Modulate $M = 2^J$ discrete messages or $J$ bits of information into amplitude of signal

If amplitude mapping changes at symbol rate of $f_{sym}$ then bit rate is $R_b = J f_{sym}$

Conventional mapping of discrete messages to $M$ uniformly space amplitudes

$$ a_i = d(2i - 1) \quad i = -\frac{M}{2} + 1, \ldots, 0, \ldots, \frac{M}{2} $$

$$ s(t) = \sum_{k=-\infty}^{\infty} a_k \delta(t - k T_{sym}) $$

No pulses overlap in time: requires infinite bandwidth
Impulse modulator

- Represent the symbol sequence by the Dirac impulse train

\[ s(t) = \sum_{k=-\infty}^{\infty} a_k \delta(t - kT_s) \]

- The impulse modulator block forms this function. This impulse train is applied to a transmit pulse shaping filter so that the signal is band limited to the channel bandwidth.
Pulse Shaping Block Diagram

- Upsampling by \( L \) denoted as \( \uparrow L \)
  - Outputs input sample followed by \( L-1 \) zeros
  - Upsampling by converts symbol rate to sampling rate
- Pulse shaping (FIR) filter \( p_{Tsym}[m] \)
  - Fills in zero values generated by upsampler
  - Multiplies by zero most of time (\( L-1 \) out of every \( L \) times)
DSP Implementation

- Random bit generation
  ...001100100111010....
- Mapping bits onto symbols, 1→1, 0→-1
- Upsampling to match the sampling rate
- Pulse shaping filter
- Send the output samples through serial port D/A
Intersymbol Interference

- If the analog pulse is wider than the time between adjacent symbols, the outputs from adjacent symbols may overlap.
  - A problem called intersymbol interference (ISI)
- What kind of pulses minimize the ISI?
- Choose a shape that is one at time $kT$ and zero at $mT$ for all $m \neq k$
- Then, the analog waveform contains only the value from the desired input symbol and no interference from other nearby input symbols.
- These are called *Nyquist Pulses*
Nyquist Pulses

- Sinc Pulse

\[ p_S(t) = \frac{\sin \pi f_0 t}{\pi f_0 t} \]

- where \( f_0 = 1/T \). Sinc is Nyquist pulse because \( p_S(0) = 1 \) and \( p_S(kT) = \sin(\pi k)/\pi k = 0 \).
- Sinc envelope decays at \( 1/t \).

- Raised-cosine pulse:

\[ p_{RC}(t) = 2f_0 \left( \frac{\sin(2\pi f_0 t)}{2\pi f_0 t} \right) \left[ \frac{\cos(2\pi f_\Delta t)}{1 - (4f_\Delta t)^2} \right] \]

- with roll-off factor \( \beta = f_\Delta/f_0 \).
- \( T = 1/2f_0 \) because \( p_{RC} \) has a sinc factor
- \( \sin(\pi k)/\pi k \) which is zero for all nonzero integers \( k \).
- Raised-cosine envelope decays at \( 1/|t^\beta| \).
- As \( \beta \to 0 \), raised-cosine \( \to \) sinc.
Frequency Domain

- Fourier transform

\[ P_{RC}(f) = \begin{cases} 
1, & |f| < f_1 \\
\frac{1 + \cos(\alpha)}{2}, & f_1 < |f| < B \\
0, & |f| > B 
\end{cases} \]

where
- \( B \) is the absolute bandwidth,
- \( f_0 \) is the 6db bandwidth,
- \( f_\Delta = B - f_0 \),
- \( f_1 = f_0 - f_\Delta \), and
- \( \alpha = \pi(|f| - f_1)/2f_\Delta \)
Spectrum

- Spectral comparison of rectangular and raised-cosine pulses
  - Note the band-limitation of raised-cosine shaping
Eye Diagram

- *Eye diagram is a popular robustness evaluation tool.*
- For 4-PAM, single-baud-wide Hamming blip with additive broadband channel noise, retriggering oscilloscope after every 2 baud intervals produces
Eye Diagrams

- Eye diagrams with raised-cosine pulse shaping with 2-PAM and 4-PAM systems