May 6, 2011

Final Exam

EE 6340: Introduction to Telecommunications Networks

NOTE: Please, complete the following table and keep record of your assignment number.

| First Name | 
| Last Name | 
| Student ID | 
| Assignment # | 0 |

**Exercise 1.** Consider the graph shown in Fig. 1. Note that a double arrow link in the figure represents two distinct directed links with opposite directions and same weight (shown next to the link). Using a graphical or matrix based representation of each intermediate iteration, find the shortest path from every node to node 1 as indicated below.

**A)** Run the first three iterations \( h = 0, 1, 2 \) of the Bellman-Ford algorithm [pt. 20].

**B)** Identify the path found from node 7 at the end of the third iteration [pt. 10].

**C)** Indicate at what iteration \( h \) the algorithm stops and what is the reason for the algorithm to stop there [pt. 20].

**Exercise 2.** Consider the flow network shown in Fig. 2. The label on the link indicates the capacity of the link. A double arrow link in the figure represents two distinct directed links with opposite directions and same capacity.

**A)** Using a graphical based representation of each intermediate iteration, find the maximum flow in the flow network from node 1 to node 5. Choose and define a shortest path approach when finding the augmenting path [pt. 20].
B) Determine the value of the maximum flow [pt. 5].

C) Identify all the minimum cut(s) between node 1 and 5 [pt. 15].

D) Assume that you are given the option to increase the capacity of one link. Assume that any link can be chosen. Which link would you upgrade to further maximize the maximum flow, what new minimum capacity value would you assign to the link to obtain such target, and what new maximum flow value would you achieve? [pt. 10].

E) Identify all the minimum cut(s) between node 1 and 5 of the latter flow network [pt. 15].

**Exercise 3.** Consider the open network of queues shown in Figure 3. It consists of two M/M/1 queues.

![Diagram of open queueing network](image)

Figure 3: Open queueing network.

Customers enter the network at rate $\lambda_1$ and $\lambda_2$, immediately reaching queue Q1 and Q2, respectively. Customers leaving Q1 (Q2) will either choose Q2 (Q1) with probability $P_1$ ($P_2$), or will depart from the system for ever. The service times at the queues are independent and exponentially distributed with mean $1/\mu_1$ and $1/\mu_2$, respectively.

A) Find the stability conditions of the network of queues, and compute the departure rate $R_1$ and $R_2$ from Q1 and Q2 respectively [note: the departure rate is measured before the customer decides to either stay in or leave the system] [pt. 20].

B) Compute the probability of finding the server of Q1 and Q2 busy, i.e., $\rho_1$ and $\rho_2$, respectively. Compute the probability that both servers are busy simultaneously ($P_{\text{both}}$) and compute the probability that at least one server is busy ($P_{\text{at least one}}$) [pt. 15].

C) Compute the expected number of customers in Q1 and Q2, i.e., $N_1$ and $N_2$, respectively [pt. 10].

D) Compute the average time spent in Q1 ($T_1$), spent in Q2 ($T_2$), total time spent in the system by any customer ($T_i$), total time spent in the system by customers arriving from $\lambda_1$ and from $\lambda_2$, i.e., $T_{i1}$ and $T_{i2}$ respectively [pt. 15].

**Exercise 4.** Consider the M/G/1/1 queue, with Poisson arrival rate $\lambda$ and service time $X$. Assume that both first and second moments of $X$ are known.

A) Determine the stability condition of the queue [pt. 5].

B) Compute $P_b$, defined as the probability that an arriving job is dropped upon arrival (finding the server busy). Compute the probability of finding the server of the queue busy [pt. 20].

C) Consider now a M/G/2/2, with two servers, one with service time $X$, the second with service time $Y$. Upon arrival, if server $X$ is available, it will be chosen for service, otherwise, server $Y$ will be chosen for service. An arrival is blocked if both servers are busy at the time of the arrival. Determine the stability condition of the queue [pt. 5].

D) For the latter system (M/G/2/2) compute the following parameters: $\rho_1$, defined as the probability of finding the $X$ server busy; $\rho_2$, defined as the probability of finding the $Y$ server busy; $P_{b1}$, defined as the probability for an arrival not to be able to access server $X$; $P_{b2}$, defined as the probability for an arrival (arriving when server $X$ is busy) not to be able to access server $Y$; $P_b$, defined as the overall probability for an arrival of being rejected as both servers are busy. [pt. 25].