or equivalently $p_+^I=\frac{h}{1-\gamma}x_\gamma$. As such, (C.30) reduces to

$$\mathcal{N}C(p)=\theta p\frac{dC(p)}{dp}=(\lambda+\theta)K_\gamma p^{\frac{\lambda+\theta}{\theta}}-\theta p,$$

which immediately implies that

$$\lambda(r-p)-(\lambda+\theta)C(p)+\mathcal{N}C(p)=\lambda(r-p)-\lambda r-(\lambda+\theta)K_\gamma p^{\frac{\lambda+\theta}{\theta}}+(\lambda+\theta)p+(\lambda+\theta)K_\gamma p^{\frac{\lambda+\theta}{\theta}}-\theta p=0,$$

establishing the result.

We next show $\mathcal{M}C(p)-C(p)\geq 0$ *for* $p\in\left[0,\frac{h}{1-\gamma}\right]$. The functional operator $\mathcal{M}$ defined by (C.17) can be rewritten as

$$\mathcal{M}C(p) := \min_{\frac{1}{1-\beta}\left(\frac{h}{1-\gamma}-\frac{\beta}{\gamma}F\right)^+\leq p_+^I\leq\min\left\{\frac{h}{1-\gamma}\frac{1-\beta/\gamma}{1-\beta}F\right\}} k+C(p_+^I)-\left[\beta/\gamma F+(1-\beta)p_+^I-p\right]\frac{k+C(p_+^I)-C(p_+)}{\beta/\gamma F+(1-\beta)p_+^I-p_+}, \tag{C.32}$$

where we eliminate decision variable q^m and slack variable z via the two equality constraints in (C.17):

$$q^m=\frac{p-p_+}{\beta/\gamma F+(1-\beta)p_+^I-p_+}=1-\frac{\beta/\gamma F+(1-\beta)p_+^I-p}{\beta/\gamma F+(1-\beta)p_+^I-p_+}\in[0,1]. \tag{C.33}$$

By (C.31), we note that

$$\frac{\partial}{\partial p_+}\frac{k+C(p_+^I)-C(p_+)}{\beta/\gamma F+(1-\beta)p_+^I-p_+}=\frac{k+C\left(p_+^I\right)-C(p_+)-\left(\beta/\gamma F+(1-\beta)p_+^I-p_+\right)\frac{dC(p_+)}{dp}}{\left[\beta/\gamma F+(1-\beta)p_+^I-p_+\right]^2}\geq 0. \tag{C.34}$$

By (C.33), there are two cases for us to consider:

- For $p \geq p_+$ and $\beta/\gamma F + (1-\beta)p_+^I - p_+ \geq \beta/\gamma F + (1-\beta)p_+^I - p \geq 0$, we have, by (C.34),

$$
\begin{aligned}
&k + C(p_+^I) - \left[\beta/\gamma F + (1-\beta)p_+^I - p\right] \frac{k + C(p_+^I) - C(p_+)}{\beta/\gamma F + (1-\beta)p_+^I - p_+} \\
\geq &k + C(p_+^I) - \left[\beta/\gamma F + (1-\beta)p_+^I - p\right] \frac{k + C(p_+^I) - C(p)}{\beta/\gamma F + (1-\beta)p_+^I - p} = C(p),
\end{aligned}
$$

establishing $\mathcal{M}C(p) - C(p) \geq 0$.

- For $p \leq p_+$ and $\beta/\gamma F + (1-\beta)p_+^I - p_+ \leq \beta/\gamma F + (1-\beta)p_+^I - p \leq 0$, we again have, by (C.34),

$$
\begin{aligned}
&k + C(p_+^I) - \left[\beta/\gamma F + (1-\beta)p_+^I - p\right] \frac{k + C(p_+^I) - C(p_+)}{\beta/\gamma F + (1-\beta)p_+^I - p_+} \\
\geq &k + C(p_+^I) - \left[\beta/\gamma F + (1-\beta)p_+^I - p\right] \frac{k + C(p_+^I) - C(p)}{\beta/\gamma F + (1-\beta)p_+^I - p} = C(p),
\end{aligned}
$$

also establishing $\mathcal{M}C(p) - C(p) \geq 0$.

Finally, we derive different cost components. The principal's cost $c_\gamma^\star$ immediately follows from (C.29). To compute the agent's cost, we denote the agent's cost-to-go function as $C_a(p) := \mathbb{E}\left[e^{-\theta(T-t)} P_T^\star \,\middle|\, P_t^\star = p, t < T\right]$. Then, letting $r = k = 0$ in (C.28) yields $-C_a(p)$, namely

$$
C_a(p) = p - \left(\frac{1-\gamma}{h}\right)^{\frac{\theta+\lambda}{\theta}} \frac{\beta\left[\frac{F}{\gamma} - \frac{h}{1-\gamma}x_\gamma\right]}{\frac{\theta+\lambda}{\theta}\frac{1-\gamma}{h}\left[\beta\frac{F}{\gamma} + (1-\beta)\frac{h}{1-\gamma}x_\gamma\right] - \frac{\lambda}{\theta} - x_\gamma^{\frac{\theta+\lambda}{\theta}}} p^{\frac{\lambda+\theta}{\theta}}. \tag{C.35}
$$

Thus, we can easily obtain the agent's cost as follows:

$$
\begin{aligned}
c_{a\gamma}^\star = C_a(p_\gamma) = &p_\gamma - \left(\frac{1-\gamma}{h}\right)^{\frac{\theta+\lambda}{\theta}} \frac{\beta\left[\frac{F}{\gamma} - \frac{h}{1-\gamma}x_\gamma\right]}{\frac{\theta+\lambda}{\theta}\frac{1-\gamma}{h}\left[\beta\frac{F}{\gamma} + (1-\beta)\frac{h}{1-\gamma}x_\gamma\right] - \frac{\lambda}{\theta} - x_\gamma^{\frac{\theta+\lambda}{\theta}}} p_\gamma^{\frac{\lambda+\theta}{\theta}} \\
= &\left[1 - \frac{\theta}{\lambda+\theta}\frac{\beta\left[\frac{F}{\gamma} - \frac{h}{1-\gamma}x_\gamma\right]}{k + \beta\left[\frac{F}{\gamma} - \frac{h}{1-\gamma}x_\gamma\right]}\right] p_\gamma = \frac{\lambda}{\lambda+\theta}p_\gamma + \frac{\theta}{\lambda+\theta}\frac{kp_\gamma}{k + \beta\left[\frac{F}{\gamma} - \frac{h}{1-\gamma}x_\gamma\right]},
\end{aligned}
$$

where the second equality follows from the first equation in (C.29). According to (A.14), we then have

$$
\frac{\lambda}{\lambda+\theta}(r - p_\gamma) = c_\gamma^\star = \frac{\lambda}{\theta+\lambda}r + \underbrace{\mathbb{E}\left[k\int_0^T e^{-\theta t} dN_t \,\middle|\, Q^\star\right]}_{a_\gamma^\star} - \underbrace{\mathbb{E}\left[e^{-\theta T} P_T^\star\right]}_{c_{a\gamma}^\star}
$$

which immediately implies that $a_\gamma^\star = c_{a\gamma}^\star - \frac{\lambda}{\lambda+\theta}p_\gamma = \frac{\theta}{\lambda+\theta}\frac{kp_\gamma}{k+\beta\left[\frac{F}{\gamma} - \frac{h}{1-\gamma}x_\gamma\right]}$. □

Proof of Theorem 4 and Corollary 3. The results in the theorem and corollary follow from Lemmas C.3 and C.4 by letting $p_0^\star = p_{\gamma^\star}$ (and hence $t_0^\star = t_{\gamma^\star}^\circ$) and $x^\star = x_{\gamma^\star}$ (and hence $t^\star = t_{\gamma^\star}$). □

Proof of Proposition 1. The "if" direction directly follows from Theorem 3. To show the "only if" part, we note that under a cyclic deterministic policy $(\bar{t}, \bar{p})$ with $q_{\tau_i}^m = 1$, $q_t^m = 0$ for $t \neq \tau_i$, and $q_t^n := 0$ for all t, (8) implies that

$$
U_{\tau_i} = U_{\tau_{i-1}+}^I e^{\theta\bar{t}}, \quad \text{for } i = 1, 2, \ldots \text{ with } \tau_0 = 0, \tag{C.36}
$$

while (7) implies that

$$
U_{\tau_i} = \beta F + (1-\beta)U_{\tau_i+}^I, \quad \text{for } i = 1, 2, \ldots. \tag{C.37}
$$

Combining (C.36) and (C.37) yields

$$
U_{\tau_i+}^I = \rho U_{\tau_{i-1}+}^I - \frac{\beta}{1-\beta}F, \quad \text{for } i = 1, 2, \ldots,
$$

with $\rho := e^{\theta\bar{t}}/(1-\beta) > 1$, which implies

$$U^I_{\tau_i+} = \left(U_0 - \frac{\beta}{1-\beta}\frac{F}{\rho-1}\right)\rho^i + \frac{\beta}{1-\beta}\frac{F}{\rho-1}, \quad \text{for } i = 1, 2, \ldots.$$

Thus, if $U_0 \gtrless \frac{\beta}{1-\beta}\frac{F}{\rho-1}$, then $U^I_{\tau_i+} \to \pm\infty$ as $i \to \infty$. Thus, for the feasibility of U_t, we must have

$$U^I_{\tau_i+} = U_0 = \frac{\beta}{1-\beta}\frac{F}{\rho-1}, \quad \text{for } i = 1, 2, \ldots.$$

implying that

$$U_t = \frac{\beta}{1-\beta}\frac{F}{\rho-1}e^{-\theta(\tau_i - t)}, \quad \text{for } t \in (\tau_{i-1}, \tau_i] \text{ and } i = 1, 2, \ldots,$$

which is proportional to P_t. Thus, (P, Q) belongs to the class of proportional policies, among which Theorem 4 (resp., Theorem 3) has shown that the optimal audit policy cannot be periodic deterministic when $h < \hat{h}(\beta)$ (resp., $h \geq \hat{h}(\beta)$ and $r < F$). □

Appendix D: Proofs in Section 9

Proof of Theorem 5. Under any policy $\mathcal{P} = (F_t, \overline{F}_t, P_t, Q_t)$, the agent's and the principal's expected discounted costs onwards after having taken an evasive action at time t (i.e., $H_t = 1$) are now modified to

$$U_t := \mathbb{E}\left[-\int_t^\infty e^{-\theta(\zeta - t)}\overline{F}_\zeta dZ_\zeta \,\middle|\, T \leq t, Z_t = 1, H_t = 1, \mathcal{I}_t, \mathcal{P}\right], \text{ and} \tag{D.1}$$

$$V_t := \mathbb{E}\left[\int_t^\infty e^{-\theta(\zeta - t)}\left\{Z_\zeta\left(cd\zeta + kdN_\zeta\right) - (r - \overline{F}_\zeta)dZ_\zeta\right\} \,\middle|\, T \leq t, Z_t = 1, H_t = 1, \mathcal{I}_t, \mathcal{P}\right]. \tag{D.2}$$

If a self-correction is conducted, then the agent's expected continuation cost would be reduced to 0 and the principal's expected continuation cost is still given by (A.6).

For any given policy $\widehat{\mathcal{P}} = \left(\widehat{F}_t, \widehat{\overline{F}}_t, \widehat{P}_t, \widehat{Q}_t\right)$ and the agent's corresponding best response $\widehat{\sigma}^\star$, we now construct an alternative policy $\mathcal{P} := \left(F_t, \overline{F}_t, P_t, \overline{P}_t, Q_t\right)_{t\in[0,\infty)}$ by letting $F_t := \widehat{F}_t$, $\overline{F}_t := \widehat{\overline{F}}_t$, $Q_t := \widehat{Q}_t$, and,

$$P_t := \mathbb{E}\left[e^{-\theta(\widehat{\sigma}^\star(t) - t)}\min\left\{\widehat{P}_{\widehat{\sigma}^\star(t)}, h + \widehat{U}_{\widehat{\sigma}^\star(t)}, r\right\}Z_{\widehat{\sigma}^\star(t)} - \int_t^{\widehat{\sigma}^\star(t)} e^{-\theta(\zeta - t)}\widehat{F}_\zeta dZ_\zeta \,\middle|\, T \leq t, Z_t = 1, H_t = 0, \mathcal{I}_t, \widehat{\mathcal{P}}\right], \quad \forall \mathcal{I}_t. \tag{D.3}$$

Clearly, $\mathcal{P}$ is well defined and in particular, $F_t, \overline{F}_t$ and P_t are all bounded above by F. Because $\overline{F}_t := \widehat{\overline{F}}_t$ and $Q_t := \widehat{Q}_t$ by construction, we immediately have $\widehat{U}_t = U_t$ and $\widehat{W}_t = W_t$.

Now we demonstrate that the above-defined policy $\mathcal{P}$ satisfies the following properties.

Property 1: (6) holds, i.e., the agent prefers disclosure over evasion or self-correction. Indeed, the optimality of $\widehat{\sigma}^\star$ in (D.3) immediately implies $P_t \leq \min\left\{\widehat{P}_t, h + \widehat{U}_t, r\right\} \leq \min\left\{h + \widehat{U}_t, r\right\} = \min\{h + U_t, r\}$.

Property 2: Prompt disclosure is the agent's best response to $\mathcal{P}$. Indeed, we can follow the same argument as in (A.11) to show

$$P_t \leq \mathbb{E}\left[e^{-\theta(s-t)}Z_s P_s - \int_t^s e^{-\theta(\zeta - t)}F_\zeta dZ_\zeta \,\middle|\, T \leq t, Z_t = 1, H_t = 0, \mathcal{I}_t, \mathcal{P}\right], \quad \forall s \geq t. \tag{D.4}$$

That is, the agent's cost of immediate disclosure is always dominated by the agent's total expected discounted cost of delaying the disclosure to any stopping time $s \geq t$ under $\mathcal{P}$. As such, the agent always prefers to disclose without delay.

Property 3: The principal is not worse off under $\mathcal{P}$ than under $\widehat{\mathcal{P}}$. We first note that since $c \geq \theta r$ and $k \geq 0$, (D.2) (applied under $\widehat{\mathcal{P}}$) implies that

$$\begin{aligned}
\widehat{V}_t =& \mathbb{E}\left[\int_t^{\infty} e^{-\theta(\zeta-t)}\left\{Z_\zeta\left(cd\zeta + kdN_\zeta\right) - (r - \widehat{\overline{F}}_\zeta)dZ_\zeta\right\}\middle| T \leq t, Z_t = 1, H_t = 1, \mathcal{I}_t, \widehat{\mathcal{P}}\right] \\
\geq& r\mathbb{E}\left[\theta\int_t^{\infty} e^{-\theta(\zeta-t)} Z_\zeta d\zeta - \int_t^{\infty} e^{-\theta(\zeta-t)} dZ_\zeta \middle| T \leq t, Z_t = 1, H_t = 1, \mathcal{I}_t, \widehat{\mathcal{P}}\right] \\
&+ \mathbb{E}\left[\int_t^{\infty} e^{-\theta(\zeta-t)}\widehat{\overline{F}}_\zeta dZ_\zeta \middle| T \leq t, Z_t = 1, H_t = 1, \mathcal{I}_t, \widehat{\mathcal{P}}\right] \\
=& r - \widehat{U}_t,
\end{aligned} \tag{D.5}$$

where the last equality follows from (A.13) and the definition in (D.1). Then, similar to (A.14), we can use (D.5) to show that $C(\mathcal{P}, T) \leq C\left(\widehat{\mathcal{P}}, \widehat{\sigma}^\star\right)$, i.e., the principal's expected cost under policy $\mathcal{P}$ is no larger than that under $\widehat{\mathcal{P}}$. In particular, we note that the principal saves on the inspection cost k and damage cost c thanks to the agent's prompt disclosure under $\mathcal{P}$.

Property 4: It is optimal for the principal to set $F_t = \overline{F}_t := F$ for all $t \geq 0$, which immediately yields the recursive representation of (A.4) and (A.11) in (7), (8), (9) and (10) by following a similar derivation as in Lemma 1 of Wang et al. (2016). As shown by the previous properties, the agent will choose to disclose without delay nor evasion under $\mathcal{P}$, and hence the principal's expected payoff is given by $C(\mathcal{P}, T) = \mathbb{E}\left[k\int_0^T e^{-\theta t} dN_t + e^{-\theta T}(r - P_T) \middle| \mathcal{P}\right]$, in which the variables F_t and $\overline{F}_t$ are absent. Variable F_t only appears on the right-hand side of the constraint (D.4) and variable $\overline{F}_t$ only on the right-hand side of (6) through U_t in (D.1). Therefore, it is optimal for the principal to relax the constraints (D.4) and (6) by setting both F_t and $\overline{F}_t$ to the maximum F.

Property 5: Policy $\mathcal{P}$ and policy $\widehat{\mathcal{P}}$ are payoff-equivalent to the agent, i.e., $C_a(\mathcal{P}, T) = C_a(\widehat{\mathcal{P}}, \widehat{\sigma}^\star)$. This is because, by construction in (D.3), P_t is the agent's minimum expected discounted cost from t onwards under policy $\widehat{\mathcal{P}}$ by following $\widehat{\sigma}^\star$ as the response. On the other hand, by Properties 1 and 2, the agent will always promptly disclose at time T under policy $\mathcal{P}$ and hence incur the same expected cost P_t, leading to the conclusion. □

Proof of Proposition 2. With an exogenous detection at rate μ, the agent's disclosure incentive is strengthen and it is straightforward to show, by replicating the proof of Theorem 1, that it is optimal for the principal to impose maximal fine F upon exogenous detection and to restrict to policies that induce the agent's prompt disclosure (and hence the penalty F will never realize). That is, the principal can optimize within the class of policies $\widetilde{\mathcal{P}} = \left(\widetilde{P}_t, \widetilde{Q}_t\right)_{t\in[0,\infty)}$ satisfying

$$\widetilde{P}_t \leq \min\{r, h + \widetilde{U}_t\}, \quad \text{for all } t \geq 0, \tag{D.6}$$

with the dynamic evolution of $\widetilde{U}_t$ and $\widetilde{P}_t$ given as follows:

$$\widetilde{U}_t = (1 - \tilde{q}_t^m)\widetilde{U}_{t+} + \tilde{q}_t^m\left(\beta F + (1-\beta)\widetilde{U}_{t+}^I\right), \quad \text{for } \tilde{q}_t^m > 0, \tag{D.7}$$

$$\frac{d\widetilde{U}_t}{dt} = (\theta + \mu)\widetilde{U}_t - \mu F - \tilde{q}_t^n\left[\beta F + (1-\beta)\widetilde{U}_{t+}^I - \widetilde{U}_t\right], \quad \text{for } \tilde{q}_t^m = 0, \tag{D.8}$$

$$\widetilde{P}_t \leq (1 - \tilde{q}_t^m)\widetilde{P}_{t+} + \tilde{q}_t^m F, \quad \text{for } \tilde{q}_t^m > 0, \quad \text{and} \tag{D.9}$$

$$\widetilde{P}_t \le \widetilde{P}_{t+}, \quad \text{or} \quad \frac{d\widetilde{P}_t}{dt} \ge (\theta+\mu)\widetilde{P}_t - \mu F - \tilde{q}_t^n\left(F - \widetilde{P}_t\right), \quad \text{for } \tilde{q}_t^m = 0, \tag{D.10}$$

where $\widetilde{U}_{t+}$ (resp., $\widetilde{U}^I_{t+}$) is the value of $\widetilde{U}_t$ right after time t in the absence (resp., presence) of an audit, and $\widetilde{P}_{t+}$ (resp., $\widetilde{P}^I_{t+}$) is the value of $\widetilde{P}_t$ right after time t in the absence (resp., presence) of an audit. Under such a policy satisfying (D.6)–(D.10), the principal's objective can be written as

$$\frac{\lambda}{\theta+\lambda} r + \min_{\widetilde{\mathcal{P}}} \mathbb{E}\left[k\int_0^T e^{-\theta t} dN_t - e^{-\theta T}\widetilde{P}_T \,\middle|\, \widetilde{\mathcal{P}}\right]. \tag{D.11}$$

Now, with slight abuse of notation, let $\widetilde{F} = \frac{\theta}{\theta+\mu}F$, $\widetilde{r} = r - \frac{\mu}{\theta+\mu}F$ (which is positive because of the assumption $\mu < \bar{\mu} := \min\{\lambda, \frac{\theta r}{(F-r)^+}\}$) and policy $\mathcal{P}$ be

$$Q_t = (q_t^m, q_t^n) := \widetilde{Q}_t = (\tilde{q}_t^m, \tilde{q}_t^n), \quad P_t := \widetilde{P}_t - \frac{\mu}{\theta+\mu}F, \quad \text{and} \quad U_t := \widetilde{U}_t - \frac{\mu}{\theta+\mu}F, \quad \text{for all} t \ge 0. \tag{D.12}$$

Then, under this variable transformation, the constraints (D.6)–(D.10) can be rewritten as

$$P_t \le \min\{\widetilde{r}, h + U_t\}, \quad \text{for all } t \ge 0, \tag{D.13}$$

$$U_t = (1-q_t^m)U_{t+} + q_t^m\left(\beta\widetilde{F} + (1-\beta)U^I_{t+}\right), \quad \text{for } q_t^m > 0, \tag{D.14}$$

$$\frac{dU_t}{dt} = (\theta+\mu)U_t - q_t^n\left[\beta\widetilde{F} + (1-\beta)U^I_{t+} - U_t\right], \quad \text{for } q_t^m = 0, \tag{D.15}$$

$$P_t \le (1-q_t^m)P_{t+} + q_t^m\widetilde{F}, \quad \text{for } q_t^m > 0, \quad \text{and} \tag{D.16}$$

$$P_t \le P_{t+}, \quad \text{or} \quad \frac{dP_t}{dt} \ge (\theta+\mu)P_t - q_t^n\left(\widetilde{F} - P_t\right), \quad \text{for } q_t^m = 0. \tag{D.17}$$

Similarly, the principal's objective can be rewritten as

$$\begin{aligned}
&\frac{\lambda}{\theta+\lambda}\widetilde{r} + \min_{\mathcal{P}} \mathbb{E}\left[k\int_0^T e^{-\theta t} dN_t - e^{-\theta T}P_T \,\middle|\, \mathcal{P}\right]\\
=&\frac{\lambda}{\theta+\lambda}\widetilde{r} + \min_{\mathcal{P}} \mathbb{E}\left[\int_0^\infty e^{-(\lambda+\theta)t}\left(k dN_t - \lambda P_t\right) \,\middle|\, \mathcal{P}\right]\\
=&\frac{\lambda}{\theta+\lambda}\widetilde{r} + \min_{\mathcal{P}} \frac{\lambda}{\lambda-\mu}\mathbb{E}\left[\int_0^\infty e^{-(\theta+\mu+\lambda-\mu)t}\left(\frac{\lambda-\mu}{\lambda}k dN_t - (\lambda-\mu)P_t\right) \,\middle|\, \mathcal{P}\right]
\end{aligned} \tag{D.18}$$

Similar to Lemma A.1, the constraints (D.13)–(D.17) imply the feasible range of (P_t, U_t) is $\widetilde{\Omega}(h,\beta) := \left\{(p,u) : 0 \le \beta p \le u \le p \le \widetilde{r} \wedge \widetilde{F},\ p \le h + u\right\}$. Therefore, the principal's problem (D.18) subject to (D.13)–(D.17) is equivalent to the principal's problem in (11) with (i) the discounting rate θ replaced by $\theta+\mu$, (ii) the limited liability F by $\widetilde{F} = \frac{\theta}{\theta+\mu}F$, the remedial cost r by $\widetilde{r} = r - \frac{\mu}{\theta+\mu}F$, (iv) the hazard rate λ by $\lambda-\mu$ (which is positive because of the assumption $\mu < \bar{\mu} := \min\{\lambda, \frac{\theta r}{(F-r)^+}\}$), and (v) the auditing cost k by $k(\lambda-\mu)/\lambda$, as shown by the last line of equation in (D.18). □

Proof of Proposition 3. First, we note that Theorem 1 still applies to (24), because Theorem 1(1)-(3) only depends on the agent's IC constraint in (24), which is the same as (5), and Theorem 1(4) also holds as the principal's cost is enlarged to account for the agent's portion. Thus, we can again restrict to the optimization among policies that induce prompt disclosure, under which the principal's problem is reformulated as (11) with the constraints unchanged but the objective function replaced by

$$\begin{aligned}
(1+\alpha)C(\mathcal{P},\sigma) + C_a(\mathcal{P},\sigma) =&\frac{\lambda}{\theta+\lambda}(1+\alpha)r + \mathbb{E}\left[(1+\alpha)k\int_0^T e^{-\theta t}dN_t - e^{-\theta T}\alpha P_T \,\middle|\, \mathcal{P}\right]\\
=&\alpha\left\{\frac{\lambda}{\theta+\lambda}(1/\alpha+1)r + \mathbb{E}\left[(1/\alpha+1)k\int_0^T e^{-\theta t}dN_t - e^{-\theta T}P_T \,\middle|\, \mathcal{P}\right]\right\},
\end{aligned}$$

which the same (up to a constant difference) as the objective function of (11) by replacing k with $(1/\alpha+1)k$.

□

Proof of Proposition 4. Straightforward calculation yields

$$\begin{aligned}
&\mathbb{E}\left[c\int_T^{\tau(T)} e^{-\theta t}dt + e^{-\theta\tau(T)}\left(k+r-\underline{F}\right)\middle|\mathcal{P}\right]\\
=&\mathbb{E}\left[e^{-\theta T}\left\{c\int_0^{\tau(T)-T} e^{-\theta s}ds + e^{-\theta(\tau(T)-T)}\left(k+r-\underline{F}\right)\right\}\middle|\mathcal{P}\right]\\
=&\mathbb{E}\left[e^{-\theta T}\left\{c/\theta + e^{-\theta(\tau(T)-T)}\left(k+r-\underline{F}-c/\theta\right)\right\}\middle|\mathcal{P}\right],
\end{aligned}$$

which reduces to a constant $\frac{\lambda}{\theta+\lambda}\frac{c}{\theta}$ if $\underline{F}=k+r-c/\theta$, rendering the objective function in (25) to

$$\frac{\lambda}{\theta+\lambda}\left[\delta r+(1-\delta)c/\theta\right]+\delta\mathbb{E}\left[k/\delta\int_0^T e^{-\theta t}dN_t - e^{-\theta T}P_T\middle|\mathcal{P}\right]. \tag{D.19}$$

Thus, the proposition follows immediately by contrasting (D.19) with (11). □

References

Brémaud P (1981) *Point Processes and Queues* (Springer-Verlag).

Davis MHA (1993) *Markov Models and Optimization* (Chapman & Hall/CRC).

Lipster RS, Shiryaev AN (2010) *Statistics of Random Processes: II. Applications* (Springer), 2 edition.

Wang S, Sun P, de Véricourt F (2016) Inducing environmental disclosures: A dynamic mechanism design approach. *Oper. Res.* 64(2):371–389.