November 30, 2007

Final Exam

EE 3302: Signals and Systems

NOTE: Please, complete the following table and keep record of your assignment number.

<table>
<thead>
<tr>
<th>First Name</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Last Name</td>
<td></td>
</tr>
<tr>
<td>Student ID</td>
<td></td>
</tr>
<tr>
<td>Assignment #</td>
<td>0</td>
</tr>
</tbody>
</table>

Exercise 1. A system is described by the following differential equation

\[ \frac{d^2 y(t)}{dt^2} + 8 \frac{d y(t)}{dt} + 15 y(t) = \frac{d^2 x(t)}{dt^2} + 7 \frac{d x(t)}{dt} + 10 x(t) \]

where \(x(t)\) is the input signal, and \(y(t)\) is the output signal. Assume that the initial rest condition is satisfied.

A) Determine the frequency response of the system [pt. 10].

B) Determine the unit impulse response of the system [pt. 10].

C) Determine the frequency response of the inverse system [pt. 5].

D) Determine the unit impulse response of the inverse system [pt. 10].

Exercise 2. Consider the continuous-time signal

\[ x(t) = \frac{\sin(\alpha t)}{t} \]

where \(\alpha\) is a positive finite value. Let \(y(t) = x^2(t)\). The following signals are sampled using a train of impulses with periodicity \(T\): \(\sum_{k=\infty}^{\infty} \delta(t - kT)\): signal \(x(t)\) is sampled to obtain \(x_c(t)\), and signal \(y(t)\) is sampled to obtain \(y_c(t)\).

A) Determine the range of values for \(T\) that allows complete recovery of \(x(t)\) from \(x_c(t)\) [pt. 10].

B) Determine the range of values for \(T\) that allows complete recovery of \(y(t)\) from \(y_c(t)\) [pt. 10].

Exercise 3. Consider the two discrete-time sequences

\[ x_1[n] = 2^n u[n + 1] \quad \text{and} \quad x_2[n] = \left(\frac{1}{3}\right)^n u[-n] \]

where \(u[n]\) is the causal unit step function. A third signal is obtained using the convolution sum, e.g., \(x[n] = x_1[n] * x_2[n]\).

A) Compute the z-transform of \(x_1[n]\) [pt. 10].

B) Compute the z-transform of \(x_2[n]\) [pt. 10].

C) Compute the z-transform of \(x[n]\) [pt. 10].

D) Does the Fourier transform of \(x[n]\) converge? [pt. 5].
**Exercise 4.** The algebraic part of the $z$-transform of a discrete-time signal $x[n]$ is

$$X(z) = \frac{1}{(1 + 2z^{-1})^2}$$

The region of convergence (RoC) is not explicitly given, but it is known that point $z = -5$ belongs to the RoC.

A) Determine the RoC of $X(z)$ from the information available [pt. 10].

B) Derive $x[n]$ using the RoC found in A) [pt. 15].

C) Does the Fourier transform of $x[n]$ converge? [pt. 5].

**Exercise 5.** Consider a discrete-time LTI system with unit impulse response $h[n] = \delta[n] - \delta[n - 1]$, where $\delta[n]$ is the unit impulse function. Let $x[n]$ and $y[n]$ be the input and output signal, respectively. Let the $z$-transform of $x[n]$ be

$$X(z) = \frac{1}{z(z^2 - \frac{2}{3}z - \frac{1}{3})} \quad |z| > 1$$

A) Derive the $z$-transform of $y[n]$ [pt. 15].

B) Derive, sketch and label carefully $y[n]$ [pt. 15].