Exercise 1. A system is described by the following differential equation

\[ \frac{d^2 y(t)}{dt^2} + 7 \frac{dy(t)}{dt} + 10 y(t) = \frac{d^2 x(t)}{dt^2} + 8 \frac{dx(t)}{dt} + 15 x(t) \]

where \( x(t) \) is the input signal, and \( y(t) \) is the output signal. Assume that the initial rest condition is satisfied.

A) Determine the frequency response of the system [pt. 10].
B) Determine the unit impulse response of the system [pt. 10].
C) Determine the frequency response of the inverse system [pt. 5].
D) Determine the unit impulse response of the inverse system [pt. 10].

Exercise 2. Consider the continuous-time signal

\[ x(t) = \frac{\sin(5t)}{5t} \]

Let \( y(t) = x^2(t) \). The following signals are sampled using a train of impulses with periodicity \( T \), \( \sum_{k=-\infty}^{+\infty} \delta(t - kT) \): signal \( x(t) \) is sampled to obtain \( x_\epsilon(t) \), and signal \( y(t) \) is sampled to obtain \( y_\epsilon(t) \).

A) Determine the range of values for \( T \) that allows complete recovery of \( x(t) \) from \( x_\epsilon(t) \) [pt. 10].
B) Determine the range of values for \( T \) that allows complete recovery of \( y(t) \) from \( y_\epsilon(t) \) [pt. 10].

Exercise 3. Consider the two discrete-time sequences

\[ x_1[n] = \left( \frac{1}{2} \right)^n u[n-1] \quad \text{and} \quad x_2[n] = \left( \frac{1}{3} \right)^n u[n+1] \]

where \( u[n] \) is the causal unit step function. A third signal is obtained using the convolution sum, e.g., \( x[n] = x_1[n] \ast x_2[n] \).

A) Compute the z-transform of \( x_1[n] \) [pt. 10].
B) Compute the z-transform of \( x_2[n] \) [pt. 10].
C) Compute the z-transform of \( x[n] \) [pt. 10].
D) Derive, sketch and label carefully \( x[n] \) [pt. 15].
Exercise 4. The algebraic part of the \( z \)-transform of a discrete-time signal \( x[n] \) is

\[
X(z) = -\frac{z^{-1}}{(1 - 5jz^{-1})^2}
\]

The region of convergence (RoC) is not explicitly given, but it is known that point \( z_1 = 1 + j \) belongs to the RoC.

A) Determine the RoC of \( X(z) \) from the information available [pt. 10].

B) Derive \( x[n] \) using the RoC found in A) [pt. 15].

C) Does the Fourier transform of \( x[n] \) converge? [pt. 5].

Exercise 5. Consider a discrete-time LTI system with unit impulse response \( h[n] = \delta[n - 1] - \delta[n - 2] \), where \( \delta[n] \) is the unit impulse function. Let \( x[n] \) and \( y[n] \) be the input and output signal, respectively. Let the \( z \)-transform of \( x[n] \) be

\[
X(z) = \frac{z}{(z^2 - \frac{5}{3}z + \frac{2}{3})}, \quad |z| > 1
\]

A) Derive the \( z \)-transform of \( y[n] \) [pt. 15].

B) Derive, sketch and label carefully \( y[n] \) [pt. 15].