Spectral Simulation of Hybrid Bodies with Deformable and Rigid Materials

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Abstract
A novel spectral framework is proposed in order to simulate a hybrid body with both deformable and rigid materials in real-time. We extend the modal warping technique from deformable models to rigid bodies and employ a new constraint strategy to eliminate the accumulation of approximation errors at the boundary. The powerful parallel computation ability of the GPU is utilized as well to update the geometry of the hybrid object. This work provides a general-purpose solution to simulating hybrid objects in real-time, even for large-scale models.

Spectral Simulation
The Euler-Lagrange equation is projected onto spectral domain by solving an eigenproblem. After that, the motion is represented as the combination of independent vibrations with different frequency. High frequency vibrations are ignored. As a result, the spectral motion equation is of much smaller size than the spectral one.

\[ \mathbf{M}_u + \mathbf{C}_u + \mathbf{K}_u = f \]

#DoFs: \(3 \times n\)

\[ \mathbf{M}_q + \mathbf{C}_q + \mathbf{K}_q = \mathbf{\Phi}^T (\mathbf{R}^T f) \]

#DoFs: \(k, k << n\)

Boundary Handling
The interface from different regions must be overlapped by imposing constraints on the nodes at the interface. Such constraints are called boundary constraints: \(u_j = u_j^r\).

- An intuitive definition
  \[ E_j^d \mathbf{R}_j \mathbf{\Phi}_d \mathbf{q}_d = E_j^d \mathbf{R}_j \mathbf{\Phi}_d \mathbf{q}_r, \]

- A fortified definition
  \[ E_j^d \mathbf{\Phi}_d \mathbf{q}_d = E_j^d \mathbf{\Phi}_d \mathbf{q}_r, \]
  \[ E_j^r \mathbf{\Psi}_r \mathbf{q}_r = E_j^r \mathbf{\Psi}_r \mathbf{q}_r. \]

The definition decomposes the boundary constraint into two subconstraints: 1) a constraint on linear deformation 2) a constraint on linear rotation.

Importing more constraints into the system, however, brings overhead. For every constrained boundary node, the number of vanished DoFs raises from 3 to 6, indicating that twice as many constraints are induced. For our DoF-reduced spectral simulator, the increased number of eliminated DoF due to boundary constraints could turn out to degenerate the system, i.e. there may not be enough available DoF to allow the body to deform properly. This causes the system to be over-constrained.

We resolve this problem by capping the number of constrained nodes at the boundary. Spectral simulator implicitly neutralizes the demand of the number of boundary constraints. So fewer boundary constraint nodes will not produce much boundary inconsistency.

Constraint Manipulation
Other users’ manipulation constraints including either position or orientation constraints are necessary as well. These constraints can be formulated in a symmetric fashion:

\[ \begin{bmatrix} E_j^d \mathbf{R}_j \mathbf{\Phi}_d & 0 \\ E_j^d \mathbf{R}_j \mathbf{\Psi}_d & 0 \\ 0 & E_j^r \mathbf{R}_j \mathbf{\Phi}_r \\ 0 & E_j^r \mathbf{R}_j \mathbf{\Psi}_r \end{bmatrix} \begin{bmatrix} \mathbf{q}_d \\ \mathbf{q}_r \end{bmatrix} = \mathbf{c} \]

Implementation and Results

<table>
<thead>
<tr>
<th>Model</th>
<th># Tetra</th>
<th># Mode</th>
<th>CPU Simulation (FPS)</th>
<th>GPU Simulation (FPS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bar1</td>
<td>323</td>
<td>100</td>
<td>269.1</td>
<td>845.2</td>
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<tr>
<td>Bar2</td>
<td>5,372</td>
<td>200</td>
<td>31.9</td>
<td>89.2</td>
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<tr>
<td>Gargoyle</td>
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<td>50</td>
<td>20.3</td>
<td>64.9</td>
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<tr>
<td>Armadillo</td>
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<td>100</td>
<td>14.3</td>
<td>43.6</td>
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<tr>
<td>Dragon</td>
<td>32,959</td>
<td>50</td>
<td>10.3</td>
<td>35.6</td>
</tr>
</tbody>
</table>

The dragon model of 32,959 tetrahedra opens its mouth with two rigid jaws (purple).

Simulate leg behavior of Armadillo model.

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