(a) Find the differential equation relating $x(t)$ and $y(t)$.
(b) Determine the frequency response of this system by considering the output of the system to inputs of the form $x(t) = e^{j\omega t}$.
(c) Determine the output $y(t)$ if $x(t) = \sin(t)$.

BASIC PROBLEMS

3.21. A continuous-time periodic signal $x(t)$ is real valued and has a fundamental period $T = 8$. The nonzero Fourier series coefficients for $x(t)$ are specified as
\[ a_1 = a_{-1} = j, a_5 = a_{-5} = 2. \]
Express $x(t)$ in the form
\[ x(t) = \sum_{k=0}^{\infty} A_k \cos(w_k t + \phi_k). \]

3.22. Determine the Fourier series representations for the following signals:
(a) Each $x(t)$ illustrated in Figure P3.22(a)–(f).
(b) $x(t)$ periodic with period 2 and
\[ x(t) = e^{-t} \text{ for } -1 < t < 1 \]

![Figure P3.22](image-url)
(c) \( x(t) \) periodic with period 4 and

\[
x(t) = \begin{cases} 
\sin \pi t, & 0 \leq t \leq 2 \\
0, & 2 < t \leq 4 
\end{cases}
\]

3.23. In each of the following, we specify the Fourier series coefficients of a continuous-time signal that is periodic with period 4. Determine the signal \( x(t) \) in each case.

(a) \( a_k = \begin{cases} 
0, & k = 0 \\
\left(j^k \frac{\sin k \pi/4}{k \pi}\right), & \text{otherwise}
\end{cases} \)

(b) \( a_k = \begin{cases} 
(-1)^k \frac{\sin k \pi/8}{2k \pi}, & a_0 = \frac{1}{16}
\end{cases} \)

(c) \( a_k = \begin{cases} 
jk, & |k| < 3 \\
0, & \text{otherwise}
\end{cases} \)

(d) \( a_k = \begin{cases} 
1, & k \text{ even} \\
2, & k \text{ odd}
\end{cases} \)

3.24. Let

\[
x(t) = \begin{cases} 
t, & 0 \leq t \leq 1 \\
2 - t, & 1 \leq t \leq 2
\end{cases}
\]

be a periodic signal with fundamental period \( T = 2 \) and Fourier coefficients \( a_k \).

(a) Determine the value of \( a_0 \).

(b) Determine the Fourier series representation of \( dx(t)/dt \).

(c) Use the result of part (b) and the differentiation property of the continuous-time Fourier series to help determine the Fourier series coefficients of \( x(t) \).
3.25. Consider the following three continuous-time signals with a fundamental period of $T = 1/2$:

$$x(t) = \cos(4\pi t),$$
$$y(t) = \sin(4\pi t),$$
$$z(t) = x(t)y(t).$$

(a) Determine the Fourier series coefficients of $x(t)$.

(b) Determine the Fourier series coefficients of $y(t)$.

(c) Use the results of parts (a) and (b), along with the multiplication property of the continuous-time Fourier series, to determine the Fourier series coefficients of $z(t) = x(t)y(t)$.

(d) Determine the Fourier series coefficients of $z(t)$ through direct expansion of $z(t)$ in trigonometric form, and compare your result with that of part (c).

3.26. Let $x(t)$ be a periodic signal whose Fourier series coefficients are

$$a_k = \begin{cases} 2, & k = 0 \\ \frac{1}{2} |k|, & \text{otherwise} \end{cases}$$

Use Fourier series properties to answer the following questions:

(a) Is $x(t)$ real?

(b) Is $x(t)$ even?

(c) Is $dx(t)/dt$ even?

3.27. A discrete-time periodic signal $x[n]$ is real valued and has a fundamental period $N = 5$. The nonzero Fourier series coefficients for $x[n]$ are

$$a_0 = 2, a_2 = a_{-2}^* = 2e^{j\pi/6}, \quad a_4 = a_{-4}^* = e^{j\pi/6}.$$ Express $x[n]$ in the form

$$x[n] = A_0 + \sum_{k=1}^{\infty} A_k \sin(\omega_k n + \phi_k) .$$

3.28. Determine the Fourier series coefficients for each of the following discrete-time periodic signals. Plot the magnitude and phase of each set of coefficients $a_k$.

(a) Each $x[n]$ depicted in Figure P3.28(a)-(c)

(b) $x[n] = \sin(2\pi n/3) \cos(\pi n/2)$

(c) $x[n]$ periodic with period 4 and

$$x[n] = 1 - \sin \frac{\pi n}{4} \quad \text{for } 0 \leq n \leq 3$$

(d) $x[n]$ periodic with period 12 and

$$x[n] = 1 - \sin \frac{\pi n}{4} \quad \text{for } 0 \leq n \leq 11$$