Active Influence in Dynamical Models of Structural Balance in Social Networks

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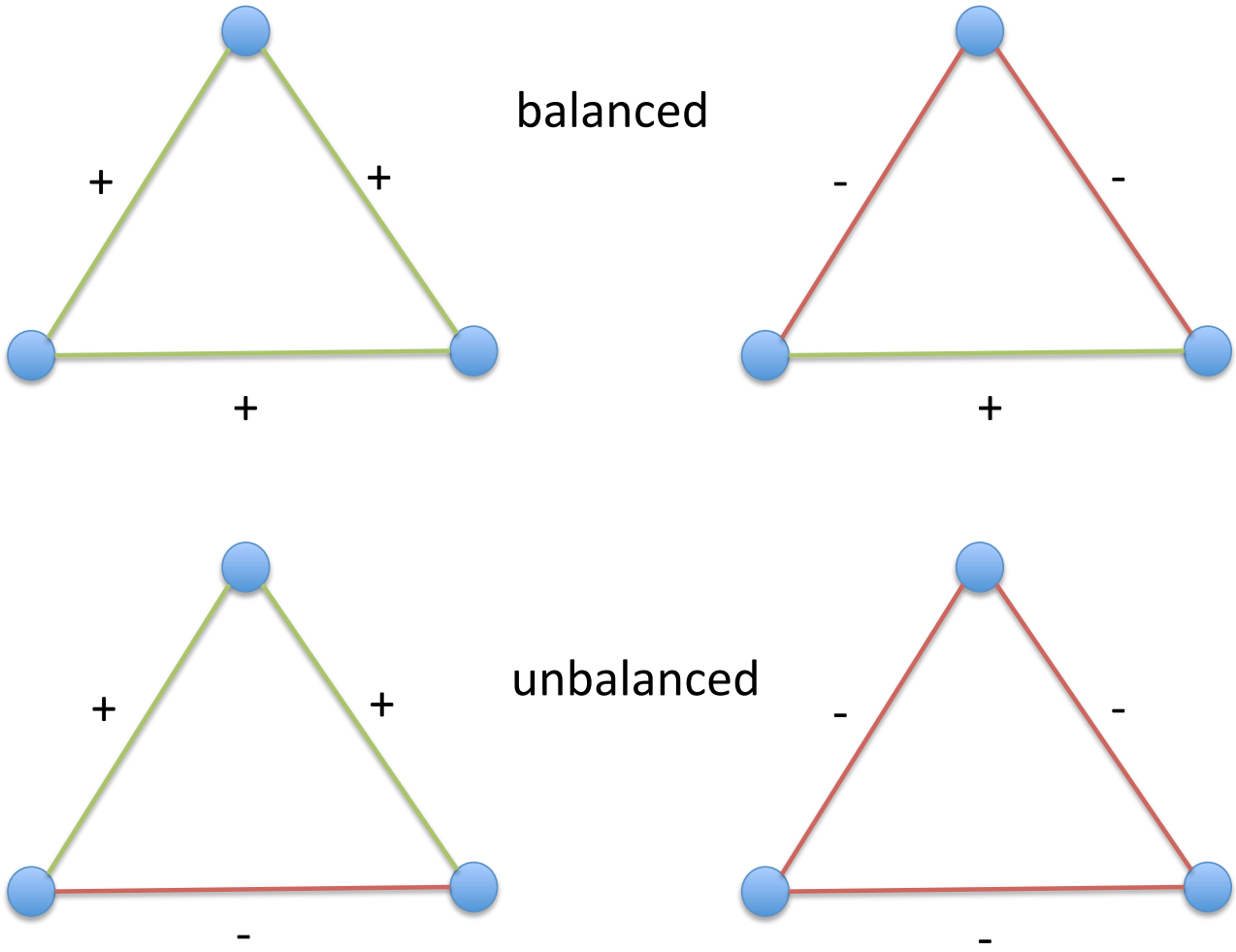
positive + negative links

- links in social networks often have positive connotations, e.g friendship, collaboration, information sharing, etc.
- links with negative connotations, e.g. antagonism, distrust, disagreement, etc., also play key role in structure and dynamics
- concept of *structural balance*
  - very old idea in social psychology (Heider 1940s, Harary & Cartwright 1950s)
  - new data in online social networks (e.g. epinions, slashdot, wikipedia)

structural balance in three agent networks



- friend of my friend is my friend
- enemy of my enemy is my friend

structural balance in n-agent networks

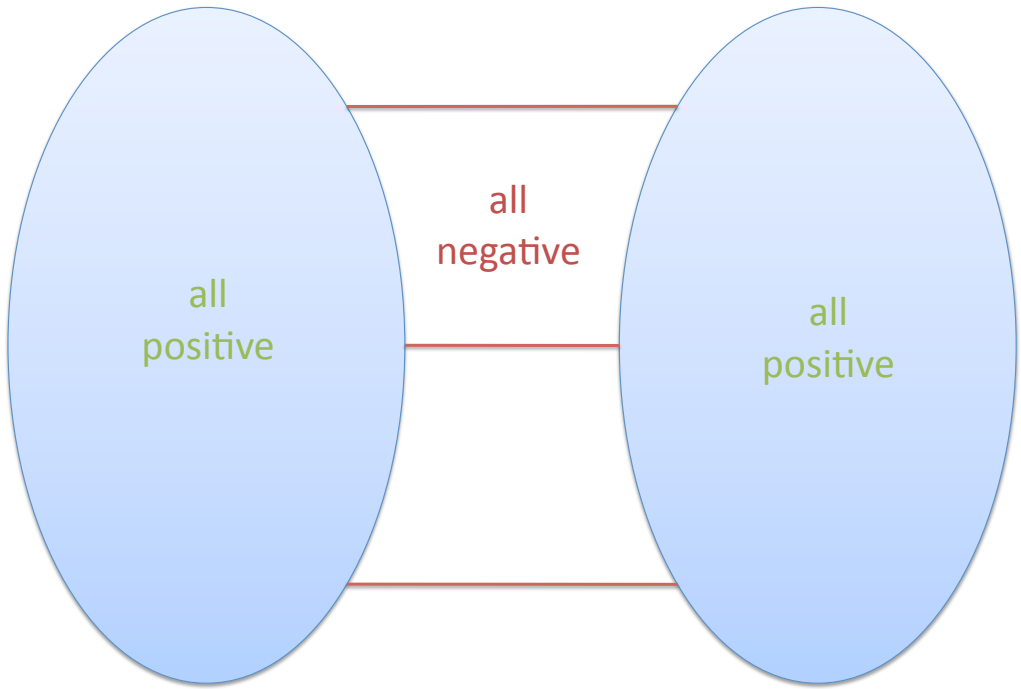
- focus on complete networks
- define as balanced if all triangles balanced

structural balance in *n*-agent networks

- focus on complete networks
- define as balanced if all triangles balanced

Theorem (Cartwright & Harary 1956)

A complete, signed network is structurally balanced iff it can be partitioned into two factions such that within each faction all relationships are positive and between the two factions all relationships are negative.



dynamical models of structural balance

- if initially unbalanced, how might network state evolve toward a balanced state?

- discrete state models, $x_{ij} \in \{+, -\}$
  - edge flipping
  - problems with "jammed states" (i.e. local equilibria)

- continuous state models (Kulakowski et al 2005, Marvel et al 2011)
  - associate "friendliness level" w/ each agent pair, $x_{ij} \in \mathbf{R}$
  - (symmetric) matrix ODE

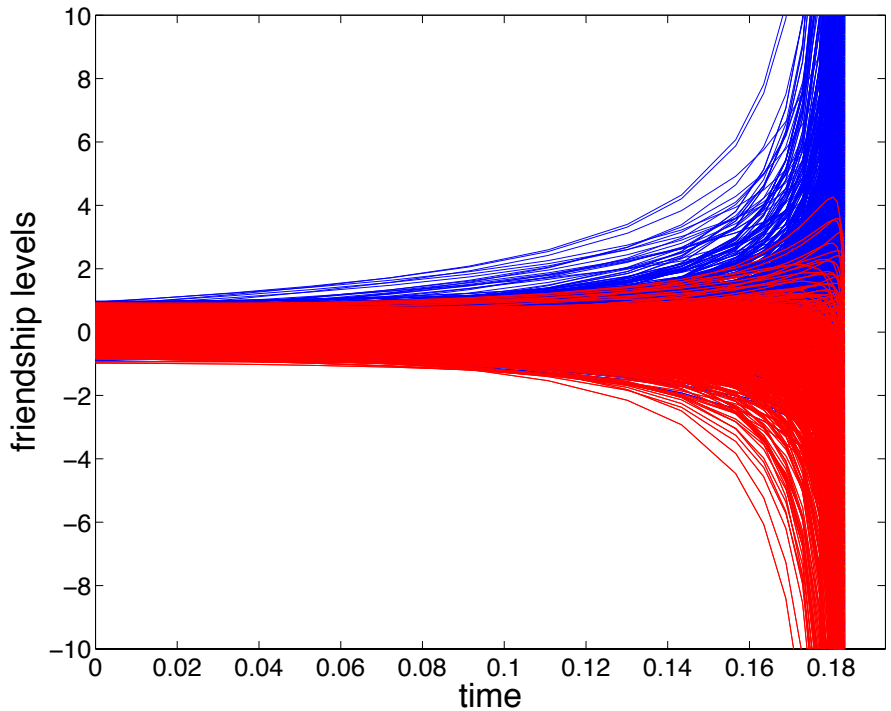
$$\dot{X} = X^2, \qquad \dot{x}_{ij} = \sum_k x_{ik} x_{kj}$$

dynamical models of structural balance: theoretical results

Theorem (Marvel et al 2011)

For almost all initial conditions, the system converges to a structurally balanced state in finite time. Furthermore, the final balanced state determined by the sign pattern of eigenvector associated with largest eigenvalue of initial state matrix.

- can *predict* eventual factions based on current state



active influence

- what if an agent actively influences the system?

$$\dot{X} = X^2 + U, \qquad U = \begin{bmatrix} u_{11} & u_1^T \\ u_1 & 0 \end{bmatrix}$$

- suppose single agent can measure/estimate initial friendliness matrix and can perturb its associated entries in initial friendliness matrix

$$\Delta X_0 = \begin{bmatrix} \delta x_{11} & \delta x_1^T \\ \delta x_1 & 0 \end{bmatrix}$$

Problem

Given X_0, find, if it exists, a ΔX_0 such that the eigenvector associated with the largest eigenvalue of $\widetilde{X}_0 = X_0 + \Delta X_0$ has a specified sign pattern.

active influence: main result

Theorem

A perturbation exists and can be computed to achieve any desired structurally balanced state given any initial state.

- straightforward linear algebra

$$(X_0 + \Delta X_0)(v + \delta v) = (\lambda + \delta\lambda)(v + \delta v)$$

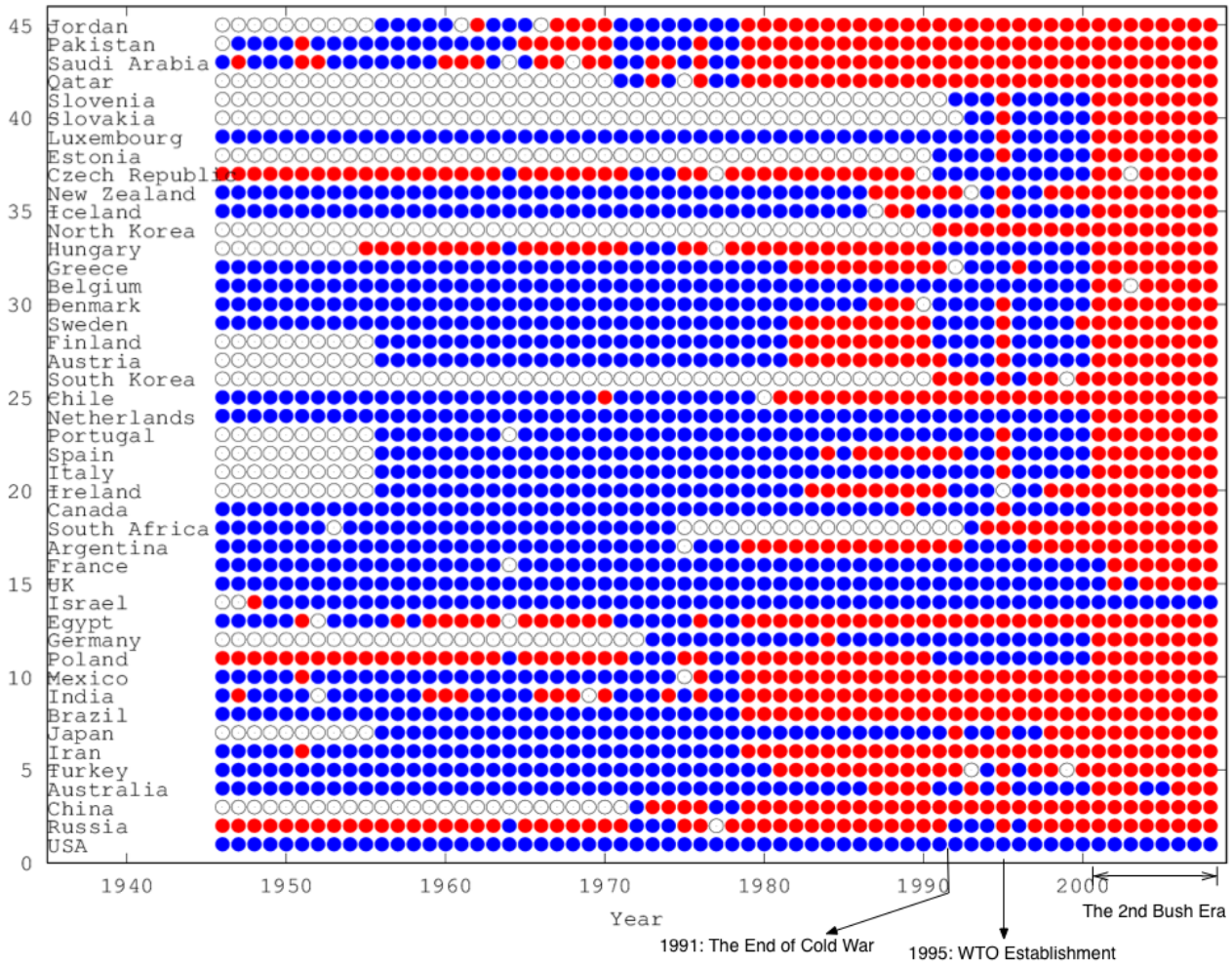
- optimization problem

$$\begin{aligned} &\text{minimize} && ||\Delta X_0|| \\ &\text{subject to} && \mathit{sign}[v_1(X_0 + \Delta X_0)] = v^* \end{aligned}$$

  - heuristic method compute "small" perturbation

active influence: international relations application

- data from UN General Assembly voting records from 1946 to 2008 to estimate friendliness levels amongst various countries



- interpretation: prescribes foreign policy investment allocation

conclusions + outlook

- active influence in dynamical models of structural balance in social networks
- single agent can perturb its own relationships to achieve any desired structurally balanced state
- future work
  - more general notion of structural balance, non-complete graphs
  - games with two or more agents with active influence
  - study online networks: epinions, slashdot, wikipedia
  - study polarization processes

questions?