$$S_1: \text{causal LTI,}$$
$$w[n] = \frac{1}{2} w[n - 1] + x[n];$$

$$S_2: \text{causal LTI,}$$
$$y[n] = \alpha y[n - 1] + \beta w[n].$$

The difference equation relating $x[n]$ and $y[n]$ is:

$$y[n] = -\frac{1}{8} y[n - 2] + \frac{3}{4} y[n - 1] + x[n].$$

(a) Determine $\alpha$ and $\beta$.
(b) Show the impulse response of the cascade connection of $S_1$ and $S_2$.

2.20. Evaluate the following integrals:
(a) $\int_{-\infty}^{\infty} u_0(t) \cos(t) \, dt$
(b) $\int_{0}^{5} \sin(2\pi t) \delta(t + 3) \, dt$
(c) $\int_{-5}^{5} u_1(1 - \tau) \cos(2\pi \tau) \, d\tau$

**BASIC PROBLEMS**

2.21. Compute the convolution $y[n] = x[n] * h[n]$ of the following pairs of signals:

(a) $x[n] = \alpha^n u[n], \quad h[n] = \beta^n u[n], \quad \alpha \neq \beta$
(b) $x[n] = h[n] = \alpha^n u[n]$
(c) $x[n] = (-\frac{1}{2})^n u[n - 4], \quad h[n] = 4^n u[2 - n]$
(d) $x[n]$ and $h[n]$ are as in Figure P2.21.

![Figure P2.21](image)

2.22. For each of the following pairs of waveforms, use the convolution integral to find the response $y(t)$ of the LTI system with impulse response $h(t)$ to the input $x(t)$. Sketch your results.

(a) $x(t) = e^{-\alpha t} u(t), \quad h(t) = e^{-\beta t} u(t)$ (Do this both when $\alpha \neq \beta$ and when $\alpha = \beta$.)
(b) \( x(t) = u(t) - 2u(t - 2) + u(t - 5) \)
\( h(t) = e^{2t}u(1 - t) \)
(c) \( x(t) \) and \( h(t) \) are as in Figure P2.22(a).
(d) \( x(t) \) and \( h(t) \) are as in Figure P2.22(b).
(e) \( x(t) \) and \( h(t) \) are as in Figure P2.22(c).

![Graphs](image)

Figure P2.22

2.23. Let \( h(t) \) be the triangular pulse shown in Figure P2.23(a), and let \( x(t) \) be the impulse train depicted in Figure P2.23(b). That is,

\[
x(t) = \sum_{k=-\infty}^{+\infty} \delta(t - kT).
\]

Determine and sketch \( y(t) = x(t) * h(t) \) for the following values of \( T \):
(a) \( T = 4 \)
(b) \( T = 2 \)
(c) \( T = 3/2 \)
(d) \( T = 1 \)
2.24. Consider the cascade interconnection of three causal LTI systems, illustrated in Figure P2.24(a). The impulse response $h_2[n]$ is

$$h_2[n] = u[n] - u[n - 2],$$

and the overall impulse response is as shown in Figure P2.24(b).

(a) Find the impulse response $h_1[n]$.
(b) Find the response of the overall system to the input

$$x[n] = \delta[n] - \delta[n - 1].$$
2.25. Let the signal

\[ y[n] = x[n] \ast h[n], \]

where

\[ x[n] = 3^n u[-n - 1] + \left( \frac{1}{3} \right)^n u[n] \]

and

\[ h[n] = \left( \frac{1}{4} \right)^n u[n + 3]. \]

(a) Determine \( y[n] \) without utilizing the distributive property of convolution.
(b) Determine \( y[n] \) utilizing the distributive property of convolution.

2.26. Consider the evaluation of

\[ y[n] = x_1[n] \ast x_2[n] \ast x_3[n], \]

where \( x_1[n] = (0.5)^n u[n] \), \( x_2[n] = u[n + 3] \), and \( x_3[n] = \delta[n] - \delta[n - 1] \).
(a) Evaluate the convolution \( x_1[n] \ast x_2[n] \).
(b) Convolve the result of part (a) with \( x_3[n] \) in order to evaluate \( y[n] \).
(c) Evaluate the convolution \( x_2[n] \ast x_3[n] \).
(d) Convolve the result of part (c) with \( x_1[n] \) in order to evaluate \( y[n] \).

2.27. We define the area under a continuous-time signal \( v(t) \) as

\[ A_v = \int_{-\infty}^{+\infty} v(t) \, dt. \]

Show that if \( y(t) = x(t) \ast h(t) \), then

\[ A_y = A_x A_h. \]

2.28. The following are the impulse responses of discrete-time LTI systems. Determine whether each system is causal and/or stable. Justify your answers.
(a) \( h[n] = \left( \frac{1}{3} \right)^n u[n] \)
(b) \( h[n] = (0.8)^n u[n + 2] \)
(c) \( h[n] = \left( \frac{1}{2} \right)^n u[-n] \)
(d) \( h[n] = (5)^n u[3 - n] \)
(e) \( h[n] = (-\frac{1}{2})^n u[n] + (1.01)^n u[n - 1] \)
(f) \( h[n] = (-\frac{1}{2})^n u[n] + (1.01)^n u[1 - n] \)
(g) \( h[n] = n(\frac{1}{2})^n u[n - 1] \)

2.29. The following are the impulse responses of continuous-time LTI systems. Determine whether each system is causal and/or stable. Justify your answers.
(a) \( h(t) = e^{-4t} u(t - 2) \)
(b) \( h(t) = e^{-6t} u(3 - t) \)
(c) \( h(t) = e^{-2t} u(t + 50) \)
(d) \( h(t) = e^{2t} u(-1 - t) \)