May 1, 2009
Final Exam
EE 6340: Introduction to Telecommunications Networks

NOTE: Please, complete the following table and keep record of your assignment number.

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| Assignment # | 0 |

Exercise 1. Consider the graph shown in Fig. 1. Note that a double arrow link in the figure represents two distinct directed links with opposite directions and same weight. Using a graphical or matrix based representation of each intermediate iteration, find the shortest path from any node to node 4 as indicated below.

A) Run the first three iterations \( h = 3 \) of the Bellman-Ford algorithm [pt. 20].

B) Identify the path found from node 1 at the end of the third iteration [pt. 10].

C) Indicate at what iteration \( h \) the algorithm stops and what is the reason for the algorithm to stop there [pt. 10].

Exercise 2. Consider the flow network shown in Fig. 2. The label on the link indicates the capacity of the link. A double arrow link in the figure represents two distinct directed links with opposite directions and same capacity.

A) Using a graphical based representation of each intermediate iteration, find the maximum flow in the flow network from node 9 to 2, and its value. Use a shortest path approach when finding the augmenting path [pt. 20].
B) Determine all the minimum cuts from node 9 to 2 [pt. 10].

C) To increment the maximum flow value you are given the opportunity to update the capacity of only one directed link in the graph. Choose the link to upgrade and the new (minimal) capacity for that link that will maximize the incremental value of the flow. Indicate the resulting (new) maximum flow from node 9 to 2 [pt. 10].

D) Determine all the minimum cuts from node 9 to 2 after the link update [pt. 10].

**Exercise 3.** Consider a closed network of two queues: Q1 and Q2. Service time at each queue is exponentially distributed with average $1/\mu_1$ and $1/\mu_2$, respectively, and $\mu_1 > \mu_2$. Upon completion of service at either Q1 or Q2, a job will choose to enter Q1 (Q2) with probability $p$ ($1 - p$). Assume that there are $K = 2$ jobs only in the closed network of queues.

A) Draw the network of queues, build the Markov chain of the network of queues, and determine its stability condition [pt. 10].

B) Compute the steady state distribution, i.e., $\pi_{ij}$, defined as the probability of having $i$ jobs in Q1 and $j$ jobs in Q2 [pt. 20].

C) Compute the same distribution ($\pi_{ij}$) using the product form approach and compare with the answer given in B) to determine whether or not the product form is applicable in this case [pt. 20].

D) Compute the departure rate from each queue, i.e., $d_1$ and $d_2$ [pt. 10].

E) Compute the value of $p$ that will make the departure rate from both queues to be equal [extra credit pt. 10].

**Exercise 4.** Consider the M/G/1 queue with the following special behavior. The job arrival stream forms a Poisson process with rate $\lambda$. Every job is assigned a unique sequence number, i.e., 1,2,3, etc. Server can only service a pair of jobs together, one being odd and the other being even. When a pair of such jobs is in the queue, the server can service the two jobs at once, with a service time $X$. First and second moments of $X$ are known.

A) Determine the server utilization ($\rho$) and the stability condition of the queue [pt. 10].

B) Compute $W_r$, $W_o$, and $W$, defined as the average waiting time for an even job, an odd job, any job, respectively [pt. 25].

C) Compute $N_{re}$, $N_{ro}$, and $N_r$, defined as the average number of waiting jobs in the queue, considering even jobs only, odd jobs only, all jobs, respectively [pt. 25].

D) Study the behavior of $N_{re}$, $N_{ro}$, and $N_r$, as $X$ tends to become a zero value (i.e., first and second moments tend to 0) and explain intuitively the result found [pt. 25].