OFDM AF Relaying Under I/Q Imbalance: Performance Analysis and Baseband Compensation

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Abstract—We analyze the outage performance of half-duplex amplify-and-forward relaying in an OFDM system with MRC detection in the presence of I/Q imbalance and compare it with that of the direct transmission mode. Both analytical and numerical results demonstrate that the direct mode can outperform the amplify-and-forward mode even under moderate levels of uncompensated I/Q imbalance. The cross-over I/Q imbalance levels are determined analytically to be inversely proportional to the cube of the signal constellation size. In addition, we present a low-complexity receiver-based digital baseband I/Q imbalance compensation scheme for the amplify-and-forward mode and analyze its EVM performance. Furthermore, we derive accurate analytical approximations for the EVM performance as a function of relay location and I/Q imbalance level with and without compensation.

Index Terms—I/Q imbalance, amplify-and-forward relay, OFDM, EVM.

I. INTRODUCTION

RELAY-ASSISTED networks have received renewed significant attention in the past few years after the pioneering work in [1]. Furthermore, relaying has been recently adopted in the long-term evolution (LTE)-advanced standard to meet the international Mobile Telecommunications (IMT)-advanced requirements [2]. Two of the most common relaying protocols are decode-and-forward (DF) and amplify-and-forward (AF). Our focus in this paper is on AF relays because they are more cost effective since the signal is only amplified and not decoded at the relay compared to DF relays. The performance of AF-based relaying has been analyzed in several papers such as [3], assuming an ideal radio frequency (RF) front-end. In practice, and for hardware implementation reasons, the AF relays typically down-convert, amplify at baseband, and then up-convert the signal (for more details, see [4]). Moreover, and most importantly, AF relays, specifically half-duplex relays, must buffer the received signal block in the first time slot until it is transmitted (after amplification) in the second time slot which is essential for synchronization. This buffering operation cannot be efficiently done in the analog domain. Instead, it is done digitally at baseband [5]. Therefore, due to the down/up conversion operations at the AF relay and the cost constraints on its RF front-end, AF relays can suffer from several major RF impairments such as carrier frequency offset (CFO), phase noise (PN) and I/Q imbalance (IQI) [6]. In [5], the authors considered CFO and analyzed its effects on AF relays, while the impact of PN on the performance of AF-based relay networks was analyzed in [7] for orthogonal frequency-division multiplexing (OFDM) systems. IQI effects in AF relay networks were investigated in [8]. In contrast to this paper, the authors in [8] did not consider OFDM systems and did not consider IQI effects at the relay itself.

We summarize our main contributions as follows. First, we compare the outage performances of the AF and direct (no relay) modes and show that the direct mode can outperform the relay mode even for moderate IQI levels. Second, we derive the IQI threshold level above which the direct mode outperforms the AF relay mode in the outage probability sense. Finally, we drive accurate analytical approximations for the error vector magnitude (EVM) of joint channel and IQI compensation scheme based on both zero-forcing (ZF) and conventional maximum ratio combings (MRC) detection. To the best of our knowledge, this is the first paper to analyze IQI effects in OFDM AF relay systems and consider IQI at the relay itself.

The rest of this paper is organized as follows. The system model is described in Section II and the performances of the AF and direct modes without IQI compensation are analyzed in Section III. We present channel estimation and digital baseband compensation algorithms in Section IV. In the same section, we analyze the EVM performance of the presented compensation algorithm. Simulation results and conclusions are given in Sections V and VI, respectively.

Notations: Unless otherwise stated, lower and upper case bold letters denote vectors and matrices, respectively. The matrices $\mathbf{I}$ and $\mathbf{F}$ denote, respectively, the identity matrix and the Fast Fourier Transform (FFT) matrix whose middle row corresponds to the DC, and their subscripts denote theirs sizes. For matrices, $\mathbf{A}$, $\mathbf{F}$, and $\mathbf{I}_N$ are square matrices of size $N$, while for vectors, $\mathbf{a}$ where $\mathbf{I}_N = \mathbf{F}^\dagger \mathbf{F}$. $\mathbf{F}^\dagger$ is the reversal (image) permutation matrix. Also, $(\cdot)^H$, $(\cdot)^*$, and $(\cdot)^T$ denote the matrix complex-conjugate transpose, complex-conjugate, and transpose operations, respectively. The operators $\mathbb{E}[]$ and $||$ denote the statistical expectation and the absolute value, respectively.

II. SYSTEM MODEL

We consider the downlink transmission scenario in relay-assisted OFDM systems with one antenna at each of the base station (BST), relay station (RS) and user equipment (UE). The RS operates in the AF mode where the transmission offset (CFO), phase noise (PN) and I/Q imbalance (IQI) [6]. We consider half-duplex amplify-and-forward relaying in an OFDM system with MRC detection in the presence of I/Q imbalance and compare it with that of the direct transmission mode. Both analytical and numerical results demonstrate that the direct mode can outperform the amplify-and-forward mode even under moderate levels of uncompensated I/Q imbalance. The cross-over I/Q imbalance levels are determined analytically to be inversely proportional to the cube of the signal constellation size. In addition, we present a low-complexity receiver-based digital baseband I/Q imbalance compensation scheme for the amplify-and-forward mode and analyze its EVM performance. Furthermore, we derive accurate analytical approximations for the EVM performance as a function of relay location and I/Q imbalance level with and without compensation.

Index Terms—I/Q imbalance, amplify-and-forward relay, OFDM, EVM.
occurs over two time slots, as shown in Fig. 1. In the first slot, the BST transmits to both the RS and UE. In the second slot, the relay amplifies its received signal and forwards it to the UE while the BST remains silent. The fading channels of the three links are assumed independent, frequency-selective and fixed over at least one OFDM symbol duration. The RF front-end of the BST is assumed IQI-free while those of the RS and UE are both IQI-impaired where the RS suffers from both transmit and receive IQI. We consider frequency-independent IQI caused by the gain and phase mismatches between the I and Q branches [6] and they are denoted, respectively, by $\epsilon_x$ and $\theta_x$ where the subscript $x$ is the terminal identifier (BST, RS or UE) and the superscript $t/r$ denotes the up/down-conversion process, respectively. The time-domain (TD) baseband representation of the IQI-impaired signal is given by $g_{t/r}(t)$ [6] where

$$ g_{t/r}(t) = \mu_{t/r} g(t) + \nu_{t/r} g^*(t) $$

(1)

where $\mu_{t/r} = \cos\left(\frac{\theta_y}{2}\right) + i \epsilon_y \sin\left(\frac{\theta_y}{2}\right)$, the minus and plus signs are for up and down conversion, respectively, $\nu_{t/r} = \epsilon_y \cos\left(\frac{\theta_y}{2}\right) - i \sin\left(\frac{\theta_y}{2}\right)$, and $g(t)$ is the TD baseband IQI-free signal and the $g^*(t)$ term arises due to IQI effects. The TD received signals in the first time slot at the RS and UE after removing the cyclic-prefix (CP) are given, respectively, by

$$ r_R = \mu_R s_R + \nu_R s^* R $$

(2)

$$ r_D = \mu_D s_D + \nu_D s^* D $$

(3)

where $S$, $R$, and $D$ denote the BST, RS and UE, respectively, and $s$ is the TD $N \times 1$ data vector with $\mathbb{E}[s^H] = \eta_0 I_N$. The $N \times N$ circulant channel matrix $\mathbf{H}_{XY}$ models the channel from node $X$ to $Y$ and its first column is $[\mathbf{h}_{XY}^T \ 0_{1 \times N-L}]^T$, where $\mathbf{h}_{XY}$ is the channel impulse response (CIR) with $L$ complex Gaussian taps. Moreover, $n_R$ and $n_D$ are the circularly symmetric complex additive white Gaussian noise (AWGN) vectors with single-sided power spectra densities of $N_o$. In the second time slot, the relay amplifies the received signal and forwards it to the destination. To keep complexity low at the relay, the channel is not estimated at the relay. Hence, the amplification factor is defined as $\alpha = \sqrt{\frac{P_x}{P_R}}$ where $P_x$ is the average energy transmitted by the RS. Therefore, the TD baseband equivalent signal received at the UE in the second time slot is given by

$$ r_D^2 = (\mu_D s_D + \nu_D s^* D) s^* + z $$

(4)

where $z$ is the composite noise vector and

$$ \mathbf{H}_1 = (\mu_R s_R + \nu_R s^* R) a_{SR} \mathbf{H}_{RD} $$

(5)

$$ \mathbf{H}_2 = (\mu_R s_R + \nu_R s^* R) a_{SR} \mathbf{H}_{RD} $$

(6)

Based on the above model, the frequency-domain (FD) received signals at the UE in the first and second time slots are obtained by taking the FFT of (3) and (4), respectively, and can be written as follows

$$ r_D^1 = \mu_D s_D + \nu_D s^* D $$

(7)

$$ r_D^2 = (\mu_D s_D + \nu_D s^* D) s^* + z $$

(8)

where $H_1$ and $H_2$ can be easily obtained from (5) and (6) by replacing each $H_{XY}$ and $H_{XY}^*$ by $H_{XY}$ and $H_{XY}^*$, respectively, where $H_{XY}$ is the $N \times N$ diagonal matrix whose diagonal elements represent the channel frequency response (CFR) from node $X$ to $Y$. Moreover, $H_{XY}^*$ is the $k^\text{th}$ element on the main diagonal of the matrix $H_{XY}$, is an exponentially-distributed random variable with mean $\alpha_{k}^* Y$ which is proportional to $(d_{XY})^{-\xi}$ where $d_{XY}$ is the distance between node $X$ and node $Y$ and $\xi$ is the pathloss exponent. In addition, $s$, $n_D^1$, and $z$ are the frequency-domain representations of $s$, $n_D^1$, and $z$, respectively. For the definition of $(\cdot)^*$, please refer to the notation definitions in Section I where complex conjugation in TD results in reversal and complex conjugation in FD. Hence, IQI results in image (mirror) interference in FD where the $k^\text{th}$ subcarrier is distorted by its image subcarrier whose index is $-k$.

III. PERFORMANCE ANALYSIS WITHOUT IQI COMPENSATION

In this section, we investigate the effect of IQI on the performance at the UE where the channel is assumed perfectly known at the UE and we use MRC, which ignores IQI, for detection. It is known that MRC achieves the maximum likelihood (ML) performance in the presence of AWGN only, i.e. no IQI, as shown in [3]. We consider two performance metrics: in Section III-A we analyze the outage probability while EVM is analyzed in Section III-B.

A. Performance Analysis: Outage Probability

In this section, we derive the outage probabilities of the direct and AF transmission modes under uncompensated IQI [10]. The AF mode provides a higher diversity gain than the direct mode; however, it suffers more from IQI due to the non-ideal RF front-end of the relay station.

1) Instantaneous SINR Expressions: The instantaneous signal-to-interference-plus-noise ratio (SINR) expressions are required for the outage probabilities calculations in Sections III-A2 and III-A3. We compute the SINR at the $k^\text{th}$ subcarrier

\[ \text{SINR}_k = \frac{P_k h_{k}^2}{\sigma^2 N_0 + P_k h_{k}^2} \]

where $P_k$ is the transmit power at node $k$, $h_k$ is the channel coefficient at subcarrier $k$, and $\sigma^2$ is the noise power.

2) Outage Probabilities: The outage probability is the probability that the SINR at the UE falls below a certain threshold $\gamma$. The outage probability for the direct and AF modes can be written as

\[ P_o = \Pr \{ \text{SINR} < \gamma \} \]

where $\Pr \{ \cdot \}$ denotes the probability that the event occurs. The outage probability for the direct and AF modes can be derived using the cumulative distribution function (CDF) of the SINR and the CDF of the noise power.
in the direct mode from (7) as follows
\[ \gamma_{d0}(k) = \frac{|\mu_D|^2 |H_{SD}(k)|^2}{|\nu_D|^2 |H_{RD}(k)|^2} \]  
(9)

Assuming MRC [11] of \( P_{D}^{(1)} \) and \( r_{D}^{(2)} \), the SINR at the \( k \)th subcarrier in the AF mode is upper-bounded by \( \gamma_{af} \) where [7]
\[ \gamma_{af}(k) = \gamma_{d0}(k) + \min (\gamma_{sk}(k), \gamma_{ko}(k)) \]  
(10)
and \( \gamma_{sk}(k) \) has the same form as \( \gamma_{d0}(k) \) with \( \mu_D^{*}, \nu_D^{*} \) and \( H_{SD} \) replaced by \( \mu_R^{*}, \nu_R^{*} \) and \( H_{SR} \), respectively. Next, \( \gamma_{ko}(k) \) is computed from (8) as follows
\[ \gamma_{ko}(k) = \frac{|\mu_R|^{2} \nu_{RD}(k) + (\nu_{R})^{*} \nu_{RD}(k)|^{2}}{|\nu_{R}|^{2} \nu_{RD}(k) + (\nu_{R})^{*} \nu_{RD}(k)|^{2}} + \frac{\Psi_{D}(|\nu_{R}|^{2} + |\nu_{D}|^{2})}{\Pi_{D}|\nu_{R}|^{2}} \]  
(11)

2) The Direct Mode Outage Probability: The direct mode outage probability is given by
\[ P_{d0}(R) = \Pr \left\{ \log_{2} (1 + \gamma_{d0}(k)) < R \right\} \]
\[ = \Pr \left\{ \gamma_{d0}(k) < 2^{R} - 1 \right\} = F_{\gamma_{d0}(k)}(2^{R} - 1) \]  
(12)
where \( R \) is the required rate and \( F_{\gamma_{d0}(k)}(u) \) is the cumulative density function (CDF) of \( \gamma_{d0}(k) \) given by
\[ F_{\gamma_{d0}(k)}(u) = \int_{0}^{\infty} \int_{0}^{\infty} f_{X,Y}(x,y)dx dy \]  
(13)
where \( X = |H_{SD}(k)|^{2}, Y = |H_{RD}(k)|^{2}, a_{D} = |\mu_{D}|^{2}, b_{D} = |\nu_{D}|^{2} \) and \( c_{D} = \frac{\Psi_{D}}{\Pi_{D}} \left( |\mu_{D}|^{2} + |\nu_{D}|^{2} \right) \). Except for center subcarriers, the correlation between \( H_{SD}(k) \) and \( H_{RD}(k) \) is small thanks to their large separation. Hence, they are considered uncorrelated and independent\(^4\), and so are \( X \) and \( Y \). Hence, \( f_{X,Y}(X,Y) \) is approximated by \( f_{X,Y}(x,y) \approx f_{X}(x)f_{Y}(y) \) and \( F_{\gamma_{d0}(k)}(u) \) is given by
\[ F_{\gamma_{d0}(k)}(u) \approx \int_{0}^{\infty} f_{Y}(y) \int_{0}^{\infty} d_{X}(b_{Y} + c_{D}) f_{X}(x) dx \]  
(14)
where \( X \) and \( Y \) are exponentially-distributed random variables with the densities \( f_{X}(x) = \frac{1}{\sigma_{D} \exp \left( \frac{x}{\sigma_{D}} \right)} \) and \( f_{Y}(y) = \frac{1}{\sigma_{D} \exp \left( \frac{y}{\sigma_{D}} \right)} \), as defined in Section II. Carrying out the integration in (14), we get
\[ F_{\gamma_{d0}(k)}(u) \approx 1 - \exp \left( \frac{-c_{D}}{a_{D} \sigma_{D}^{2}} u \right) \left( 1 + \frac{b_{D} \sigma_{D}^{2}}{a_{D} \sigma_{D}^{2}} u \right)^{-1} \]  
(15)
Substituting \( 2^{R} - 1 \) for \( u \) in (15), we get
\[ P_{d0}(R) \approx 1 - \exp \left( \frac{-c_{D}(2^{R} - 1)}{a_{D} \sigma_{D}^{2}} \right) \left( 1 + \frac{b_{D} \sigma_{D}^{2}}{a_{D} \sigma_{D}^{2}} (2^{R} - 1) \right)^{-1} \]  
(16)
\(^4\)Because they are jointly Gaussian.

3) The AF Mode Outage Probability: The AF mode outage probability is lower-bounded by [7]
\[ P_{a0}(R) \geq \int_{0}^{2^{2R-1}} F_{\gamma_{d0}(k)}(2^{2R} - 1 - u) f_{\gamma_{d0}(k)}(u) du \]  
(17)
where \( f_{\gamma_{d0}(k)}(u) = \frac{\partial F_{\gamma_{d0}(k)}(u)}{\partial u} \) is the probability density function (PDF) of \( \gamma_{d0}(k) \). Also, \( R \) is multiplied by 2 in (17) to compare the outage probabilities of both modes at the same rate since the MRC is performed over two time slots in the AF mode [7]. Furthermore, \( F_{\gamma_{d0}(k)}(u) \) is the CDF of \( \gamma_{d0}(k) \) \( \triangleq \min (\gamma_{sk}(k), \gamma_{ko}(k)) \) given by [7]
\[ F_{\gamma_{d0}(k)}(u) = 1 - \left[ 1 - F_{\gamma_{sk}(k)}(u) \right] \left[ 1 - F_{\gamma_{ko}(k)}(u) \right] \]  
(18)
where \( F_{\gamma_{sk}(k)}(u) \) and \( F_{\gamma_{ko}(k)}(u) \) are the CDFs of \( \gamma_{sk}(k) \) and \( \gamma_{ko}(k) \), respectively. \( F_{\gamma_{d0}(k)}(u) \) has the same form as \( F_{\gamma_{d0}(k)}(u) \) in (15) with the UE parameters replaced by those of the RS. Next, we compute \( F_{\gamma_{d0}(k)}(u) \) where, for notational convenience, we re-write \( \gamma_{ko}(k) \) in (11) as follows
\[ \gamma_{ko}(k) = \left\{ \begin{array}{ll}
Z^{2} & \text{if} \quad W + 1 N_{1} \leq I_{1} \\
0 & \text{otherwise}
\end{array} \right. \]  
(19)
where \( W \) and \( Z \) are not exponentially-distributed because \( W \) and \( Z \) are not proper random variables [12], \( E \left[ \frac{W^{2}}{2} \right] = 2c_{1}d_{1}\rho_{k} \neq 0 \) and \( E \left[ \frac{Z^{2}}{2} \right] = 2a_{1}b_{1}\rho_{k} \neq 0 \) where \( \rho_{k} \approx E \left[ H_{RD}(k)H_{RD}^{*}(k) \right] \). The PDF of \( Z \) is given by [13]
\[ f_{Z}(z) = \sum_{j=0}^{\infty} c_{j} z^{j} \exp \left( \frac{z}{2 \beta_{z}} \right) \]  
(21)
where \( \beta_{z} = 2\lambda_{1}^{2} \lambda_{2}^{2}/(\lambda_{1}^{2} + \lambda_{2}^{2}) \), \( c_{0} = \beta_{z} \sqrt{\lambda_{1}^{2} \lambda_{2}^{2}} \)
\[ c_{j} = \frac{1}{2j} \sum_{m=0}^{j-1} d_{j-m} c_{m}^{2}, \quad j > 0 \]  
(22)
\[ d_{j} = \left( 1 - \frac{\beta_{1}}{\lambda_{1}^{2}} \right)^{j} + \left( 1 - \frac{\beta_{2}}{\lambda_{2}^{2}} \right)^{j} \]  
(23)
where \( \lambda_{1}^{2} \) and \( \lambda_{2}^{2} \) are the eigenvalues of the correlation matrix \( V_{Z} = E \left[ ZZ^{T} \right] \) where \( Z = \left[ Z_{R} \ Z_{I} \right]^{T} \) and \( Z_{R} \) and \( Z_{I} \) denote the real and imaginary parts of \( Z \), respectively. The integral \( I_{1} \) in (20) is given by [14]
\[ I_{1} = \sum_{j=0}^{\infty} \frac{c_{j}^{2}}{j!(2\beta_{z})^{j+1}} \int_{0}^{(u+N_{1})/t} \left[ -1/2 \beta_{z}^{2} \right] dz \]  
(24)
\[ = \sum_{j=0}^{\infty} c_{j} \left( 1 - \exp \left( -\frac{u}{2\beta_{z}} \right) \right) \sum_{t=0}^{\infty} \frac{u^{t}(w + N_{1})^{t}}{t!(2\beta_{z})^{t}} \]  
(25)
Substituting \( I_1 \) back into (20) and writing \( f_{\mathcal{W}}(w) \) in a form similar to \( f_Z(z) \) in (21), we get

\[
F_{\gamma_0(k)}(u) \simeq \sum_{j=0}^{\infty} \sum_{l=0}^{\infty} c_j^w c_l^w \left\{ \int_0^\infty u^l \exp \left( -\frac{1}{2\beta_w} w \right) dw \right\} \left( \frac{u}{2\beta_w} \right)^t l!
\]

where \( \int_0^\infty w^t \exp \left( -\frac{1}{2\beta_w} w \right) dw = l!(2\beta_w)^{t+1} \) [14]. To simplify the second integral in (25), we approximate \( q_j \) by

\[
q_j \simeq \exp \left( -\frac{w(N_1 + N_t)}{2\beta_w} \right) \sum_{t=0}^{\infty} \frac{\left( \frac{w}{\beta_w} \right)^t N_1}{t!} \left( 1 + \frac{\beta_w}{\beta_w} \right)^{t+1}
\]

where we used \( (1+\frac{N_1}{N_t})^t \approx 1+\frac{N_1}{N_t} \) as a valid approximation at high SNR where \( N_1 < \frac{1}{\lambda} \). Using \( q_j \) in (26), we get

\[
F_{\gamma_0(k)}(u) \simeq 1 - \sum_{j=0}^{\infty} \sum_{l=0}^{\infty} c_j^w c_l^w \left( \frac{u}{2\beta_w} \right)^t l!
\]

To derive an expression for the EVM at the \( k^{th} \) subcarrier at medium and high SNR where IQI dominates the performance, we exploit the CFR independence between the three links \((S \to D, S \to R, \text{and } R \to D)\). Moreover, the \( k^{th} \) subcarrier and its image are assumed uncorrelated with equal variance. In addition, to simplify the EVM expression, henceforth, we make the following assumptions without loss of generality: 1) we assume the same IQI level at all nodes, 2) we normalize \( \sigma_{SD} = 1 \), 3) we set \( \sigma_{SR} = \sigma_{RD} = \sigma = \left( \frac{d}{\sigma_{SD}} \right)^{-\frac{1}{2}} \). To reach the expression in (28), we approximate terms of the form \((1+x)^{-n}\) by \( \exp(-nx) \) where \( x < < 1 \).

4. Direct vs AF Transmission Under Uncompensated-IQI: We compare the outage probabilities of the direct and AF modes at high SNR (\( \text{SNR} \to \infty \)) where IQI effects dominate performance. Furthermore, we assume the same IQI levels \( (\epsilon \text{ and } \theta) \) at the RS and UE and that \( \sigma_{k}^{SD} = \sigma_{-k}^{SD}, \sigma_{k}^{SR} = \sigma_{-k}^{SR} \) and \( \sigma_{k}^{RD} = \sigma_{-k}^{RD} \) which is the case when the CIR coefficients are uncorrelated. Denoting by \( \approx \) the asymptotic equivalence at high SNR, we have

\[
P_{\text{AF}}(R) \approx 1 - \left( 1 + \frac{b(2R - 1)}{a} \right)^{-1} \approx \frac{b(2R - 1)}{a}
\]

where \( \frac{b}{a} = \left| \frac{\varepsilon}{\mu} \right|^2 \ll 1 \) for practical IQI levels with the subscripts of \( b \) and \( a \) omitted because the UE and RS are assumed to have the same IQI levels. Although \( P_{\text{AF}}(R) \) in (28) has infinite terms, the first term \((j = 0)\) is the most significant for non-center subcarriers where \( \mathbf{H}_{RD}(-k) \) are almost uncorrelated. Noting that \( \omega_2 \approx 0 \) and considering only the first term in (28), we write

\[
P_{\text{AF}}(R) \approx \frac{b}{a} R - \frac{c_0^w c_0^w}{a} \left( R - \left( \frac{b}{a} + \frac{\beta_w}{\beta_z} \right) \right) R^2
\]

In [15], we showed that \( c_0^w \approx 1, c_0^w \approx 1 \) and \( \frac{\beta_w}{\beta_z} \approx \frac{b}{a} \).

Hence,

\[
P_{\text{AF}}(R) \approx \frac{b}{a}^2 \left( 2R - 1 \right)^2
\]

Comparing (29) and (31), we calculate the maximum IQI level (i.e. \( \frac{b}{a} = \left| \frac{\mu}{\nu} \right|^2 \)) for which the AF mode outage probability is still lower than that of the direct mode as follows

\[
\frac{P_{\text{AF}}(R)}{P_{\text{DS}}(R)} \leq 1 \Rightarrow \left| \frac{\nu}{\mu} \right|^2 \leq \frac{b}{a}^2 \left( 2R - 1 \right)^2 \approx 1 \Rightarrow 3 \times \frac{3}{2R - 1} \leq \delta
\]

Hence, if \( \left| \frac{\nu}{\mu} \right|^2 \) exceeds the threshold \( \delta \) in (32), the AF mode is outperformed by the direct mode. The threshold \( \delta \) is inversely proportional to the cube of the signal constellation size \( 2^R \). Hence, the bigger the signal constellation size is, the earlier the AF-Direct modes crossover point occurs.

B. Performance Analysis: EVM

To derive an expression for the EVM at the \( k^{th} \) subcarrier at medium and high SNR where IQI dominates the performance, we exploit the CFR independence between the three links \((S \to D, S \to R, \text{and } R \to D)\). Moreover, the \( k^{th} \) subcarrier and its image are assumed uncorrelated with equal variance. In addition, to simplify the EVM expression, henceforth, we make the following assumptions without loss of generality: 1) we assume the same IQI level at all nodes, 2) we normalize \( \sigma_{SD} = 1 \), 3) we set \( \sigma_{SR} = \sigma_{RD} = \sigma = \left( \frac{d}{\sigma_{SD}} \right)^{-\frac{1}{2}} \). To reach the expression in (28), we approximate terms of the form \((1+x)^{-n}\) by \( \exp(-nx) \) where \( x < < 1 \).

\[
\hat{s}(k) = \psi_{k} \hat{s}(k) + \lambda_{k} \hat{s}^*(-k) + n_k
\]

where \( \psi_{k} \) and \( \lambda_{k} \) represent the combined effects of the channel and IQI given by

\[
\psi_{k} = \frac{\mu \sqrt{\frac{\nu}{N_o}} \mathbf{H}_{SD}(k)^2}{N_o} + \frac{\alpha \sqrt{\frac{\mu}{N_o}} \mathbf{H}_{SR}(k) \mathbf{H}_{RD}(k)}{N_o(1 + a^2 |\mathbf{H}_{RD}(k)|^2)} \mathbf{C}(k)
\]

\[
\lambda_{k} = \frac{\nu \sqrt{\frac{\nu}{N_o}} \mathbf{H}_{SD}(k) \mathbf{H}_{SD}^*(-k) + \alpha \sqrt{\frac{\mu}{N_o}} \mathbf{H}_{SR}(k) \mathbf{H}_{RD}(k)}{N_o(1 + a^2 |\mathbf{H}_{RD}(k)|^2)} \mathbf{D}(k)
\]

The CFR has a uniform power distribution over subcarriers if the CIR taps are uncorrelated which is the case in practice for uncorrelated scattering.
Furthermore, the noise term in (33) is given by

\[ n_k = \frac{\sqrt{\eta} H_{SD}(k)}{N_o} \left( \mu n_D^{(1)}(k) + \nu \left( n_D^{(2)}(k) \right)^* \right) + \frac{a \sqrt{\eta} H_{SR}(k) H_{RD}(k)}{N_o(1 + a^2 |H_{RD}(k)|^2)} \left( \mu n_D^{(2)}(k) + \nu \left( n_D^{(1)}(k) \right)^* \right) + \frac{a \sqrt{\eta} H_{SR}(k) H_{RD}(k)}{N_o(1 + a^2 |H_{RD}(k)|^2)} \left( \mu A(k) + \nu^* B(k) \right) n_R(k) + (\nu A(k) + \mu^* B(k)) n_R^*(k) \]

(36)

where \( C(k) \) and \( D(k) \) are given by

\[ C(k) = \mu H_{SR}(k) A(k) + \nu^* H_{SR}(k) B(k) \]

(37)

\[ D(k) = \nu H_{SR}^*(-k) A(k) + \mu^* H_{SR}^*(-k) B(k) \]

(38)

\[ A(k) = a \left( |\mu|^2 H_{RD}(k) + |\nu|^2 H_{RD}^*(-k) \right) \]

(39)

\[ B(k) = a \nu \left( H_{RD}(k) + H_{RD}^*(-k) \right) \]

(40)

Denoting the error term at the \( k \)-th subcarrier by \( \epsilon_{MRC}(k) = \tilde{s}(k) - \psi_k s(k) \), the EVM of this subcarrier is defined as follows

\[ \text{EVM}_{MRC}(k) = \sqrt{\mathbb{E} \left[ |\epsilon_{MRC}(k)|^2 \right]} \]

(41)

For practical IQI levels and using (34) and (37), \( \mathbb{E} \left[ |\psi_k|^2 \right] \) is approximated by

\[ \mathbb{E} \left[ |\psi_k|^2 \right] \approx \frac{0.4}{\eta_0} \left( 5 |\mu|^2 \sigma_o^2 + 2 |\mu|^4 \sigma_o^2 + |\nu|^2 \sigma_o^2 \right) \]

(42)

where \( \gamma_o = \frac{N_0}{\eta_0} \) is the input SNR, and the proof is provided in Appendix A.

Since the noise and transmitted symbols are assumed independent, \( \mathbb{E} \left[ |\epsilon_{MRC}(k)|^2 \right] = \eta_0 \mathbb{E} \left[ |\lambda_k|^2 \right] + \mathbb{E} \left[ |n_k|^2 \right] \). Therefore, from (35) and (38), we get

\[ \mathbb{E} \left[ |\lambda_k|^2 \right] \approx (5 |\mu|^2 \sigma_o^2 + 0.84 |\nu|^2 |\mu|^4 \sigma_o^2) / \eta_0 \]

(43)

\[ \mathbb{E} \left[ |n_k|^2 \right] \approx |\mu|^2 \gamma_o + 0.19 |\mu|^2 \sigma_o^2 + 0.11 |\mu|^4 \sigma_o^2 + 0.4 |\nu|^2 |\mu|^4 \sigma_o^2 \]

(44)

with the proof given in Appendix A. Substituting (42)-(44) into (41), we get the EVM expression in (45) (shown on the next page) as a function of the IQI level, input SNR, and relay location. Furthermore, we compute the EVM floor by considering the high SNR scenario (\( \gamma_o \rightarrow \infty \)) as follows

\[ \text{EVM}_{MRC}(k) \approx \frac{1}{|\mu|^4} \left( 1 + 0.84 |\mu|^2 \sigma_o^2 \right) \sqrt{\frac{2 + 0.8 |\mu|^2 \sigma_o^2 + 0.4 |\mu|^4 \sigma_o^2}{|\mu|^4 \left( 1 + 0.84 |\mu|^2 \sigma_o^2 \right)}} \]

(47)

Setting \( \sigma = 1 \), i.e. the nodes form an equivalent triangle, and substituting back into (47), we get \( \text{EVM}_{MRC}(k) \approx \frac{1}{|\mu|^4} \sqrt{\frac{2 + 0.8 |\mu|^2 \sigma_o^2 + 0.4 |\mu|^4 \sigma_o^2}{|\mu|^4 \left( 1 + 0.84 |\mu|^2 \sigma_o^2 \right)}} \), which shows the dependence of EVM on the IQI parameters.

IV. DIGITAL BASEBAND COMPENSATION

The EVM analysis in Section III-B showed that MRC detection for AF relays results in an error floor in the presence of IQI. This motivates us to implement a more efficient detection scheme which compensates for IQI effects. In Section IV-A, we describe a low-complexity approach for joint channel and IQI parameters estimation in the TD. Doing the estimation in the TD requires one OFDM symbol, as opposed to two OFDM symbols for performing the estimation in the FD. We describe a low-complexity detection approach in Section IV-B and analyze its EVM performance in Section IV-C.

A. Pilot-Aided Channel and IQI Imbalance Estimation

Since \( H_{SD} \) is a circulant matrix and for the training phase, Equation (3) can be re-written in the matrix form as in Equation (46) (shown on the next page) where \( \tilde{S}^{(i)} \) is the \( N \times L \) circulant training matrix whose first column is the training sequence transmitted in the first time slot. The LS estimate of \( \hat{H}_{SD,eq} \) is \( (X^{(1)H} X^{(1)})^{-1} X^{(1)H} X^{(1)} \) [17] and the estimation error, \( e = \hat{H}_{SD,eq,LS} - \hat{H}_{SD,eq} \), variance is proportional to \( \text{tr}(X^{(1)H} X^{(1)} \text{I}_L) \) [17]. Moreover, to minimize the error variance subject to a power constraint \( E_e \) over the transmitted training signal, the condition \( X^{(1)H} X^{(1)} = E_e \text{I}_{2L \times 2L} \) needs to be satisfied [17]. This condition translates to the training signal design criteria \( S^{(1)} = S^{(1)T} (S^{(1)})^{-1} = \text{I}_L \) and \( S^{(1)}H (S^{(1)})^{-1} = S^{(1)T} S = 0_{L \times L} \). Hence, the matrix inversion of \( X^{(1)H} X^{(1)} \) is no longer needed and \( \hat{H}_{SD,eq,LS} \) is simplified to \( \hat{H}_{SD,eq,LS} = \frac{1}{E_e} \left[ X^{(1)} (X^{(1)})^{-1} X^{(1)T} S^{(1)} \right] \).

In the second time slot, since the equivalent channel matrices are circular, Equation (4) can be written as follows

\[ \tilde{F}^{(2)} = \left[ \tilde{S}^{(2)} (\tilde{S}^{(2)})^{-1} \right] \left[ \mu \tilde{h}_1 + \nu \tilde{h}_2 \right] + \hat{h}_{D,eq} \]

(48)

where \( \tilde{h}_1 \) and \( \tilde{h}_2 \) are the first columns of \( \tilde{H}_1 \) and \( \tilde{H}_2 \), respectively, each of length 2L−1. Moreover, \( \tilde{S}^{(2)} \) is the \( N \times (2L-1) \) training matrix whose first column is transmitted in the second time slot. Similarly, LS estimator of \( \hat{H}_{SD,eq} \) is given by \( \hat{H}_{SD,eq,LS} = (X^{(2)H} X^{(2)})^{-1} X^{(2)H} X^{(2)} \) and the optimality conditions on \( S^{(2)} \) are: \( S^{(2)H} S^{(2)} = S^{(2)T} S^{(2)} = 0_{2L-1} \) and \( S^{(2)H} (S^{(2)})^{-1} = S^{(2)T} S^{(2)} = 0_{2L-1 \times 2L-1} \). Hence, the LS estimator of \( \hat{H}_{SD,eq,LS} \) is simplified to \( \hat{H}_{SD,eq,LS} = \frac{1}{E_e} \left[ \tilde{F}^{(2)} (\tilde{F}^{(2)})^{-1} \right] \). Several optimal designs of \( S^{(i)} \), \( i \in \{1, 2\} \), were proposed in [18]. Below, we describe one of these optimal pilot designs in the FD for \( N = 64 \)

\[ S^{(i)} = F^{(i)} P^{(i)} \]

(49)

where \( P \) is a diagonal matrix with the diagonal entries \( p_{k,k} \) and \( F^{(1)} \) and \( F^{(2)} \) denote the first L and \( 2L-1 \) columns of \( F \), respectively.

B. Zero-Forcing Joint Equalization and IQI Compensation

We present a zero-forcing approach for joint channel equalization and IQI compensation where each subcarrier and its image are jointly processed for data detection [19]. The ZF approach has a low complexity and does not require SNR knowledge. Furthermore, we show in Section V that its performance is very close to the IQI-free case. Hence, there

\( \text{The line combination of circulant matrices is also circulant.} \)
\begin{align}
\text{EVM}_{MRC}(k) & \approx \frac{1}{|\mu|} \left(1 + 0.84|\mu|^2 |\sigma|^2\gamma_0 + \frac{0.15}{|\mu|^2} (1 + 0.19\sigma + 0.21|\mu|^2 (|\mu|^2 + 4|\mu|^2) \sigma) \right) \\
& \quad (2 + 0.8|\mu|^4 \sigma + 0.4|\mu|^4 |\sigma|^2 \gamma_0) \\
\hat{v}_D^{(1)} &= \left[ \mathbf{S}^{(1)} \right] \left( \mathbf{S}^{(1)} \right)^* \left[ \mu_D \mathbf{h}_{SD} \right] \left( \nu_D \mathbf{h}_{SD} \right]^* + \left[ \mu_D \mathbf{I}_N \right] \left( \nu_D \mathbf{I}_N \right)^* \\
& \quad \left( \mathbf{t}_D^{(1)} \right)^* \mathbf{X}^{(1)}_{SD,eq} + \mathbf{n}_D \\
\tilde{\mathbf{r}}_D^{(1)}(k) &= \left[ \Gamma_k \right]^{-1} \hat{v}_D^{(1)}(k) \\
\tilde{\mathbf{r}}_D^{(2)}(k) &= \left[ \Gamma_k \right]^{-1} \left[ \begin{array}{c}
\nu_D^2 \mathbf{H}_{SD}(k) \\
\nu_D \mathbf{H}_{SD}(k) \\
\mathbf{H}_1(k) \\
\mathbf{H}_2(k)
\end{array} \right]
\end{align}

is no need for the more complex MMSE or ML detection techniques. Denoting the \( k \)th entry of the vector \( \mathbf{x} \) by \( \mathbf{x}(k) \), we stack \( \mathbf{r}_D^{(1)}(k), \mathbf{r}_D^{(2)}(k), \mathbf{r}_D^{(-1)}(k) \) and \( \mathbf{r}_D^{(-2)}(k) \) into the vector \( \mathbf{y}_k \), defined in Equation (50) (on the top of this page), where \( \mathbf{n} \) denotes the composite noise vector given by (51) (on the top of this page). Furthermore, \( \mathbf{y}_k \) is the \( 2 \times 2 \) all-zero matrix while \( \mathbf{U} \) and \( \mathbf{V} \) are given by

\begin{align}
\mathbf{U} &= \left[ \begin{array}{cc}
\mu & \nu \\
\nu^* & \mu^*
\end{array} \right] \\
\mathbf{V} &= \left[ \begin{array}{cc}
\frac{\mu H_1(k) + \nu H_2(k)}{H_{SR}(k)} & \frac{\nu H_1(k) + \mu H_2(k)}{H_{SR}(k)} \\
\frac{\nu H_1(k) + \mu H_2(k)}{H_{SR}(k)} & \frac{\mu H_1(k) + \nu H_2(k)}{H_{SR}(k)}
\end{array} \right]
\end{align}

Given the CSI and IQI parameters estimates at the UE, \( \mathbf{s}(k) \) and \( \mathbf{s}^*(-k) \) are jointly detected using the following ZF equalizer followed by a slicer

\begin{align}
\hat{s}(k) &= \mathbf{y}_k^H \mathbf{r}_D^{(1)}(k) \\
\mathbf{s}^*(-k) &= \mathbf{y}_k^H \mathbf{r}_D^{(-1)}(k)
\end{align}

where the matrix inversion in (52) is easily implemented because \( \mathbf{y}_k^H \mathbf{y}_k \) is a \( 2 \times 2 \) matrix. Moreover, for practical IQI levels, \( \mathbf{y}_k^H \mathbf{y}_k \) exists with high likelihood [15].

C. EVM Performance Analysis of ZF Compensation

Since the received symbol at the \( k \)th subcarrier is decoded interference-free, thanks to the ZF equalizer, the EVM of the \( k \)th subcarrier is defined as \( \text{EVM}_{ZF}(k) = \frac{|\sigma^2 + \alpha^2| |\mu|^2 (|\mu|^2 + 4|\mu|^2) \sigma}{\gamma_0} \), where \( \gamma_0 = \text{EVM} \) \( \mathbf{S}^{(-1)} \mathbf{e}_1 \). From Equations (50 and 52), it can be easily shown that (see [17], Equation (12.29)) \( \mathbb{E} \left[ |\mathbf{S}^{(k)}(k)|^2 \right] = \mathbb{E} \left[ \mathbf{S}^{(-1)} \mathbf{e}_1 \right] \mathbf{e}_1 \) \( \mathbf{e}_1 = \left[ \begin{array}{c}1 \end{array} \right] \) and \( \Sigma = \mathbf{y}_k^H \mathbf{R}_{\mathbf{n}_m} \mathbf{y}_k \). Moreover, from the definition of \( \mathbf{n} \) in (51), \( \mathbf{R}_{\mathbf{n}_m} \) (the covariance matrix of the composite noise vector) has the following block diagonal form

\begin{align}
\mathbf{R}_{\mathbf{n}_m} = \mathbb{N}_o \mathbb{E} \left[ \mathbf{n}_m^H \right] = \mathbb{N}_o \mathbb{E} \left[ \begin{array}{cc}
\mathbf{U} \mathbf{U}^H & 0 \\
0 & \mathbf{V} \mathbf{V}^H
\end{array} \right]
\end{align}

For practical IQI levels, it can be shown that \( \mathbf{R}_{\mathbf{n}_m} \) is a diagonally-dominant matrix, then \( \mathbf{R}_{\mathbf{n}_m} \) can be approximated by a diagonal matrix whose main diagonal is \( \mathbb{N}_o [\sigma_n(1) \sigma_n(1) \sigma_n(2) \sigma_n(2)] \), where \( \sigma_n(1) = |\mu|^2 \) and \( \sigma_n(2) = |\mu|^2 + a^2 |\mu|^4 \) and \( \sigma_n(2) = |\mu|^2 + a^2 |\mu|^4 \) and \( \sigma_n(2) = |\mu|^2 + a^2 |\mu|^4 \) and \( \sigma_n(2) = |\mu|^2 + a^2 |\mu|^4 \) and \( \sigma_n(2) = |\mu|^2 + a^2 |\mu|^4 \). Now, to simplify computing \( \mathbb{E} \left[ \mathbf{S}^{(-1)} \right] \), we used the following linear approximation of the matrix inverse [21]

\begin{align}
\mathbf{S}^{(-1)} & \approx \frac{1}{\mathbf{I} - \frac{\mathbf{I}}{\mathbf{S}}}
\end{align}

Moreover, by exploiting the subcarrier CFR independence between the three links and between the \( k \)th subcarrier and its image of the same link, \( \text{EVM}_{ZF}(k) \) is approximated as follows (the accuracy of this approximation will be verified in the simulations section)

\begin{align}
\text{EVM}_{ZF}(k) & \approx \frac{2}{\mathbb{N}_o} \left( \mathbf{I} - \frac{\mathbf{I}}{\mathbf{S}} \right) \mathbf{e}_1
\end{align}

Because of the independence assumption between the CFRs at the \( k \)th subcarrier and its image, it can be easily shown that \( \mathbb{E} \left[ \mathbf{S}^{(-1)} \mathbf{e}_1 \right] \mathbf{e}_1 = \frac{2}{\mathbb{N}_o} \mathbb{E} \left[ \mathbf{S} \right] \mathbf{e}_1 \). After straightforward algebra, we get

\begin{align}
\mathbb{E} \left[ \mathbf{S} \right] & \approx \frac{2}{\mathbb{N}_o} \left( \mathbb{N}_o + 1 - \frac{|\mu|^2 a^2 \sigma^2}{1 + |\mu|^2 a^2 \sigma^2} \right)
\end{align}

For medium-to-high SNR where \( a^2 \approx \frac{1}{\gamma_0} \), and substituting (56) into (55), we get

\begin{align}
\text{EVM}_{ZF}(k) & \approx \frac{2}{\gamma_0} \left( \frac{1}{1 + |\mu|^4 + |\mu|^4 \gamma_0} \right)
\end{align}

which shows that our presented ZF compensation scheme eliminates the error floor due to IQI.

V. Numerical Results

We start by verifying the outage probability expressions in Section III-A numerically for the direct and AF modes. We assume the same IQI levels at the RF front-ends of the UE and RS, i.e. \( \mu_D^* = \nu_R^* = (\mu_R^*)^* = \mu \) and \( \nu_D^* = \nu_R^* = \nu \). However, the derived expressions in Section III-A are applicable for any IQI levels. We use a multi-path channel model with 8 uncorrelated paths and a uniform power-delay profile. The FFT size is \( N = 64 \), and the subcarrier index of interest
is $k = 15$. We set $\eta_0 = \eta_1 = 1$ and SNR=1/$N_o$. To compute $P_{\text{w}}(R)$ in (28), we truncated each infinite summation to the first 15 non-zero terms since $c_j^2 = c_l^2 = 0$ for odd $j$ and $l$. Fig. 2 shows the numerical and analytical outage probabilities for both the direct and AF modes under various IQI levels for $R = 2$ bits/sec/Hz. We note that the numerical results and analytical expressions are very close to each other especially in the direct mode where they coincide under different IQI levels. Furthermore, we observe that the AF mode becomes worse than the direct mode even under moderate IQI levels such as $|\nu| = 0.15$ which corresponds to $10\log_{10}(1+\epsilon)=0.6$ dB gain imbalance and $\theta=2^\circ$ phase imbalance. In Fig. 3, we compare $P_{\text{w}}(R)$ and $P_{\text{bw}}(R)$ versus $|\nu|$ for different rates ($R = 2$, 3 and 4 bits/sec/Hz) where the marked AF-Direct crossover point occurs at smaller $|\nu|$ values for higher rates as predicted by (32).

In Figs. 4 and 5, we investigate the accuracy of the EVM expressions for the MRC detector for different relay positions and IQI levels, respectively. Although the EVM expression in (45) was derived for medium-to-high SNR, Figs. 4 and 5 demonstrate its accuracy even for low SNR. As expected, a lower sigma (when the relay is farther away from the BST and UE) is worse at low SNR because the relay is essentially amplifying and re-sending noise which degrades the performance. However, a higher sigma (when the relay is closer to the BST and UE) is worse at high SNR because the link ($S \rightarrow R \rightarrow D$) has more IQI, due to both transmit and receive IQI at the relay, than the direct link; hence, the relay contributes more interference. We found that for practical IQI levels, where the phase imbalance is less than $5^\circ$, EVM depends mainly on gain imbalance while its variation with phase imbalance is negligible. Therefore, in Fig. 5, we fixed the phase imbalance at its worst-case level of $5^\circ$ and varied the gain imbalance to verify the accuracy of our analytical expression for different IQI levels. Fig. 6 demonstrates the accuracy of our EVM approximation for the ZF equalizer in Equation (57) at high SNR for different relay positions at 1 dB amplitude imbalance. Specifically, the simulation result supports the main conclusion from (57) that the ZF equalizer eliminates the EVM floor due to IQI. Finally, in Fig. 7, we compare the BER performance of the conventional MRC detector where the IQI is ignored with that of our ZF-based IQI compensation scheme. We observe that the performance of the ZF detector is very close to the IQI-free case while the MRC technique suffers from significant performance loss. It is interesting to observe that, with perfect CSI, the ZF detector slightly outperforms the IQI-free system at high SNR thanks to the diversity gain provided by transmit IQI at the RS (see [22]). In the same figure, we show the impact of channel estimation inaccuracies using the approach described in Section IV-A. Although the ZF detector experiences about 3dB SNR loss due to channel estimation inaccuracies, it still significantly outperforms the MRC detector.

This follows from definitions of $\mu$ and $\nu$ where $|\mu| \approx 1$ and $|\nu| \approx \epsilon$ for small $\theta$. 
Fig. 6. Analytical (solid lines) and simulated (dashed lines) EVM of ZF for different values of $\sigma$ and IQI level of $(10\log_{10}(1+\epsilon)=1\text{ dB and } \theta=5^\circ)$.  

Fig. 7. Uncoded BER of ZF and MRC under IQI $(10\log_{10}(1+\epsilon)=1\text{ dB and } \theta=5^\circ)$ with perfect (solid lines) and estimated (dashed lines) CSI as compared to the IQI-free scenario with perfect CSI.

VI. CONCLUSION

We analyzed the EVM and outage probability of relay-assisted AF transmission and MRC detection in the presence of IQI and verified the accuracy of the derived expressions numerically. Comparing the outage performances of the AF and direct modes, we found that the former loses its performance advantage to the latter when the IQI level exceeds a certain threshold. This threshold was derived analytically and found to be inversely proportional to the cube of the signal constellation size. Hence, bigger signal constellations are much more sensitive to IQI in the AF mode compared to the direct mode. Furthermore, we presented a low-complexity pilot-aided digital baseband IQI estimation and compensation scheme for AF relay systems and analyzed its EVM performance at high SNR. Simulation results showed that the BER performance of our proposed pilot-aided compensation scheme is significantly better than conventional MRC detection schemes which do not take IQI effects into account.

APPENDIX A

To calculate $\mathbb{E}[|\psi_k|^2]$, $\mathbb{E}[|\lambda_k|^2]$ and $\mathbb{E}[|\eta_k|^2]$ as defined in Equations (34)-(36), we need to evaluate $\mathbb{E}\left[\frac{v^2}{(1+a^2v)^2}\right]$ and $\mathbb{E}\left[\frac{v}{\sigma_v}\exp\left(-\frac{v}{\sigma_v}\right)\right]$. Thus, we have

$$\mathbb{E}\left[\frac{v^2}{(1+a^2v)^2}\right] = \frac{1}{\sigma_v^2} \int_0^\infty \frac{u}{u^2} \exp\left[-\frac{1}{2\sigma_v^2}\right] du$$

Making the change of variables $u = 1 + a^2 v$, we get

$$\mathbb{E}\left[\frac{v^2}{(1+a^2v)^2}\right] = \frac{1}{a^6 \sigma_v^2} \int_0^\infty \frac{(u-1)^2}{u^2} \exp\left[-\frac{(u-1)}{2\sigma_v^2}\right] du$$

$$= \frac{1}{a^6 \sigma_v^2} \exp\left[-\frac{1}{a^2 \sigma_v^2}\right] \int_0^\infty \frac{u^2 - 2u + 1}{u^2} \exp\left[-\frac{(u-1)}{a^2 \sigma_v^2}\right] du$$

$$= \frac{1}{a^6 \sigma_v^2} \left(1 + a^4 \sigma_v^2\right) \exp\left[-\frac{1}{a^2 \sigma_v^2}\right] \left(1 + a^2 \sigma_v^2\right)$$

$$\mathbb{E}\left[\frac{v}{\sigma_v}\exp\left(-\frac{v}{\sigma_v}\right)\right] \triangleq \int_0^\infty \exp\left[-\frac{v}{\sigma_v}\right] dv$$

where $\mathbb{E}[-pw] \triangleq \int_0^\infty \exp[-px] dx$ is the exponential integral function [14]. Similarly,

$$\mathbb{E}\left[\frac{v}{(1+a^2v)^2}\right] = -\frac{1}{a^4 \sigma_v^2} \left(1 + a^2 \sigma_v^2\right) \exp\left[-\frac{1}{a^2 \sigma_v^2}\right] \left(1 + a^2 \sigma_v^2\right)$$

Moreover, using the definitions of $A(k)$ and $B(k)$ in Equations (39) and (40), $|A(k)|^2$, $|B(k)|^2$ and $\text{Re} \left[\mu A(k)B(k)^*\right]$ are expanded as in Equations (61) to (63), (on the next page).

Observing that $\mathbf{H}_{RD}(k)$ and $\mathbf{H}_{RD}(-k)$ are independent Gaussian random variables with zero mean and using Equations (59) to (63), we define the following expectations which will be used later in the derivation

$$\mathbb{E}\left[\frac{1}{(1+a^2\mathbf{H}_{RD}(k))^2}\right] = \frac{1}{a^4} X - \frac{1}{a^2} Y$$

Starting with the evaluation of $\mathbb{E}[|\psi_k|^2]$, from (34) and (37), we have

$$\mathbb{E}[|\psi_k|^2] = \frac{\mu^2 \eta_0}{N_0^2} \mathbb{E}[|\mathbf{H}_{SD}(k)|^4]$$

$$+ 2\eta_0 a \frac{N_0^2}{N_o^2} \mathbb{E}\left[|\mathbf{H}_{SD}(k)|^2 \text{Re} \left[\mu \mathbf{H}_{SR}(k)\mathbf{H}_{RD}(k)\mathbf{C}(k)^*\right]\right]$$

$$+ a^2 \eta_0 \frac{N_0^2}{N_o^2} \mathbb{E}\left[|\mathbf{H}_{SR}(k)|^2 |\mathbf{H}_{RD}(k)|^2 \text{Re} \left[\mu A(k)B(k)^*\right] + |\mathbf{B}|^2\right]$$

In the sequel, we compute the three statistical expectations in the right hand side (RHS) of (67). Given that $\sigma_{SD} = 1$, the $\mathbb{E}[|\mathbf{H}_{SD}(k)|^4]$ is computed as follows

$$\mathbb{E}[|\mathbf{H}_{SD}(k)|^4] = \int_0^\infty v^2 \exp[-v] dv = 2$$

Next, exploiting independence between the different links and between the $k^{th}$ subcarrier and its image, we write

$$\mathbb{E}\left[|\mathbf{H}_{SD}(k)|^2 \text{Re} \left[\mu \mathbf{H}_{SR}(k)\mathbf{H}_{RD}(k)\mathbf{C}(k)^*\right]\right]$$

$$\approx a \mu |\mathbf{H}_{RD}(k)|^2 \mathbb{E}\left[|\mathbf{H}_{RD}(k)|^2\right]$$

$$= a \mu |\mathbf{H}_{RD}(k)|^2$$

$$\approx a \mu |\mathbf{H}_{RD}(k)|^2$$
where \( \mathbb{E}\left[\frac{|H_{RD}(k)|^2}{(1 + a^2|H_{RD}(k)|^2)}\right] \) is calculated from [14, P341] as

\[
\mathbb{E}\left[\frac{|H_{RD}(k)|^2}{(1 + a^2|H_{RD}(k)|^2)}\right] = \frac{1}{a^2\sigma^2} \int_0^\infty \frac{v}{(1 + a^2v)} \exp\left[\frac{-v}{\sigma^2}\right] dv
\]

Substituting (70) into (69), we get

\[
\mathbb{E}\left[|H_{SD}(k)|^2|H_{SR}(k)H_{RD}(k)|C(k)^*\right] = \frac{|\mu|^4}{a} \frac{1}{a^2\sigma^2} \exp\left[\frac{1}{a^2\sigma^2}\right] E_{\left[\frac{1}{\sigma^2}\right] + \sigma}
\]

The third statistical expectations in the RHS of (67) is computed as follows

\[
\mathbb{E}\left[|H_{SR}(k)|^2|H_{RD}(k)|^2\right] \left[\frac{(1 + a^2|H_{RD}(k)|^2)}{(1 + a^2|H_{RD}(k)|^2)}\right]
\]

\[
\times \left(|\mu|^2|A(k)|^2 + 2\text{Re}\left[\mu|A(k)|B(k)^*\right] + |\nu|^2|B|^2\right)
\]

\[
= \frac{2|\mu|^2|\sigma|^2}{a^2} \left(|\mu|^4\hat{X} - |\nu|^4Y\right) + \frac{2|\mu|^4|\sigma|^2}{a^2} \left(\hat{X} - \hat{Y}\right)
\]

\[
+ \frac{4|\mu|^2|\sigma|^2}{a^2} \left(|\mu|^2\hat{X} - |\nu|^2\hat{Y}\right)
\]

\[
\approx (2|\mu|^2 + 4|\nu|^2) \frac{|\mu|^4|\sigma|^2}{a^2} \hat{X}
\]

where \( \hat{X} = \frac{1}{a^2\sigma^2} X \) and the approximation in (72) is valid for practical IQI levels under the assumptions in Section IV-B where \( \hat{X} > \hat{Y} \). Thus, substituting Equations (68), (71), and (72) into (67), we get

\[
\eta_o \mathbb{E}[|\psi_k|^2] \approx 2|\mu|^2\gamma_o^2 + 2|\mu|^4|\sigma|^2 \hat{X}
\]

\[
+ 2|\mu|^4|\sigma|^2 \frac{1}{a^2\sigma^2} \exp\left[-\frac{1}{a^2\sigma^2}\right] E_{\left[\frac{1}{a^2\sigma^2}\right] + \sigma}
\]

For medium-to-high SNR where IQI dominates noise, \( a^2 = \frac{1}{\sigma^2} \), thus \( \hat{X} = 2 + 3 \exp[1] \mathbb{E}[-1] = 0.21 \) and \( \eta_o \mathbb{E}[|\psi_k|^2] \) can be simplified as shown in (42). Next, from (35) and (38), \( \mathbb{E}[|\lambda_k|^2] \) can be written as follows

\[
\mathbb{E}[|\lambda_k|^2] = \frac{|\nu|^2}{N_o^2} \mathbb{E}[|H_{SD}(k)|^2|H_{RD}(k)|^2]
\]

\[
+ \frac{a^2}{N_o^2} \mathbb{E}[|H_{SR}(k)|^2|H_{SR}(k)|^2|H_{RD}(k)|^2|H_{RD}(k)|^2]
\]

\[
\times \left(|\mu|^2|A(k)|^2 + 2\text{Re}\left[\mu|A(k)|B(k)^*\right] + |\nu|^2|B|^2\right)
\]

The first statistical expectation in the RHS of (74) can be easily computed because of the independence assumption between the \( k \)th subcarrier and its image; hence,

\[
\mathbb{E}[|H_{SD}(k)|^2|H_{SD}(k)|^2] = 1.
\]

Using Equations (64) to (66), the second statistical expectation term in the RHS of Equation (74) is as follows

\[
\mathbb{E}\left[\frac{H_{SR}(k)|H_{SR}(k)|^2|H_{RD}(k)|^2}{(1 + a^2|H_{RD}(k)|^2)^2}\right]
\]

\[
\times \left(|\mu|^2|A(k)|^2 + 2\text{Re}\left[\mu|A(k)|B(k)^*\right] + |\nu|^2|B|^2\right)
\]

\[
= \frac{|\nu|^2}{a^2} \left(|\mu|^4\hat{X} - |\nu|^4\hat{Y}\right) + \frac{|\mu|^4|\nu|^2}{a^2} \left(\hat{X} - \hat{Y}\right)
\]

\[
+ \frac{2|\mu|^2|\nu|^2}{a^2} \left(|\mu|^2\hat{X} - |\nu|^2\hat{Y}\right) \approx \frac{4|\mu|^4|\nu|^2}{a^2} \hat{X}
\]

Substituting \( \mathbb{E}[|H_{SD}(k)|^2|H_{SD}(k)|^2] = 1 \) and (75) into (74), and considering medium-to-high SNR levels, Equation (74) can be written as in (43). Then, using (36), \( \mathbb{E}[|n_k|^2] \) is

\[
\mathbb{E}[|n_k|^2] = \frac{\eta_o}{N_o} (|\mu|^2 + |\nu|^2) \mathbb{E}[|H_{SD}(k)|^2]
\]

\[
+ \frac{\eta_o a^2}{N_o} (|\mu|^2 + |\nu|^2) \mathbb{E}[|H_{SR}(k)|^2|H_{RD}(k)|^2]
\]

\[
+ \frac{\eta_o a^2}{N_o} \mathbb{E}[|H_{SR}(k)|^2|H_{RD}(k)|^2]
\]

\[
\times (|\mu|^2|A(k)|^2 + 2\text{Re}\left[\mu|A(k)|B(k)^*\right] + |\nu|^2|B|^2)
\]

where \( \mathbb{E}[|H_{SD}(k)|^2] = 1 \) and \( \mathbb{E}[|H_{SR}(k)|^2|H_{RD}(k)|^2] \approx \frac{1}{\sigma^2} \hat{Y} \). For practical IQI levels where \( |\mu|^2 \gg |\nu|^2 \), the following approximation can be used

\[
|\mu|A(k) + \nu^*B(k)^2 + |\nu|A(k) + \mu^*B(k)^2 \approx (|\mu|^2|A(k)|^2 + 4\text{Re}\left[\mu|A(k)|B(k)^*\right] + |\nu|^2|B|^2)
\]

Hence, the third expectation term in the RHS of Equation (76) is computed as follows

\[
\mathbb{E}\left[\frac{H_{SR}(k)|H_{SR}(k)|^2|H_{RD}(k)|^2}{(1 + a^2|H_{RD}(k)|^2)^2}\right]
\]

\[
\times (|\mu|^2|A(k)|^2 + 2\text{Re}\left[\mu|A(k)|B(k)^*\right] + |\nu|^2|B|^2)
\]

\[
\approx \frac{|\mu|^2}{a^2} \left(|\mu|^4\hat{X} - |\nu|^4\hat{Y}\right) + \frac{|\mu|^4|\nu|^2}{a^2} \left(\hat{X} - \hat{Y}\right)
\]

\[
+ \frac{4|\mu|^2|\nu|^2}{a^2} \left(|\mu|^2\hat{X} - |\nu|^2\hat{Y}\right) \approx \frac{|\mu|^4|\nu|^2}{a^2} \hat{X}
\]

Using (78) and the relationships \( \mathbb{E}[|H_{SD}(k)|^2] = 1 \) and \( \mathbb{E}[|H_{SR}(k)|^2|H_{RD}(k)|^2] \approx \frac{1}{\sigma^2} \hat{Y} \), we approximate \( \mathbb{E}[|n_k|^2] \) in (76) as follows

\[
\mathbb{E}[|n_k|^2] \approx |\mu|^2\gamma_o - \frac{|\mu|^2}{a^2}\gamma_o Y + (|\mu|^2 + 4|\nu|^2) |\mu|^4\gamma_o \hat{X}
\]
For medium-to-high SNR levels, substituting $Y = 1 + 2 \exp[1] Ei[-1] = -0.19$ and $\bar{X} = 0.21$ into (79), we arrive at the final result in (44).

REFERENCES


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