Centralized Pricing Versus Delegating Pricing to the Salesforce Under Information Asymmetry

Birendra K. Mishra
Anderson Graduate School of Management, University of California, Riverside, California 92521, and
School of Management, The University of Texas at Dallas, Richardson, Texas 75083-0688, barrymishra@yahoo.com

Ashutosh Prasad
School of Management, The University of Texas at Dallas, Richardson, Texas 75083-0688, aprasad@utdallas.edu

The issue of delegating pricing responsibility to the salesforce is of interest to marketing academics and practitioners. It has been shown by Lal (1986) that under certain situations with information asymmetry, it is more profitable for the firm to delegate pricing authority to the salesforce than to have centralized pricing. In this paper we re-examine situations where information asymmetry exists and analyze its effect on the decision of the firm to set a price or delegate pricing responsibility to the salesforce. Using contract theory, we find that when the salesperson’s private information can be revealed to the firm through contracting, centralized pricing performs at least as well as price delegation. We derive the optimal centralized pricing contract under a set of standard assumptions used in the economics and business literature.

Key words: pricing research; compensation; salesforce; delegation

History: This paper was received January 22, 2002, and was with the authors 2 months for 2 revisions; processed by Chakravarthi Narasimhan.

1. Introduction
An important issue that has attracted the interest of several scholars in marketing involves the following question: Should pricing decisions be made by the firm or delegated to the salesforce (Bhardwaj 2001, Stephenson et al. 1979, Joseph 2001, Lal 1986, Weinberg 1975)? There appears to be no unconditional answer to this question because one can find examples of both in marketing practice. Theoretical recommendations, however, have tended to favor price delegation (e.g., Lal 1986, Weinberg 1975). In contrast, this paper shows that for a broad class of situations it is no worse for the firm to set prices, referred to as centralized pricing, than to delegate the pricing responsibility to the salesforce. We describe these situations by examining the pricing decision in the informational and contracting framework of agency theory.

In agency theory, the firm, or a representative of the firm such as the sales manager, hires the salesperson to sell its product in return for compensation as specified in a legally binding contract that both have agreed upon. This modeling framework has been used extensively in marketing for exploring the salesforce-firm relationship (Bergen et al. 1992, Coughlan and Sen 1989). The firm designs an optimal contract that maximizes its expected profit, while it is cognizant of the fact that the salesperson will expend effort on selling that maximizes his or her own expected utility (Basu et al. 1985). Ideally, compensation should be tied to the selling effort, but because the firm cannot directly observe the salesperson’s effort and because it lacks the knowledge that would enable it to indirectly infer the effort applied by the salesperson, the two parties must contract on observable variables such as the realized sales and the prices charged.

In a key paper, Lal (1986) showed that price delegation is appropriate when information asymmetry exists, i.e., when the salesperson has relevant private information that is not available to the firm. One can intuitively see that a salesperson would have better local sales knowledge emerging from established relationships, interaction with customers, and expertise in the selling arena compared to the sales manager. In some cases, the private information may simply be the salesperson’s selling ability, known to the salesperson but hidden from the firm (Lal and Staelin 1986, Rao 1990). Lal (1986) proved the optimality of price delegation under one type of information asymmetry. However, there exist several approaches to information asymmetry corresponding to different situations faced by different firms at different times. We list these approaches below. Note that other than Lal (1986), the sequences of events listed below are not adapted for understanding the price delegation issue.
(a) The salesperson first signs the contract, then exerts effort, then observes the private information, and then selects price (e.g., Lal 1986).

(b) The salesperson first signs the contract, then observes the private information, and then exerts effort (e.g., Holmstrom 1979, §6).

(c) The salesperson first observes the private information, then signs the contract, and then exerts effort (e.g., Chen 1990; Harris et al. 1982; Lal and Staelin 1986; Mantrala et al. 1994; McAfee and McMillan 1988; Mishra and Vaysman 2001; Myerson 1979, 1982; Rao 1990).

In sequence (a), the firm is unable to extract the private information from the salesperson because the contract is signed prior to the private information being realized. Thus, with centralized pricing the firm chooses price without knowing the information and does worse than under price delegation, where the salesperson has better information with which to make the pricing decision.

Sequence (b) is equivalent to sequence (c) when the salesperson is free to quit upon observing the private information. This is because only those salespeople whose private information shows that it is rational to do so will continue to participate and exert effort. In contrast, in sequence (a) the salesperson will not quit on learning the private information because effort is a sunk cost by the time it is learnt.

We follow sequence (c), which is the standard approach to asymmetric information problems in the economics and business literature where the agent has superior information and the principal offers the contract. In the salesforce context, this approach is prevalent in participative sales forecasting and quota setting. In general, participative budgeting and compensation negotiations are widely used as a means to capture the agents’ private information during the formal planning process.

A frequently used method for sales forecasting is the sales composite, where input from the salesforce is used to create a bottom-up forecast. More than 65% of companies, particularly industrial goods firms, use this approach on a regular basis (Mentzer and Cox 1984, Moon and Mentzer 1999, Wotruba and Thurlow 1976). To ensure that salespeople reveal their information truthfully, it is well understood that compensation should be based on forecast accuracy in such a manner that salespeople find it more profitable to make a truthful forecast rather than an optimistic or pessimistic one (Gonik 1978, Mantrala and Raman 1990). Gonik (1978) describes IBM Brazil’s implementation of such a quota-based compensation scheme where salespeople self-select their quota based on their forecast. Mantrala and Raman (1990) list other examples.

The following example is provided by Keenan (1995):

Sales Forecasting at Dow Chemical’s Basic Chemicals Division takes a balanced approach, making use of both historical data and current sales input...Dow also controls salespeople’s optimism and gives them a thorough grounding in reality by tying the sales forecast number back into their sales quota and performance goals.

Because salespeople cannot be expected to have the ability to forecast accurately over a long horizon, the sales composite method, not surprisingly, is used primarily for short- to medium-term forecasting (Mentzer and Cox 1984). Mantrala et al. (1994) specify the setting as one where the salesforce operates in a repetitive buying environment, promotes a limited set of familiar products, and is geographically specialized. Moon and Mentzer (1999) include situations where the firm has a few large buyers that buy sporadically. Under such situations, the salesforce has useful information to contribute. The salesperson and regional sales manager arrive at a negotiated quota that the salesperson finds achievable given his or her private information. Each time the contract is negotiated, which may be as frequently as a month or a quarter, salespersons have better information about the potential demand in their territory and have the opportunity to convey this private information. For the contract that is finally agreed upon, the contract particulars specify the compensation plan, i.e., the compensation for different sales levels or gross margins realized, and the price list if the contract is a centralized pricing contract.

The actual nature of the private information is not relevant to the conclusions. Thus, sequence (c) is also appropriately applied when the private information concerns the salesperson’s own characteristics, as in the literature on compensating a heterogeneous salesforce (e.g., Lal and Staelin 1986, Rao 1990, Mantrala et al. 1994).

We show that if the salesperson can contract with the firm after knowledge of private information, as in sequence (c), centralized pricing achieves the upper bound on the firm’s expected profit. The rest of the paper is organized as follows: In the next section we present the model and provide the main result. In §3 we derive the optimal centralized pricing contract under a set of standard assumptions. Section 4 closes with concluding remarks.

2. The Upper Bound on the Firm’s Expected Profit

The firm would like the salesperson to exert the maximum selling effort and generate sales for the firm’s product. However, the salesperson, for whom effort is costly, has a disutility from selling and will make an effort only if adequately compensated. The firm
knows that the sales of the product are determined by the price of the product, the effort of the salesperson, and a stochastic demand parameter \( t \in \Theta \), which is the private information known to the salesperson but not known to the firm.

The timeline for the game is as follows. First, the firm designs and offers the contract to the salesperson. Next, the salesperson can accept or reject the contract. If the contract is rejected, the salesperson gets an exogenously specified reservation utility \( U_0 \). Finally, the salesperson expends effort and sets a price if required by the contract, sales are realized, and the compensation is awarded.

Sales, \( x \in [0, \infty) \), is a random variable with probability distribution function \( f(x) \). The gross margins for the firm are \((p - c)x\), where \( p \) is the price charged and \( c \) is a constant marginal cost. The compensation from the firm to the salesperson is denoted \( S(\cdot) \), and its utility to the salesperson is given by \( U(S) \), where \( U(\cdot) \) is a concave utility function. The variable \( a \in [0, \infty) \) represents the unobservable, costly effort exerted by the salesperson. The cost of effort is described by the strictly increasing, convex function \( V(a) \). Sales, \( x \), is a function of the salesperson’s private information, \( t \), the price, \( p \), and effort, \( a \).

There is a large number of contracts that the firm can potentially offer. However, an important result known as the revelation principle greatly simplifies the analysis of these types of contracting problems (Myerson 1982). Contract design takes the form of a menu of contracts designed so that the salesperson finds it in the best interest to reveal his or her true type. Laffont and Tirole (1998, p. 82) provide a detailed explanation of why optimal contracts in the form of a menu make practical sense. They note that we tend to observe mainly the final contract that is signed, and not the bargaining process that gives rise to the contract. In addition, they note, as do Lal and Staelin (1986), that explicit menus of managerial incentive schemes are starting to be used in corporations.

The firm’s profit function is the expected gross margins minus the expected compensation computed using the probability densities of sales \( f(x) \) and private information \( \gamma \). The second equation ensures that the salesperson will find it worthwhile to be hired, while Equation (3) is the requirement that the salesperson will act optimally in the choice of the amount of effort to expend. Equation (4) is the incentive compatibility condition that ensures that the salesperson of type \( t \), i.e., one who observes private information \( t \), will prefer the contract \( S(x, t), p(t) \) to any other contract, thereby revealing the private information truthfully to the firm. Once a contract, e.g., \( S(x, t), p(t) \), has been selected, the salesperson must charge price \( p(t) \) to consumers.

For delegated pricing, the problem formulation is as follows:

**MPD1:**

\[
\max_{S(x, p(t), t)} \sum_{t} \gamma_t \int ((p(t) - c)x - S(x, p(t), t)) \cdot f(x \mid a, p(t), t) dx,
\]

s.t.

\[
\int U(S(x, t))f(x \mid a, p(t), t) dx - V(a) \geq U_0,
\]

\[
a, p(t) \in \arg \max \int U(S(x, p(t), t)) \cdot f(x \mid a, p(t), t) dx - V(a).
\]

Equation (1) is the expected gross profit of the firm. It is equal to the expected gross margins minus expected compensation computed using the probability densities of sales \( f(x) \) and private information \( \gamma \). The second equation ensures that the salesperson will find it worthwhile to be hired, while Equation (3) is the requirement that the salesperson will act optimally in the choice of the amount of effort to expend. Equation (4) is the incentive compatibility condition that ensures that the salesperson of type \( t \), i.e., one who observes private information \( t \), will prefer the contract \( S(x, t), p(t) \) to any other contract, thereby revealing the private information truthfully to the firm. Once a contract, e.g., \( S(x, t), p(t) \), has been selected, the salesperson must charge price \( p(t) \) to consumers.

The following mathematical problem formulation is for centralized pricing:

**MPC1:**

\[
\max_{S(x, p(t), t)} \sum_{t} \gamma_t \int ((p(t) - c)x - S(x, t))f(x \mid a, p(t), t) dx,
\]

s.t.

\[
\int U(S(x, t))f(x \mid a, p(t), t) dx - V(a) \geq U_0,
\]

\[
a \in \arg \max \int U(S(x, t))f(x \mid a, p(t), t) dx - V(a)
\]

\[
\int U(S(x, t))f(x \mid a, p(t), t) dx - V(a) \geq \int U(S(x, \theta))f(x \mid a, p(\theta), t) dx - V(a)
\]

\[\forall t, \theta \in \Theta.\]

**Proposition 1.** MPC1 does no worse than MPD1 for the firm. In other words, the expected profit from centralized pricing is at least as high as the profit from price delegation and may be strictly higher.
Proof. The proof is as follows. Under MPD1, the profits obtained by the firm from a type \( t \) salesperson depend upon the choices \( a^* \) and \( p^* \). In MPC1 we can replicate this profit by setting a price \( p^* \) and a compensation \( S(x, t) = S(x, p^*, t) \) for the type \( t \) salesperson. It cannot be shown in reverse that MPD1 will produce the same profit for the firm as MPC1. Let us suppose that \( a^* \) and \( p^* \) generate the optimal expected profit for the firm under MPC1. Let us also suppose that the firm in MPD1 designs a contract \( S(x, p, t) \) that makes the salesperson choose \( a^* \) and \( p^* \). However, there is no guarantee that \( S(x, p, t) = S(x, t) \) under MPC1 implying that the same profit cannot be guaranteed. This, along with the observation above that MPC1 does no worse than MPD1, gives us the desired result.

A requirement for our results to hold is that the firm should be free to design an optimal compensation plan. If a functional form for the compensation plan is specified a priori, for example, the straight commission-based plans in Weinberg (1975) and Joseph (2001), then it is indeed possible that delegation may occasionally do better. A justification for the latter approach is that firms do tend to use simple compensation schemes in practice. An optimal contract, however, allows one to measure the extent of a firm’s suboptimal behavior. Once known, it may lend itself to being approximated in a simplified manner (Basu and Kalyanaram 1990).

3. Deriving an Optimal Contract

The models discussed in the previous section (MPC1, MPD1) are known as hybrid models of hidden action and hidden information (Mas-Colell et al. 1995, pp. 501–502). This means that standard approaches toward solving moral hazard or private information problems cannot be used, and in general, there is no way to completely solve these problems. Not surprisingly, therefore, neither MPC1 nor the model of Lal (1986) can exactly be solved for an optimal contract. Consequently, we obtain the optimal contract for the model MPC1 only when the following three main assumptions apply.

Assumption (A1) (Mas-Colell et al.). The demand function is stochastic for the firm, because it knows only the distribution of \( t \), denoted \( \Gamma(t) \), but is nonstochastic for the salesperson. In other words, the salesperson can correctly estimate the sales for the choice of effort and the price after observing the private information.

Assumption (A2) (Fudenberg and Tirole 1993). The salesperson is assumed to have a quasi-linear utility function.

This is consistent with the literature on private information, and it is interesting to contrast this with the literature on moral hazard where risk aversion is a primary consideration (e.g., Holmstrom 1979). The solution of selling the firm to a risk-neutral agent given moral hazard does not apply in private information settings, because what fee could be assessed for the firm when it is the salesperson that has the private information that determines the valuation of the firm? Analysis with risk-averse agents is significantly more difficult.

Assumption (A3) (Fudenberg and Tirole 1993). We replace the discrete probability distribution on \( t \) with a continuous density function \( \gamma(t) \) having support on \( \Theta = [L, \bar{L}] \). We make the assumption that the inverse hazard rate of the probability distribution of \( t \), \( (1 - \Gamma(t))/\gamma(t) \), is decreasing in \( t \). This property is satisfied by many distributions, including the uniform, the normal, the pareto, the logistic, and the exponential.

We begin by converting the private-information and hidden-action situation into a pure private-information problem by eliminating the effort variable \( a \). To accomplish this, note that the effort choice of the salesperson is a function of \( (x(t), p(t), t) \). Inserting this into \( V(a) \), we get a new unobservable cost function \( \bar{V}(x(t), p(t), t) \) with

\[
\frac{\partial}{\partial t} \bar{V}(x(t), p(t), t) < 0, \quad \frac{\partial}{\partial x} \bar{V}(x(t), p(t), t) > 0
\]

and

\[
\frac{\partial}{\partial p} \bar{V}(x(t), p(t), t) > 0. \quad (9)
\]

The centralized pricing contract takes the form \( S(t), x(t), p(t) \) rather than \( S(x(t), t), p(t) \) because from (A1), the salesperson can commit to a specific sales outcome. Note that the menu of contracts contains a large number of contracts designed with the intention that a salesperson observing private information \( t_1 \) will pick \( S(t_1), x(t_1), p(t_1) \), a salesperson observing \( t_2 \) will pick \( S(t_2), x(t_2), p(t_2) \), etc. Consider the contract specification \( S(t), x(t), p(t) \). It means that the salesperson is required to sell \( x(t) \) at the specified price \( p(t) \) and will receive a compensation \( S(t) \) for doing so. One may think of \( x(t) \) as a quota that the salesperson knows is attainable, and thus the compensation \( S(t) \) will be made. To find the optimal centralized contract, the firm solves the following problem:

\[
\text{MPC2:} \quad \max_{S(t), x(t), p(t)} \int_{\underline{t}}^{\bar{t}} [(p(t) - c)x(t) - S(t)] \cdot \gamma(t) \, dt, \quad (10)
\]

1 For more on risk aversion and for a detailed derivation of Proposition 2, please refer to the technical appendix. It shows that risk aversion is detrimental to the firm’s sales and profits and obtains the necessary conditions for the optimal contract using optimal control theory.
subject to:

For all $t \in \Theta$,

$$S(t) - \overline{V}(x(t), p(t), t) \geq 0,$$

(11)

$$S(t) - \overline{V}(x(t), p(t), t) \geq \tilde{S}(\theta) - \overline{V}(x(\theta), p(\theta), t)$$

$$\forall \theta \in \Theta,$$

(12)

where, without loss of generality, $U_{0} = 0$. To solve MPC2 we first solve for the payment $S(t)$ as a function of $\overline{V}(x(t), p(t), t)$ that gives (details of the proof are in the appendix):

$$S(t) = \overline{V}(x(t), p(t), t) - \int^{t}_{\overline{t}} \frac{\partial}{\partial t} \overline{V}(x(\theta), p(\theta), t) d\theta.$$  

(13)

A characteristic of the solution is that the salesperson earns positive profits, as does the firm, and the contracting arrangement is a win-win situation for the two parties. Following the substitution of $S(t)$ in the firm’s objective function, we can obtain the following proposition that summarizes the payments, the required sales level, and price that solve MPC2.

**Proposition 2.** Let $S^{*}(t)$, $x^{*}(t)$, $p^{*}(t)$ represent the solution to MPC2. Then,

(i) $(x^{*}(t), p^{*}(t)) \in \arg \max \left\{ (p(t) - c)x(t) - \overline{V}(x(t), p(t), t) \right\}$

$$+ \frac{1 - \Gamma(t)}{\gamma(t)} \frac{\partial}{\partial t} \overline{V}(x(t), p(t), t) \right\}.$$

(ii) $S^{*}(t) = \overline{V}(x^{*}(t), p^{*}(t), t)$

$$- \int^{t}_{\overline{t}} \frac{\partial}{\partial t} \overline{V}(x^{*}(\theta), p^{*}(\theta), t) d\theta.$$

(iii) The upper bound on the firm’s expected profit is

$$\int^{t}_{\overline{t}} \left\{ (p^{*}(t) - c)x^{*}(t) - \overline{V}(x^{*}(t), p^{*}(t), t) \right\} \gamma(t) dt.$$

This proposition provides a general solution to the problem. We proceed to examine a parametric specification for illustration purposes to obtain a better understanding of the form of the compensation plan.

**Illustration.** Consider the following specifications:

$$x = ta(1 - p), \quad V(a) = a^2/2,$$

and

$$\gamma(t) = \frac{1}{t - \overline{t}} \Rightarrow \frac{1 - \Gamma(t)}{\gamma(t)} = t - \overline{t}.$$

These specifications satisfy the required properties discussed earlier. Substituting the sales function into the effort function, we obtain

$$\overline{V}(x, p, t) = \frac{x^2}{2t^2(1 - p)^2}.$$  

(14)

Hence, from Proposition 2, we can write the maximization problem as follows.

$$\max_{x, p} (p - c)x - \frac{x^2}{2t^2(1 - p)^2} - (t - \overline{t}) \frac{x^2}{t^3(1 - p)^2}.$$  

(15)

Solving the first-order conditions for maximum simultaneously, we get the optimal solution

$$\left( p^* = \frac{c + 1}{2}, \quad x^* = \frac{t^3(1 - c)^3}{8(2t - \overline{t})} \right).$$  

(16)

Using this and Proposition 2, the compensation can be calculated

$$S(t) = \frac{2x^{*}(t)^2}{t^2(1 - c)^2} + \int^{t}_{\overline{t}} \frac{2x^{*}(\theta)^2}{\theta^3(1 - c)^2} d\theta$$

$$= \frac{(1 - c)^4}{32} \left[ t^4 \frac{16t^3(t - \overline{t})}{(2t - \overline{t})^2} + \frac{(2t - \overline{t})(2t - \overline{t})}{(2t - \overline{t})} \right]$$

$$+ 24t^2 \ln \frac{2\overline{t} - t}{2\overline{t} - \overline{t}} + (t - \overline{t})(8t + \overline{t}).$$  

(17)

Figure 1 is a plot of compensation against type $c = [0.2, 0.4, 0.6]$ corresponding to prices $p^* = [0.6, 0.7, 0.8]$, respectively, and $[\overline{t}, \overline{t}] = [0, 1]$. Using these same numerical values, one can also find that the sales target $x^*$ is increasing with type in a convex manner. Given the sales function $x = ta(1 - p)$, this seems intuitive. If a sales rep reveals her state to be twice as good, her output would be twice as high if
effort and price were held constant. However, effort is higher because compensation is increasing in type. As a result, sales increases in a convex manner with type.

Several other insights of the analysis are noteworthy. From Figure 1, compensation is increasing in type (if it were not, the incentive compatibility condition would be violated). From Equation (17), compensation is increasing with \( x^* \), thereby providing the motivation to work harder. Also from Figure 1, observe that for a given type, compensation is decreasing in \( p^* \). This occurs because high marginal cost induces a higher profit-maximizing price and a lower selling quota for each type. The sales are sufficiently lower that the compensation is reduced. To conclude, compensation may be quite a complicated function of the sales and price, and clearly not a function of only the sales or gross margins. Thus, simpler contracts based on sales or margins will, in general, be suboptimal.

4. Conclusion

This paper investigates the issue of delegating pricing to the salesforce versus centralized pricing. There are incentive and informational asymmetry issues that exist when the firm depends on a salesforce to sell its products to consumers. The firm has to compensate the salesperson for their effort, and at the same time it has to make the best use of the private information in the hands of the salesperson. Both of these objectives are accomplished through the design of an optimal contract.

We begin by describing the setting in which the salesperson’s private information can be communicated to the firm through the choice of contract. The private information may be about the changing demand conditions in the local market, with which the salesperson is better acquainted due to experience, field presence, or contacts; or about the characteristics of the salespersons themselves. Using participative sales forecasting and quota setting, the firm can learn the private information of the salesperson. We show that the firm can achieve its upper bound on profits through centralized pricing by explicitly contracting on price and sales (Proposition 1).

This result suggests that price delegation will be seen primarily in those situations where there is no time to communicate the information or the information is not communicable, as in the sequence of Lal (1986). This conforms to the situations listed by Weinberg (1975) where price delegation is seen, e.g., (1) where the product is perishable such as perishable agricultural products, (2) sales involving trade-ins where the salesman has control over the evaluation of the trade-in, and (3) where the product offering is complex, such as systems selling in which the salesperson has a wide latitude in specifying the combination of services to be offered. In all these situations the decisions must be rather quickly taken due to dynamic market conditions, and therefore, there is no opportunity for the salesman to contact the sales manager.

We derive the optimal contract in Proposition 2. This should facilitate experimental investigation and implementation of such contracts in managerial practice (e.g., Ghosh and John 2000, Mantrala et al. 1994). We find from the illustration following Proposition 2 that the optimal contract can depend on demand conditions in a complex manner. Even though the results may be complex, they provide a useful benchmark for simpler compensation plans to aim towards or alternative compensation schemes, such as tournaments, to be compared against (Basu and Kalyanaram 1990, Kalra and Shi 2001).

The main result in this paper holds for general utility functions, demand functions, and demand and type distributions. However, as in most agency models, competition is excluded. This is appropriate given sufficient product or geographic differentiation (Tirole 1988, p. 279), or nonstrategic competitors (Noble and Gruca 1999). As examples, in software and high-tech industries, the offering can be complex and highly differentiated; while in pharmaceuticals, product patents provide some monopoly power. However, situations with intense price competition, where the need for strategic commitment is strong, fall outside the purview of our results. We speculate that in such situations there may be parameter spaces where price delegation is optimal, given the results of Bhardwaj (2001), who finds that for LEN (linear incentives, exponential utility, and normal distribution) models with no information asymmetry, price delegation is appropriate under intense price competition. Future research should try to increase generality by incorporating competition into the present analysis.

Acknowledgments

The authors thank S. Raghunathan; Suresh Sethi; Max Stinchcombe; the editor-in-chief, Steve Shugan; the area editor; and three anonymous reviewers for their helpful suggestions. Remaining errors are our own.

Appendix

Outline of Proof of Proposition 2. Starting from (10)–(12), let \( W(\theta, t) = S(\theta) - \bar{V}(x(\theta), p(\theta), t) \) represent the salesperson’s utility when his private information is \( t \) and he reports his type to be \( \theta \). We define

\[
W(t, \theta) \equiv \max_{\theta \in \Theta} [S(\theta) - \bar{V}(x(\theta), p(\theta), t)]. \tag{A.1}
\]

We begin by focusing on local incentive compatibility. From the envelope theorem,

\[
\frac{\partial W}{\partial t} \bigg|_{\theta=\theta^*} = -\frac{\partial}{\partial t} \bar{V}(x(t), p(t), t). \tag{A.2}
\]
Integrating (A.2) with respect to \( t \), and noting that the lowest type receives its reservation wage only, i.e., \( W(0,0) = 0 \), we obtain

\[
W(t,t) = - \int_{\frac{1}{2}}^{t} \frac{\partial}{\partial \theta} \bar{V}(x(\theta), p(\theta), \theta) d\theta. \tag{A.3}
\]

As we defined, \( W(\theta, t) = S(\theta) - \bar{V}(x(\theta), p(\theta), t) \), (A.3) implies

\[
S(t) = \bar{V}(x(t), p(t), t) - \int_{\frac{1}{2}}^{t} \frac{\partial}{\partial \theta} \bar{V}(x(\theta), p(\theta), \theta) d\theta. \tag{A.4}
\]

Hence, we have the following optimization problem for the firm:

\[
\text{Max}_{x(t), p(t)} \int_{\frac{1}{2}}^{t} \left[ (p(t) - c)x(t) - \bar{V}(x(t), p(t), t) + \int_{\frac{1}{2}}^{t} \frac{\partial}{\partial \theta} \bar{V}(x(\theta), p(\theta), \theta) d\theta \right] \gamma(t) dt. \tag{A.5}
\]

To solve (A.5), it can be shown by using integration-by-parts that

\[
\int_{\frac{1}{2}}^{t} \left[ \frac{\partial}{\partial \theta} \bar{V}(x(\theta), p(\theta), \theta) \right] \gamma(t) dt = \int_{\frac{1}{2}}^{t} \left[ -1 + \frac{\Gamma(t)}{\gamma(t)} \frac{\partial}{\partial \theta} \bar{V}(x(t), p(t), t) \right] \gamma(t) dt. \tag{A.6}
\]

Thus, the firm’s maximization problem is given by

\[
\text{Max}_{x(t), p(t)} \int_{\frac{1}{2}}^{t} \left[ (p(t) - c)x(t) - \bar{V}(x(t), p(t), t) + \frac{1}{\gamma(t)} \frac{\partial}{\partial \theta} \bar{V}(x(t), p(t), t) \right] \gamma(t) dt. \tag{A.7}
\]

We can perform pointwise optimization to determine \( (x^*(t), p^*(t)) \). If the integrand is concave, the first-order conditions are sufficient (Stole 1995).

**References**


