Exercise 1. Consider the open network of queues shown in Figure 1. It consists of three M/M/1 queues.

![Figure 1: Open queueing network.](image)

Customers enter the network at rate $\lambda$ immediately reaching queue 1. Customers leaving queue 1 will choose queue 2 with probability $p$, and queue 3 with probability $(1-p)$. All customers leaving queue 2 return to queue 1. All customers leaving queue 3 leave the network forever. The service times at the queues are independent and exponentially distributed with mean $1/\mu_1 = 1/\mu$, $1/\mu_2 = 1/\mu$, and $1/\mu_3 = 1/\mu$.

A) Find the stability conditions of the network of queues [pt. 5].

B) Find the expected total number of customers in the entire network [pt. 15].

C) Find the average time spent in the system by a customer [pt. 15].

Exercise 2. Consider the M/G/1 queue with the following special behavior. Each arrival consists of a pair of jobs. Let $\lambda$ be the pair arrival rate. One job is of type 1, the other is of type 2. Arriving pairs are serviced FCFS. A type 1 job requires a service time $X$. A type 2 job requires a service time $Y$. $X$ and $Y$ are random variables with general distribution.

Two service strategies are defined. Strategy $R$ randomly ($50\% + 50\%$) chooses one of the two jobs in the pair to be serviced first. Strategy $D$ always chooses job of type 1 to be serviced first.

Define $W_p$, $W_1$, $W_2$, and $W$ as the waiting time for the pair, type 1 job, type 2 job, and any job, respectively. Define $T_1$, $T_2$, and $T$ as the total time in the system for type 1 job, type 2 job, and any job, respectively.

A) For strategy $R$, determine the stability condition, $W_p$, $W_1$, $W_2$, $T_1$, $T_2$, and $T$ [pt. 25].

B) For strategy $D$, determine the stability condition, $W_1$, $W_2$, $T_1$, $T_2$, and $T$ [pt. 25].

C) Assume that $X + Y = d$ for any given job pair is a constant value, and $X$ and $Y$ are random variables distributed over $(0,d)$. Determine $\overline{X}$ (and $\overline{Y}$) that will minimize $T$ in strategy $R$ and $D$, respectively [pt. 15].