May 1, 2006
Final Exam
EE 3302: Signals and Systems

NOTE: Please, complete the following table and keep record of your assignment number.

| First Name |  |
| Last Name |  |
| Student ID |  |
| Assignment # | 0 |

**Exercise 1.** A system is described by the following differential equation

\[
\frac{d^2 y(t)}{dt^2} + 10 \frac{dy(t)}{dt} + 21 y(t) = \frac{d^2 x(t)}{dt^2} + 8 \frac{dx(t)}{dt} + 15 x(t)
\]

where \(x(t)\) is the input signal, and \(y(t)\) is the output signal. Assume that the initial rest condition is satisfied.

A) Determine the frequency response of the system [pt. 10].
B) Determine the unit impulse response of the system [pt. 10].
C) Determine the frequency response of the inverse system [pt. 5].
D) Determine the unit impulse response of the inverse system [pt. 10].

**Exercise 2.** Consider the continuous-time signal

\[
x(t) = \frac{\sin(\omega_1 t)}{t}
\]

where \(\omega_1\) is a positive finite value. Let \(y(t) = x(t) \cdot e^{-j\omega_0 t}\), where \(\omega_0\) is a positive finite value. The following signals are sampled using a train of impulses with periodicity \(T\), \(\sum_{k=-\infty}^{\infty} \delta(t - kT)\); signal \(x(t)\) is sampled to obtain \(x_c(t)\), and signal \(y(t)\) is sampled to obtain \(y_c(t)\).

A) Determine the range of values for \(T\) that allows complete recovery of \(x(t)\) from \(x_c(t)\) [pt. 5].
B) Determine the range of values for \(T\) that allows complete recovery of \(y(t)\) from \(y_c(t)\) [pt. 10].

**Exercise 3.** Consider the two discrete-time sequences

\[x_1[n] = \alpha^{2n} u[n]\]

and

\[x_2[n] = -\beta^n u[-n - 1],\]

where \(u[n]\) is the causal unit step function, \(\alpha\) and \(\beta\) are two positive real constants. A third signal is obtained using the convolution sum, e.g., \(x[n] = x_1[n] * x_2[n]\).

A) Compute the z-transform of \(x_1[n]\) [pt. 5].
B) Compute the z-transform of \(x_2[n]\) [pt. 5].
C) Compute the z-transform of \(x[n]\) [pt. 10].
D) Determine under what condition on \(\alpha\) and \(\beta\) the z-transform of \(x[n]\) exists [pt. 10].
E) Determine under what condition on \(\alpha\) and \(\beta\) the discrete-time Fourier transform of \(x[n]\) exists [pt. 5].
Exercise 4. Consider a discrete-time LTI system with unit impulse response $h[n]$. Let $x[n]$ and $y[n]$ be the input and output signal, respectively. Let the $z$-transform of $h[n]$ be

$$H(z) = \frac{z}{z - 1} \quad \text{if } |z| > 1.$$ 

Let the $z$-transform of $x[n]$ be

$$X(z) = 1 + z^{-1} \quad \text{if } |z| \neq 0.$$ 

A) Derive the $z$-transform of $y[n]$ [pt. 5].

B) Derive, sketch and label carefully $y[n]$ [pt. 15].

Exercise 5. A discrete-time signal $x[n]$ has the following $z$-transform

$$X(z) = \frac{1}{1 + z^{-1}} \quad \text{if } |z| > 1.$$ 

A) Derive the $z$-transform of $y[n] = x^2[n]$ [pt. 15].