New model for fiberoptic Raman gain spectrum and response function

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BACKGROUND

- The FPWE for the Stokes or signal field $\mathcal{E}_S(z, t)$ in the SVEA:

\[
\left( \frac{\partial}{\partial z} + \beta, 1 \frac{\partial}{\partial t} + \frac{i}{2} \beta, 2 \frac{\partial^2}{\partial t^2} - \frac{1}{6} \beta, 3 \frac{\partial^3}{\partial t^3} \right) \mathcal{E}_S(z, t) = -\frac{\alpha}{2} \mathcal{E}_S(z, t) + \frac{2\pi i \omega_0^2}{\beta_0 c^2} P_S(z, t)
\]

\[\text{where} \quad \beta,j = \left. \frac{d^j \beta}{d \omega^j} \right|_{\omega=\omega_0}\]

- Nonlinear polarization

\[P_S = P_{S,i} + P_{S,R}\]

- Instantaneous (electronic) term:

\[P_{S,i} \propto (1 - f_R) \mathcal{E}_S \left[ |\mathcal{E}_S|^2 + 2 |\mathcal{E}_L|^2 \right]\]

where $f_R$ is the Raman fraction
TIME-DELAYED POLARIZATION

- Time-delayed (vibrational) term:

\[ P_{S,R}(t) \propto i f_R \mathcal{E}_S(z, t) \int_{-\infty}^{\infty} h(t-t''', 0) e^{-i(\omega_L-\omega_S)(t-t''')} \mathcal{E}^*_L(z, t''') \mathcal{E}_S(z, t''') \, dt'''

where \( \mathcal{E}_L \) is the laser or pump field envelope

▷ Convolution of the Raman response function \( h \) with the force that drives the vibrational oscillator

▷ Provides for non-instantaneous (transient) buildup of Raman gain

▷ Can compute integral in time domain (signal space)

▷ Can also compute integral in transform space as product of transfer function and convolution of \( \mathcal{E}^*_L \) and \( \mathcal{E}_S \)
RAMAN RESPONSE FUNCTION (1)

• Blow and Wood ("Theoretical description of transient stimulated Raman scattering in optical fibers", JQE 25, 2665–2673 (1989)) used a simple harmonic oscillator approach
  ▶ Single damped oscillator
  \[ h_R(t) \propto e^{-t/\tau_2} \sin(t/\tau_1) \]
  where
  \[ \tau_1 = 1/\omega_v = 12.2 \text{ fs} \]
  \[ \tau_2 = 1/\gamma = 32 \text{ fs} \]

• Raman gain spectrum can be obtained from the Raman response function via
  \[ g_R(\Delta \omega) = \frac{\omega_0}{cn_0} f_R \chi^{(3)} \text{Im} \left[ \tilde{h}_R(\Delta \omega) \right] \]
  ▶ yielding a Lorentzian
  \[ g_R(\Delta \omega) \propto \frac{2\omega \gamma}{(\omega^2 - \omega_v^2)^2 + (2\omega \gamma)^2} \]
Raman response function (2)

\[ h_R(t) \propto e^{-t/\tau_2} \sin \left( \frac{t}{\tau_1} \right) \]

(t-t') fs

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$g_R(\Delta \omega) \propto \frac{2\omega \gamma}{(\omega^2 - \omega_0^2)^2 + (2\omega \gamma)^2}$

RAMAN GAIN SPECTRUM (1)
RAMAN GAIN SPECTRUM ANALYSIS

BROADENING

- Inhomogeneous
  - Random perturbations of vibrational frequencies
  - Gaussian via central limit theorem
- Homogeneous
  - Single vibrational frequency
  - Lorentzian
- Combined inhomogeneous and homogeneous
13–MODE MODEL

- Response function:
  \[ h(t, 0) = \sum_{i=1}^{13} \frac{A'_i}{\omega_{v,i}} e^{-\gamma_{it} e^{-\Gamma^2_{it}^2/4}} \sin(\omega_{v,i}t) \theta(t) \]

- Spectrum:
  \[ s(\omega) = \sum_{\ell=1}^{13} \frac{A'_{\ell}}{2\omega_{v,\ell}} \int_0^\infty \left\{ \cos \left[ (\omega_{v,\ell} - \omega)t \right] - \cos \left[ (\omega_{v,\ell} + \omega)t \right] \right\} e^{-\gamma_{\ell} t} e^{-\Gamma^2_{\ell} t^2/4} dt \]
RAMAN RESPONSE FUNCTION (3)

- Experimental
- Computed

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RAMAN GAIN SPECTRUM (2)

The graph shows the Raman gain spectrum with wavenumbers on the x-axis and the gain on the y-axis. The spectrum is divided into experimental and computed data, with the experimental data represented by a solid blue line and the computed data by a dotted red line.
CONCLUSIONS

- Detailed, spectroscopically accurate model of Raman gain and response function in silica fibers
- Different mode amplitudes for each fiber segment
  - Low-frequency (bending) modes are most likely to vary from fiber to fiber
  - Dopants may create extra modes or modify existing ones