Quiz 5 Solution

1. A certain company makes 12-volt car batteries. After many years of product testing, the company knows that the average life of a battery is normally distributed, with a mean of 44 months and a standard deviation of 7 months. If the company does not want to make refunds for more than 10% of its batteries under the full-refund guarantee policy, for how long should the company guarantee the batteries (to the nearest month)?

Solution: X: Life of a battery. $X \sim N(44, 7^2)$

\[ P(X \leq x) = 0.10 \]

Converting into z-scores, $z = \frac{x - 44}{7}$, $x = 44 + 7 \times z$

\[ P(Z \leq z) = 0.10 \]

Using Normal probability table, we get, $z = -1.285$ ($-1.28 + (-1.29)$, so, $x = 44 - 7 \times 1.285 \approx 35$.

So the answer is 35 months. (This is same as Q. 30, section 7.3, see notes)

2-3. The average weight in a large population of bears is 980 lbs. The population standard deviation is 85 lbs. Suppose that a simple random sample of 100 bears is selected from this population. Let $\overline{X}$ denotes the average weight of 100 bears.

2. Find the mean and the standard deviation of the sampling distribution of average weight of 100 bears.

Solution:

$X$: Weight of a bear and let $\overline{X}$ is the average weight of 100 bears.

$\mu_{\overline{X}} = \mu = 980$

and $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{85}{\sqrt{100}} = 8.5$

3. Determine the probability that falls within 10 lbs of the population mean (i.e. between 970 lbs and 990 lbs).

\[ P(970 \leq X \leq 990) = P(X \leq 990) - P(X \leq 970) \]
\[ = P(Z < \frac{990 - 980}{8.5}) - P(Z < \frac{970 - 980}{8.5}) \]
\[ = P(Z < 1.18) - P(Z < -1.18) \]
\[ = 0.891 - 0.119 = 0.772 \]

4-6. Consider the number of televisions per household (X) and their probabilities from US survey data

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(x)</td>
<td>0.008</td>
<td>0.096</td>
<td>0.384</td>
<td></td>
</tr>
</tbody>
</table>

4. Find $P(X=3)$?

\[ P(X = 3) = 1 - [P(X = 0) + P(X = 1) + P(X = 2)] \]
\[ = 1 - [0.008 + 0.096 + 0.384] = 0.512. \]

5. What is the probability there is atmost two television in any given household?

\[ P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2) \]
\[ = 0.008 + 0.096 + 0.384 = 0.488. \]
6. What is the expected number of television in a household?

Solution:

\[
\text{Expectation} = 0 \times P(X = 0) + 1 \times P(X = 1) + 2 \times P(X = 2) + 3 \times P(X = 3)
\]
\[
= 1 \times 0.096 + 2 \times 0.384 + 3 \times 0.512 = 2.4
\]

So, we expect approximately 2 television in the household.

(See notes titled Note6.1-7.1-7.2.docx).