(c) Plot the power spectral density.
(d) Check the validity of Parseval’s relation for this signal.

4.2 Compute and sketch the magnitude and phase spectra for the following signals \(a > 0\),

(a) \(x_a(t) = \begin{cases} \text{A}e^{-at}, & t \geq 0 \\ 0, & t < 0 \end{cases}\)

(b) \(x_b(t) = A\text{e}^{-\alpha|t|}\)

4.3 Consider the signal

\[ x(t) = \begin{cases} 1 - |t|/\tau, & |t| \leq \tau \\ 0, & \text{elsewhere} \end{cases} \]

(a) Determine and sketch its magnitude and phase spectra, \(|X_a(F)|\) and \(\angle X_a(F)\), respectively.

(b) Create a periodic signal \(x_p(t)\) with fundamental period \(T_p \geq 2\tau\), so that \(x(t) = x_p(t)\) for \(|t| < T_p/2\). What are the Fourier coefficients \(c_k\) for the signal \(x_p(t)\)?

(c) Using the results in parts (a) and (b), show that \(c_k = (1/T_p)X_a(k/T_p)\).

4.4 Consider the following periodic signal:

\[ x(n) = \{\ldots, 1, 0, 1, 2, 3, 2, 1, 0, 1, \ldots\} \]

(a) Sketch the signal \(x(n)\) and its magnitude and phase spectra.

(b) Using the results in part (a), verify Parseval’s relation by computing the power in the time and frequency domains.

4.5 Consider the signal

\[ x(n) = 2 + 2\cos \frac{\pi n}{4} + \cos \frac{\pi n}{2} + \frac{1}{2} \cos \frac{3\pi n}{4} \]

(a) Determine and sketch its power density spectrum.

(b) Evaluate the power of the signal.

4.6 Determine and sketch the magnitude and phase spectra of the following periodic signals.

(a) \(x(n) = 4\sin \frac{\pi(n-2)}{3}\)

(b) \(x(n) = \cos \frac{2\pi}{3}n + \sin \frac{2\pi}{5}n\)

(c) \(x(n) = \cos \frac{2\pi}{3}n \sin \frac{2\pi}{5}n\)

(d) \(x(n) = \{\ldots, -2, -1, 0, 1, 2, -2, -1, 0, 1, 2, \ldots\}\)

(e) \(x(n) = \{\ldots, -1, 2, 1, 2, -1, 0, -1, 2, 1, 2, \ldots\}\)

(f) \(x(n) = \{\ldots, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, \ldots\}\)

(g) \(x(n) = 1, -\infty < n < \infty\)

(h) \(x(n) = (-1)^n, -\infty < n < \infty\)

4.7 Determine the periodic signals \(x(n)\), with fundamental period \(N = 8\), if their Fourier coefficients are given by:

(a) \(c_k = \cos \frac{k\pi}{4} + \sin \frac{3k\pi}{4}\)
(b) \( c_k = \begin{cases} \sin \frac{k\pi}{3}, & 0 \leq k \leq 6 \\ 0, & k = 7 \end{cases} \)

(c) \( \{ c_k \} = \{ \ldots, 0, \frac{1}{2}, 1, 2, 1, \frac{1}{2}, 0 \ldots \} \)

4.8 Two DT signals, \( s_1(n) \) and \( s_2(n) \), are said to be orthogonal over an interval \([N_1, N_2]\) if

\[
\sum_{n=N_1}^{N_2} s_1(n)s_2^*(n) = \begin{cases} A_k, & k = l \\ 0, & k \neq l \end{cases}
\]

If \( A_k = 1 \), the signals are called orthonormal.

(a) Prove the relation

\[
\sum_{n=0}^{N-1} e^{j2\pi kn/N} = \begin{cases} N, & k = 0, \pm N, \pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}
\]

(b) Illustrate the validity of the relation in part (a) by plotting for every value of \( k = 1, 2, \ldots, 6 \), the signals \( s_k(n) = e^{j2\pi kn/N} \), \( n = 0, 1, \ldots, 5 \). [Note: For a given \( k \), \( \pi \) the signal \( s_k(n) \) can be represented as a vector in the complex plane.]

(c) Show that the harmonically related signals

\( s_k(n) = e^{j2\pi kn/N}\)

are orthogonal over any interval of length \( N \).

4.9 Compute the Fourier transform of the following signals.

(a) \( x(n) = u(n) - u(n - 6) \)

(b) \( x(n) = 2^n u(-n) \)

(c) \( x(n) = \left( \frac{1}{2} \right)^n u(n + 4) \)

(d) \( x(n) = (\alpha^n \sin \alpha n) u(n) \quad |\alpha| < 1 \)

(e) \( x(n) = \alpha^n \sin \alpha n \quad |\alpha| < 1 \)

(f) \( x(n) = \begin{cases} 2 - \left( \frac{1}{2} \right)n, & |n| \leq 4 \\ 0, & \text{elsewhere} \end{cases} \)

(g) \( x(n) = \begin{cases} A(2M + 1 - |n|), & |n| \leq M \\ 0, & |n| > M \end{cases} \)

Sketch the magnitude and phase spectra for parts (a), (f), and (g).

4.10 Determine the signals having the following Fourier transforms.

(a) \( X(\omega) = \begin{cases} 0, & 0 \leq |\omega| \leq \omega_0 \\ 1, & \omega_0 < |\omega| \leq \pi \end{cases} \)

(b) \( X(\omega) = \cos^2 \omega \)

(c) \( X(\omega) = \begin{cases} 1, & \omega_0 - \delta \omega/2 \leq |\omega| \leq \omega_0 + \delta \omega/2 \\ 0, & \text{elsewhere} \end{cases} \)

(d) The signal shown in Fig. P4.10.

4.11 Consider the signal

\( x(n) = \{ 1, 0, -1, 2, 3 \} \)

\( \uparrow \)

Figure P4.10

\( X(\omega) = X_f(\omega) + j X_i(\omega) \). Determine and sketch the signal \( y(n) \) with Fourier transform

\( Y(\omega) = X_f(\omega) + X_i(\omega) e^{j\omega} \)

4.12 Determine the signal \( x(n) \) if its Fourier transform is as given in Fig. P4.12.
4.13 In Example 4.3.3, the Fourier transform of the signal

\[ x(n) = \begin{cases} 
1, & -M \leq n \leq M \\
0, & \text{otherwise} 
\end{cases} \]

was shown to be

\[ X(\omega) = 1 + 2 \sum_{n=1}^{M} \cos \omega n \]

Show that the Fourier transform of

\[ x_1(n) = \begin{cases} 
1, & 0 \leq n \leq M \\
0, & \text{otherwise} 
\end{cases} \]

and

\[ x_2(n) = \begin{cases} 
1, & -M \leq n \leq -1 \\
0, & \text{otherwise} 
\end{cases} \]

are, respectively,

\[ X_1(\omega) = \frac{1 - e^{-j\omega(M+1)}}{1 - e^{-j\omega}} \]

\[ X_2(\omega) = \frac{e^{j\omega} - e^{j\omega(M+1)}}{1 - e^{j\omega}} \]

Thus prove that

\[ X(\omega) = X_1(\omega) + X_2(\omega) = \frac{\sin(M + \frac{1}{2})\omega}{\sin(\omega/2)} \]

and therefore,

\[ 1 + 2 \sum_{n=1}^{M} \cos \omega n = \frac{\sin(M + \frac{1}{2})\omega}{\sin(\omega/2)} \]

4.14 Consider the signal

\[ x(n) = \{-1, 2, -3, 2, -1\} \]

with Fourier transform \( X(\omega) \). Compute the following quantities, without explicitly computing \( X(\omega) \):

(a) \( X(0) \)  
(b) \( \int X(\omega) \, d\omega \)  
(c) \( \int_{-\pi}^{\pi} X(\omega) \, d\omega \)  
(d) \( X(\pi) \)  
(e) \( \int_{-\pi}^{\pi} |X(\omega)|^2 \, d\omega \)

4.15 The center of gravity of a signal \( x(n) \) is defined as

\[ c = \frac{\sum_{n=-\infty}^{\infty} nx(n)}{\sum_{n=-\infty}^{\infty} x(n)} \]

and provides a measure of the “time delay” of the signal.

4.16 Consider the Fourier transform pair

\[ a^n u(n) \leftrightarrow \frac{1}{1 - ae^{-j\omega}} \quad |a| < 1 \]

Use the differentiation in frequency theorem and induction to show that

\[ x(n) = \frac{n!}{n!(n-1)!} a^n u(n) \leftrightarrow X(\omega) = \frac{1}{(1 - ae^{-j\omega})^2} \]

4.17 Let \( x(n) \) be an arbitrary signal, not necessarily real-valued, with Fourier transform \( X(\omega) \). Express the Fourier transforms of the following signals in terms of \( X(\omega) \).

(a) \( x^*(n) \)  
(b) \( x(-n) \)  
(c) \( y(n) = x(n) - x(n-1) \)  
(d) \( y(n) = \sum_{k=-\infty}^{n} x(k) \)  
(e) \( y(n) = x(2n) \)  
(f) \( y(n) = \begin{cases} 
x(n/2), & n \text{ even} \\
0, & n \text{ odd} 
\end{cases} \)

4.18 Determine and sketch the Fourier transforms \( X_1(\omega), X_2(\omega), \) and \( X_3(\omega) \) of the following signals.

(a) \( x_1(n) = [1, 1, 1, 1, 1] \)  
(b) \( x_2(n) = [1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1] \)  
(c) \( x_3(n) = [1, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1] \)  
(d) Is there any relation between \( X_1(\omega), X_2(\omega), \) and \( X_3(\omega) \)? What is its physical meaning?

(e) Show that if

\[ x_k(n) = \begin{cases} 
x \left( \frac{n}{k} \right), & \text{if } n/k \text{ integer} \\
0, & \text{otherwise} 
\end{cases} \]

then

\[ X_k(\omega) = X(k\omega) \]

4.19 Let \( x(n) \) be a signal with Fourier transform as shown in Fig. P4.19. Determine and sketch the Fourier transforms of the following signals.
4.20 Consider an aperiodic signal \( x(n) \) with Fourier transform \( X(\omega) \). Show that the Fourier series coefficients \( C_k \) of the periodic signal

\[
y(n) = \sum_{k=-\infty}^{\infty} x(n - kN)
\]

are given by

\[
C_k = \frac{1}{N} \int_{-\pi}^{\pi} X(\omega) e^{-j\omega k} d\omega
\]

4.21 Prove that

\[
X_N(\omega) = \sum_{k=-\infty}^{\infty} \sin(\omega N + \theta)/\pi N
\]

may be expressed as

\[
X_N(\omega) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \sin[(2N + 1)(\omega - \theta)/2] d\theta
\]

4.22 A signal \( x(n) \) has the following Fourier transform:

\[
X(\omega) = \frac{1}{1 - e^{-j\omega}}
\]

Determine the Fourier transforms of the following signals:
(a) \( x(2n + 1) \)
(b) \( x(-2n) \)
(c) \( x(n) * x(n - 1) \)
(d) \( x(n) * x(-n) \)

4.23 From a discrete-time signal \( x(n) \) with Fourier transform \( X(\omega) \), shown in Fig. 4.23, determine and sketch the Fourier transform of the following signals:
(a) \( y_1(n) = x(n) \), \( n \) even
(b) \( y_2(n) = x(2n) \)
(c) \( y_3(n) = x(n/2) \), \( n \) even
(d) \( y_4(n) = x(n/2) \), \( n \) odd

Note that \( y_i(n) = x(n) s(n) \), where \( s(n) = \{ \ldots 0, 1, 0, 1, 0, 1, 0, 1, \ldots \} \)

4.24 The following input-output pairs have been observed during the operation of various systems:
(a) \( x(n) = (1/2)^n u(n) \rightarrow y(n) = (1/2)^n u(n) \)
(b) \( x(n) = (1/2)^n u(n) \rightarrow y(n) = (1/2)^n u(n) \)
(c) \( x(n) = e^{j\pi n} \rightarrow y(n) = 2e^{j\pi n} \)
(d) \( x(n) = e^{j\pi n} u(n) \rightarrow y(n) = 3e^{j\pi n} u(n) \)
(e) \( x(n) = x(n + N_1) \rightarrow y(n) = y(n + N_1) \), \( N_1 \neq N_2 \), \( N_1, N_2 \) prime

Determine their frequency response if each of the above systems is LTI.

4.25 (a) Determine and sketch the Fourier transform \( W_2(\omega) \) of the rectangular sequence

\[
w_2(n) = \begin{cases} 1, & 0 \leq n \leq M \\ 0, & \text{otherwise} \end{cases}
\]

(b) Consider the triangular sequence

\[
w_T(n) = \begin{cases} n, & 0 \leq n \leq M/2 \\ M - n, & M/2 < n \leq M \\ 0, & \text{otherwise} \end{cases}
\]

Determine and sketch the Fourier transform \( W_T(\omega) \) of \( w_T(n) \) by expressing it as the convolution of a rectangular sequence with itself.

(c) Consider the sequence

\[
w_f(n) = \frac{1}{2} (1 + \cos \frac{2\pi n}{N}) u(n)
\]

Determine and sketch \( W_f(\omega) \) by using \( W_2(\omega) \).

4.26 Consider an LTI system with impulse response \( h(n) = (1/2)^n u(n) \).
(a) Determine and sketch the magnitude and phase response \( |H(\omega)| \) and \( \angle H(\omega) \), respectively.
(b) Determine and sketch the magnitude and phase spectra for the input and output signals for the following inputs:
(1) \( x(n) = \cos \frac{2\pi n}{10}, \quad -\infty < n < \infty \)
(2) \( x(n) = \{ \ldots, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, \ldots \} \)

4.27 Determine and sketch the magnitude and phase response of the following systems:
(a) \( y(n) = \frac{1}{2}[x(n) + x(n - 1)] \)
(b) \( y(n) = \frac{1}{2}[x(n) - x(n - 1)] \)
(c) \( y(n) = \frac{1}{2}[x(n + 1) - x(n - 1)] \)
(d) \( y(n) = \frac{1}{2} [x(n+1) + x(n-1)] \)
(e) \( y(n) = \frac{1}{2} [x(n) + x(n-2)] \)
(f) \( y(n) = \frac{1}{2} [x(n) - x(n-2)] \)
(g) \( y(n) = \frac{1}{2} [x(n) + x(n-1) + x(n-2)] \)
(h) \( y(n) = x(n) - x(n-1) \)
(i) \( y(n) = 2x(n-1) - x(n-2) \)
(j) \( y(n) = \frac{1}{2} [x(n) + x(n-1) + x(n-2) + x(n-3)] \)
(k) \( y(n) = \frac{1}{2} [x(n) + 3x(n-1) + 3x(n-2) + x(n-3)] \)
(l) \( y(n) = x(n) \)
(m) \( y(n) = x(n+1) \)
(n) \( y(n) = \frac{1}{2} [x(n) - 2x(n-1) + x(n-2)] \)

4.28 An FIR filter is described by the difference equation

\[ y(n) = x(n) + x(n-10) \]

(a) Compute and sketch its magnitude and phase response.
(b) Determine its response to the inputs

1. \[ x(n) = \cos \left( \frac{\pi n}{10} \right) + 3 \sin \left( \frac{\pi n}{3} + \frac{\pi}{10} \right) \]
   \(-\infty < n < \infty\)

2. \[ x(n) = 10 + 5 \cos \left( \frac{2\pi n}{5} + \frac{\pi}{2} \right) \]
   \(-\infty < n < \infty\)

4.29 Determine the transient and steady-state responses of the FIR filter shown in Fig. P.4.29 to the input signal \( x(n) = 10 e^{-\pi n/2} u(n) \). Let \( b = 2 \) and \( y(-1) = y(-2) = y(-3) = y(-4) = 0 \).

![Figure P.4.29](image)

4.30 Consider the FIR filter \( y(n) = x(n) + x(n-4) \)

(a) Compute and sketch its magnitude and phase response.
(b) Compute its response to the input

\[ x(n) = \cos \left( \frac{\pi n}{2} \right) + \cos \left( \frac{\pi n}{4} \right) \]

\(-\infty < n < \infty\)

(c) Explain the results obtained in part (b) in terms of the magnitude and phase responses obtained in part (a).

4.31 Determine the steady-state and transient responses of the system \( y(n) = \frac{1}{2} [x(n) - x(n-2)] \)

4.32 From our discussions it is apparent that an LTI system cannot produce frequencies at its output that are different from those applied in its input. Thus, if a system creates "new" frequencies, it must be nonlinear and/or time varying. Determine the frequency content of the outputs of the following systems to the input signal

\[ x(n) = A \cos \left( \frac{\pi n}{4} \right) \]

(a) \( y(n) = x(2n) \)
(b) \( y(n) = x^2(n) \)
(c) \( y(n) = (\cos \pi n) x(n) \)

4.33 Determine and sketch the magnitude and phase response of the systems shown in Fig. P4.33(a) through (c).

![Figure P.4.33](image)

4.34 Determine the magnitude and phase response of the multipath channel \( y(n) = x(n) + x(n-M) \)

At what frequencies does \( H(\omega) = 0 \)?

4.35 Consider the filter\( y(n) = 0.9 y(n-1) + bx(n) \)

(a) Determine \( b \) so that \(|H(0)| = 1\).
(b) Determine the frequency at which \(|H(\omega)| = 1/\sqrt{2}\).
(c) Is this filter lowpass, bandpass, or highpass?
(d) Repeat parts (b) and (c) for the filter $y(n) = -0.9y(n-1) + 0.1x(n)$.

4.36* Harmonic distortion in digital sinusoidal generators

An ideal sinusoidal generator produces the signal

$$x(n) = \cos 2\pi fn$$

which is periodic with fundamental period $N$ if $f_0 = k_0/N$ and $k_0, N$ are relatively prime numbers. The spectrum of such a “pure” sinusoid consists of two lines at $k = k_0$ and $k = N - k_0$ (we limit ourselves in the fundamental interval $0 \leq k \leq N - 1$). In practice, the approximations made in computing the samples of a sinusoid of relative frequency $f_0$ result in a certain amount of power falling into other frequencies. This spurious power results in distortion, which is referred to as harmonic distortion. Harmonic distortion is usually measured in terms of the total harmonic distortion (THD), which is defined as the ratio

$$\text{THD} = \frac{\text{spurious harmonic power}}{\text{total power}}$$

(a) Show that

$$\text{THD} = 1 - 2 \frac{|c_{k_0}|^2}{P_x}$$

where

$$c_{k_0} = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}$$

$$P_x = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2$$

(b) By using the Taylor approximation

$$\cos \phi = 1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} - \frac{\phi^6}{6!} + \cdots$$

compute one period of $x(n)$ for $f_0 = 1/96, 1/32, 1/256$ by increasing the number of terms in the Taylor expansion from 2 to 8.

(c) Compute the THD and plot the power density spectrum for each sinusoid in part (b) as well as for the sinusoids obtained using the computer cosine function. Comment on the results.

4.37* Measurement of the total harmonic distortion in quantized sinusoids

Let $x(n)$ be a periodic sinusoidal signal with frequency $f_0 = k_0/N$, that is,

$$x(n) = \sin 2\pi f_0 n$$

(a) Write a computer program that quantizes the signal $x(n)$ into $b$ bits or equivalently into $L = 2^b$ levels by using rounding. The resulting signal is denoted by $x_q(n)$.

(b) For $f_0 = 1/50$ compute the THD of the quantized signals $x_q(n)$ obtained by using $b = 4, 6, 8, 16$ bits.

(c) Repeat part (b) for $f_0 = 1/100$.

(d) Comment on the results obtained in parts (b) and (c).

4.38* Consider the discrete-time system

$$y(n) = ay(n-1) + (1 - a)x(n)$$

where $a = 0.9$ and $y(-1) = 0$.

(a) Compute and sketch the output $y(n)$ of the system to the input signals

$$x(n) = \sin 2\pi fn$$

where $f_1 = \frac{1}{4}, f_2 = \frac{1}{6}, f_3 = \frac{1}{8}, f_4 = \frac{1}{5}$.

(b) Compute and sketch the magnitude and phase response of the system and use these results to explain the response of the system to the signals given in part (a).

4.39* Consider an LTI system with impulse response $h(n) = \left(\frac{1}{2}\right)^{|n|}$

(a) Determine and sketch the magnitude and phase response $H(\omega)$ and $\angle H(\omega)$, respectively.

(b) Determine and sketch the magnitude and phase spectra for the input and output signals for the following inputs:

1. $x(n) = \cos \frac{3\pi n}{8}$, $-\infty < n < \infty$
2. $x(n) = [\ldots, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1, \ldots]$ \uparrow

4.40* Time-domain sampling

Consider the continuous-time signal

$$x_c(t) = \begin{cases} e^{-j2\pi fn}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

(a) Compute analytically the spectrum $X_c(F)$ of $x_c(t)$.

(b) Compute analytically the spectrum of the signal $x(n) = x_c(nT)$, $T = 1/F_c$.

(c) Plot the magnitude spectrum $|X_c(F)|$ for $F_c = 10$ Hz.

(d) Plot the magnitude spectrum $|X(F)|$ for $F_c = 10, 20, 40, \text{and} 100$ Hz.

(e) Explain the results obtained in part (d) in terms of the aliasing effect.

4.41 Consider the digital filter shown in Fig. P4.41.

(a) Determine the input–output relation and the impulse response $h(n)$.

(b) Determine and sketch the magnitude $|H(\omega)|$ and the phase response $\angle H(\omega)$ of the filter and find which frequencies are completely blocked by the filter.

(c) When $\omega_0 = \pi/2$, determine the output $y(n)$ to the input

$$x(n) = 3 \cos \left(\frac{\pi}{3} n + 30^\circ\right)$$

$$-\infty < n < \infty$$

4.42 Consider the FIR filter

$$y(n) = x(n) - x(n-4)$$

(a) Compute and sketch its magnitude and phase response.

(b) Compute its response to the input

$$x(n) = \cos \frac{\pi}{2} n + \cos \frac{\pi}{4} n$$

$$-\infty < n < \infty$$
(c) Explain the results obtained in part (b) in terms of the answer given in part (a).

4.43 Determine the steady-state response of the system

\[ y(n) = \frac{1}{2}[x(n) - x(n - 2)] \]

to the input signal

\[ x(n) = 5 + 3 \cos \left( \frac{\pi}{2} n + 60^\circ \right) + 4 \sin(\pi n + 45^\circ) \quad -\infty < n < \infty \]

4.44 Recall from Problem 4.32 that an LTI system cannot produce frequencies at its output that are different from those applied in its input. Thus if a system creates "new" frequencies, it must be nonlinear and/or time varying. Indicate whether the following systems are nonlinear and/or time varying and determine the output spectra when the input spectrum is

\[ X(\omega) = \begin{cases} 
1, & |\omega| \leq \pi/4 \\
0, & \pi/4 < |\omega| \leq \pi
\end{cases} \]

(a) \( y(n) = x(2n) \)
(b) \( y(n) = x^2(n) \)
(c) \( y(n) = \cos(\pi n) x(n) \)

4.45 Consider an LTI system with impulse response

\[ h(n) = \left( \frac{1}{4} \right)^n \cos \left( \frac{\pi}{4} n \right) u(n) \]

(a) Determine its system function \( H(z) \).
(b) Is it possible to implement this system using a finite number of adders, multipliers, and unit delays? If yes, how?
(c) Provide a rough sketch of \( |H(\omega)| \) using the pole-zero plot.
(d) Determine the response of the system to the input

\( x(n) = \frac{1}{2} y u(n) \)

4.46 An FIR filter is described by the difference equation

\[ y(n) = x(n) - x(n - 10) \]

(a) Compute and sketch its magnitude and phase response.
(b) Determine its response to the inputs

1. \( x(n) = \cos \frac{\pi}{10} n + 3 \sin \left( \frac{\pi}{3} n + \frac{\pi}{10} \right) \quad -\infty < n < \infty \)
2. \( x(n) = 5 + 6 \cos \left( 2\pi \frac{3}{5} n + \frac{\pi}{2} \right) \quad -\infty < n < \infty \)

4.47 The frequency response of an ideal bandpass filter is given by

\[ H(\omega) = \begin{cases} 
0, & |\omega| \leq \pi/8 \\
1, & \pi/8 < |\omega| < 3\pi/8 \\
0, & 3\pi/8 \leq |\omega| \leq \pi
\end{cases} \]

4.48 Consider the system described by the difference equation

\[ y(n) = \frac{1}{2} y(n - 1) + x(n) + \frac{1}{2} x(n - 1) \]

(a) Determine its impulse response.
(b) Determine its frequency response:
   (1) From the impulse response
   (2) From the difference equation
(c) Determine its response to the input

\[ x(n) = \cos \left( \frac{\pi}{2} n + \frac{\pi}{4} \right) \quad -\infty < n < \infty \]

4.49 Sketch roughly the magnitude \( |X(\omega)| \) of the Fourier transforms corresponding to the pole-zero patterns given in Fig. P4.49.

![Figure P4.49](image)

4.50 Design an FIR filter that completely blocks the frequency \( \omega_0 = \pi/4 \) and then compute its output if the input is

\[ x(n) = \left( \sin \frac{\pi}{4} n \right) u(n) \]

for \( n = 0, 1, 2, 3, 4 \). Does the filter fulfill your expectations? Explain.
4.51 A digital filter is characterized by the following properties:
   (1) It is highpass and has one pole and one zero.
   (2) The pole is at a distance $r = 0.9$ from the origin of the $z$-plane.
   (3) Constant signals do not pass through the system.
(a) Plot the pole-zero pattern of the filter and determine its system function $H(z)$.
(b) Compute the magnitude response $|H(\omega)|$ and the phase response $\angle H(\omega)$ of the filter.
(c) Normalize the frequency response $H(\omega)$ so that $|H(\pi)| = 1$.
(d) Determine the input-output relation (difference equation) of the filter in the time domain.
(e) Compute the output of the system if the input is
   $$ x(n) = 2\cos\left(\frac{\pi}{6} n + 45^\circ\right) \quad -\infty < n < \infty $$
   (You can use either algebraic or geometrical arguments.)

4.52 A causal first-order digital filter is described by the system function
   $$ H(z) = b_0 \frac{1 + bz^{-1}}{1 + az^{-1}} $$
   (a) Sketch the direct form I and direct form II realizations of this filter and find the corresponding difference equations.
(b) For $a = 0.5$ and $b = -0.6$, sketch the pole-zero pattern. Is the system stable? Why?
(c) For $a = -0.5$ and $b = 0.5$, determine $b_0$ so that the maximum value of $|H(\omega)|$ is equal to 1.
(d) Sketch the magnitude response $|H(\omega)|$ and the phase response $\angle H(\omega)$ of the filter obtained in part (c).
(e) In a specific application it is known that $a = 0.8$. Does the resulting filter amplify high frequencies or low frequencies in the input? Choose the value of $b$ so as to improve the characteristics of this filter (i.e., make it a better lowpass or a better highpass filter).

4.53 Derive the expression for the resonant frequency of a two-pole filter with poles at $p_1 = re^{j\theta}$ and $p_2 = r^{-1}e^{-j\theta}$, given by (4.5.25).

4.54 Determine and sketch the magnitude and phase responses of the Hanning filter characterized by the (moving average) difference equation
   $$ y(n) = \frac{1}{2}y(n) + \frac{1}{2}y(n - 1) + \frac{1}{2}x(n - 2) $$

4.55 A causal LTI system excited by the input
   $$ x(n) = \frac{1}{2}y(n) + u(-n - 1) $$
produces an output $y(n)$ with $z$-transform
   $$ Y(z) = \frac{-\frac{1}{2}z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 + z^{-1})} $$
(a) Determine the system function $H(z)$ and its ROC.
(b) Determine the output $y(n)$ of the system.
   (Hint: Pole cancellation increases the original ROC.)

4.56 Determine the coefficients of a linear-phase FIR filter
   $$ y(n) = b_0 x(n) + b_1 x(n - 1) + b_2 x(n - 2) $$
   such that:
   (a) It rejects completely a frequency component at $\omega_0 = 2\pi/3$.
   (b) Its frequency response is normalized so that $H(0) = 1$.
   (c) Compute and sketch the magnitude and phase response of the filter to check if it satisfies the requirements.

4.57 Determine the frequency response $H(\omega)$ of the following moving average filters:
   (a) $y(n) = \frac{1}{2M + 1} \sum_{k=-M}^{M} x(n - k)$
   (b) $y(n) = \frac{1}{4M} x(n + M) + \frac{1}{2M} \sum_{k=-M+1}^{M-1} x(n - k) + \frac{1}{4M} x(n - M)$
Which filter provides better smoothing? Why?

4.58 The convolution $x(t)$ of two continuous-time signals $x_1(t)$ and $x_2(t)$, from which at least one is nonperiodic, is defined by
   $$ x(t) \triangleq x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\lambda)x_2(t - \lambda)d\lambda $$
   (a) Show that $X(F) = X_1(F)X_2(F)$, where $X_1(F)$ and $X_2(F)$ are the spectra of $x_1(t)$ and $x_2(t)$, respectively.
   (b) Compute $x(t)$ if $x_1(t) = x_2(t) = \begin{cases} 1, & |t| < \tau/2 \\ 0, & \text{elsewhere} \end{cases}$
   (c) Determine the spectrum of $x(t)$ using the results in part (a).

4.59 Compute the magnitude and phase response of a filter with system function
   $$ H(z) = 1 + z^{-1} + z^{-2} + \cdots + z^{-8} $$
   If the sampling frequency is $F_s = 1$ kHz, determine the frequencies of the analog sinusoids that cannot pass through the filter.

4.60 A second-order system has a double pole at $p_1 = 0.5$ and two zeros at $z_1, z_2 = e^{\pm j\pi/4}$.
   Using geometric arguments, choose the gain $G$ of the filter so that $|H(0)| = 1$.

4.61 In this problem we consider the effect of a single zero on the frequency response of a system. Let $z = re^{j\theta}$ be a zero inside the unit circle ($r < 1$). Then
   $$ H_z(\omega) = 1 - re^{j\omega}e^{-j\theta} $$
   $$ = 1 - r\cos(\omega - \theta) + jr\sin(\omega - \theta) $$
(a) Show that the magnitude response is
   $$ |H_z(\omega)| = \sqrt{1 - 2r\cos(\omega - \theta) + r^2} $$
   or, equivalently,
   $$ 20\log_{10}|H_z(\omega)| = 10\log_{10}[1 - 2r\cos(\omega - \theta) + r^2] $$
(b) Show that the phase response is given as
\[ \Theta_2(\omega) = \tan^{-1} \frac{r \sin(\omega - \theta)}{1 - r \cos(\omega - \theta)} \]

(c) Show that the group delay is given as
\[ \tau^g_2(\omega) = \frac{r^2 - r \cos(\omega - \theta)}{1 + r^2 - 2r \cos(\omega - \theta)} \]

(d) Plot the magnitude $|H(\omega)|$, the phase $\Theta(\omega)$ and the group delay $\tau_2(\omega)$ for $r = 0.7$ and $\theta = 0, \pi/2, \text{ and } \pi$.  

4.62 In this problem we consider the effect of a single pole on the frequency response of a system. Hence, we let
\[ H_2(\omega) = \frac{1}{1 - re^{j\omega}} \quad r < 1 \]

Show that
\[ |H(\omega)|_{db} = 10\log|1 + r^2 - 2r \cos(\omega - \theta)| + 10\log|1 + r^2 - 2r \cos(\omega + \theta)| \]

4.63 In this problem we consider the effect of complex-conjugate pair of poles and zeros on the frequency response of a system. Let
\[ H_2(\omega) = 1 - re^{j\omega}(1 - re^{-j\omega}) \]

(a) Show that the magnitude response in decibels is
\[ |H(\omega)|_{db} = 10\log|1 + r^2 - 2r \cos(\omega - \theta)| + 10\log|1 + r^2 - 2r \cos(\omega + \theta)| \]

(b) Show that the phase response is given as
\[ \Theta_2(\omega) = \tan^{-1} \frac{r \sin(\omega - \theta)}{1 - r \cos(\omega - \theta)} + \tan^{-1} \frac{r \sin(\omega + \theta)}{1 - r \cos(\omega + \theta)} \]

(c) Show that the group delay is given as
\[ \tau^g_2(\omega) = \frac{r^2 - r \cos(\omega - \theta)}{1 + r^2 - 2r \cos(\omega - \theta)} + \frac{r^2 - r \cos(\omega + \theta)}{1 + r^2 - 2r \cos(\omega + \theta)} \]

(d) If $H_2(\omega) = 1/H_3(\omega)$, show that
\[ |H(\omega)|_{db} = -|H(\omega)|_{db} \]
\[ \Theta_2(\omega) = -\Theta_3(\omega) \]
\[ \tau^g_2(\omega) = -\tau^g_3(\omega) \]

(e) Plot $|H_2(\omega)|$, $\Theta_2(\omega)$ and $\tau^g_2(\omega)$ for $r = 0.9$, and $\theta = 0, \pi/2$.  

Chap. 4 Problems

4.64 Determine the 3-dB bandwidth of the filters ($0 < a < 1$)
\[ H_1(z) = \frac{1-a}{1-az^{-1}} \]
\[ H_2(z) = \frac{1-a}{2(1-az^{-1})} \]

Which is a better lowpass filter?

4.65 Design a digital oscillator with adjustable phase, that is, a digital filter which produces the signal
\[ y(n) = \cos(\omega_0 n + \theta)u(n) \]

4.66 This problem provides another derivation of the structure for the coupled-form oscillator by considering the system
\[ y(n) = ay(n-1) + x(n) \]

for $a = e^{i\omega_0}$.

Let $x(n)$ be real. Then $y(n)$ is complex. Thus
\[ y(n) = y_e(n) + jy_i(n) \]

(a) Determine the equations describing a system with one input $x(n)$ and the two outputs $y_e(n)$ and $y_i(n)$.

(b) Determine a block diagram realization

(c) Show that if $x(n) = \delta(n)$, then
\[ y_e(n) = \cos(\omega_0 n)u(n) \]
\[ y_i(n) = \sin(\omega_0 n)u(n) \]

(d) Compute $y_e(n)$, $y_i(n)$, $n = 0, 1, \ldots, 9$ for $\omega_0 = \pi/6$. Compare these with the true values of the sine and cosine.

4.67 Consider a filter with system function
\[ H(z) = b_0 \frac{(1-e^{j\omega_0}z^{-1})(1-e^{-j\omega_0}z^{-1})}{(1-re^{j\omega_0}z^{-1})(1-re^{-j\omega_0}z^{-1})} \]

(a) Sketch the pole-zero pattern.

(b) Using geometric arguments, show that for $r \approx 1$, the system is a notch filter and provide a rough sketch of its magnitude response if $\omega_0 = 60^\circ$.

(c) For $\omega_0 = 60^\circ$, choose $b_0$ so that the maximum value of $|H(\omega)|$ is 1.

(d) Draw a direct form II realization of the system.

(e) Determine the approximate 3-dB bandwidth of the system.

4.68 Design an FIR digital filter that will reject a very strong 60-Hz sinusoidal interference contaminating a 200-Hz useful sinusoidal signal. Determine the gain of the filter so that the useful signal does not change amplitude. The filter works at a sampling frequency $F_s = 500$ samples/s. Compute the output of the filter if the input is a 60-Hz sinusoid or a 200-Hz sinusoid with unit amplitude. How does the performance of the filter compare with your requirements?

4.69 Determine the gain $b_0$ for the digital resonator described by (4.5.28) so that $|H(\omega_0)| = 1$. 
4.70 Demonstrate that the difference equation given in (4.5.52) can be obtained by applying the trigonometric identity
\[ \cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2} \]
where \( \alpha = (n+1)\omega_0 \) and \( \beta = (n-1)\omega_0 \). Thus show that the sinusoidal signal \( y(n) = A \cos n\omega_0 \) can be generated from (4.5.52) by using the initial conditions \( y(-1) = A \cos \omega_0 \) and \( y(-2) = A \cos 2\omega_0 \).

4.71 Use the trigonometric identity in (4.5.53) with \( \alpha = n\omega_0 \) and \( \beta = (n-2)\omega_0 \) to derive the difference equation for generating the sinusoidal signal \( y(n) = A \sin n\omega_0 \). Determine the corresponding initial conditions.

4.72 Using the z-transform pairs 8 and 9 in Table 3.3, determine the difference equations for the digital oscillators that have impulse responses \( h(n) = A \cos n\omega_0 u(n) \) and \( h(n) = A \sin n\omega_0 u(n) \), respectively.

4.73 Determine the structure for the coupled-form oscillator by combining the structure for the digital oscillators obtained in Problem 4.72.

4.74 Convert the highpass filter with system function
\[ H(z) = \frac{1 - z^{-1}}{1 - \alpha z^{-1}} \quad \alpha < 1 \]
into a notch filter that rejects the frequency \( \omega_0 = \pi/4 \) and its harmonics.

(a) Determine the difference equation.
(b) Sketch the pole-zero pattern.
(c) Sketch the magnitude response for both filters.

4.75 Choose \( L \) and \( M \) for a lunar filter that must have narrow passbands at \( k \Delta F \) cycles/day, where \( k = 1, 2, 3, \ldots \) and \( \Delta F = 0.067726 \).

4.76 (a) Show that the systems corresponding to the pole-zero patterns of Fig. 4.58 are all-pass.
(b) What is the number of delays and multipliers required for the efficient implementation of a second-order all-pass system?

4.77 A digital notch filter is required to remove an undesirable 60-Hz hum associated with a power supply in an ECG recording application. The sampling frequency used is \( F_s = 500 \) samples/s. (a) Design a second-order FIR notch filter and (b) a second-order pole-zero notch filter for this purpose. In both cases choose the gain \( h_0 \) so that \( |H(\omega)| = 1 \) for \( \omega = 0 \).

4.78 Determine the coefficients \( h(n) \) of a highpass linear phase FIR filter of length \( M = 4 \) which has an antisymmetric unit sample response \( h(n) = -h(M - 1 - n) \) and a frequency response that satisfies the condition
\[ \left| H \left( \frac{\pi}{4} \right) \right| = \frac{1}{2}, \quad \left| H \left( \frac{3\pi}{4} \right) \right| = 1 \]

4.79 In an attempt to design a four-pole bandpass digital filter with desired magnitude response
\[ |H_4(\omega)| = \begin{cases} 1, & \frac{\pi}{2} \leq \omega \leq \frac{\pi}{2} \\ 0, & \text{elsewhere} \end{cases} \]

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we select the four poles at
\[ p_{1,2} = 0.8 \exp \left( \pm j \frac{\pi}{4} \right) \]
\[ p_{3,4} = 0.8 \exp \left( \pm j \frac{3\pi}{4} \right) \]
and four zeros at
\[ z_1 = 1, \quad z_2 = -1, \quad z_{3,4} = \exp \left( \pm j \frac{\pi}{4} \right) \]

(a) Determine the value of the gain so that
\[ \left| H \left( \frac{5\pi}{12} \right) \right| = 1 \]
(b) Determine the system function \( H(z) \).
(c) Determine the magnitude of the frequency response \( H(\omega) \) for \( 0 \leq \omega \leq \pi \) and compare it with the desired response \( |H(\omega)| \).

4.80 A discrete-time system with input \( x(n) \) and output \( y(n) \) is described in the frequency domain by the relation
\[ Y(\omega) = \exp^{-j2\pi\omega} X(\omega) + \frac{dX(\omega)}{d\omega} \]

(a) Compute the response of the system to the input \( x(n) = \delta(n) \).
(b) Check if the system is LTI and stable.

4.81 Consider an ideal lowpass filter with impulse response \( h(n) \) and frequency response
\[ H(\omega) = \begin{cases} 1, & |\omega| \leq \omega_0 \\ 0, & \omega_0 < |\omega| < \pi \end{cases} \]

What is the frequency response of the filter defined by
\[ g(n) = \begin{cases} h \left( \frac{n}{2} \right), & n \text{ even} \\ 0, & n \text{ odd} \end{cases} \]

4.82 Consider the system shown in Fig. P4.82. Determine its impulse response and its frequency response if the system \( H(\omega) \) is:
(a) Lowpass with cutoff frequency \( \omega_0 \).
(b) Highpass with cutoff frequency \( \omega_0 \).

![Figure P4.82](image)

4.83 Frequency inverters have been used for many years for speech scrambling. Indeed, a voice signal \( x(n) \) becomes unintelligible if we invert its spectrum as shown in Fig. P4.83.

(a) Derive how frequency inversion can be performed in the time domain.
(b) Design an unscrambler. (Hint: The required operations are very simple and can easily be done in real time.)
4.84 A lowpass filter is described by the difference equation
\[ y(n) = 0.9y(n - 1) + 0.1x(n) \]
(a) By performing a frequency translation of \( \pi/2 \), transform the filter into a bandpass filter.
(b) What is the impulse response of the bandpass filter?
(c) What is the major problem with the frequency translation method for transforming a prototype lowpass filter into a bandpass filter?

4.85 Consider a system with a real-valued impulse response \( h(n) \) and frequency response
\[ H(\omega) = |H(\omega)|e^{j\theta(\omega)} = \frac{1}{2} \]
The quantity
\[ D = \sum_{n=-\infty}^{\infty} n^2 |h(n)|^2 \]
provides a measure of the "effective duration" of \( h(n) \).
(a) Express \( D \) in terms of \( H(\omega) \).
(b) Show that \( D \) is minimized for \( \theta(\omega) = 0 \).

4.86 Consider the lowpass filter
\[ y(n) = ay(n - 1) + bx(n) \quad 0 < a < 1 \]
(a) Determine \( b \) so that \( |H(0)| = 1 \).
(b) Determine the 3-dB bandwidth \( \omega_3 \) for the normalized filter in part (a).
(c) How does the choice of the parameter \( a \) affect \( \omega_3 \)?
(d) Repeat parts (a) through (c) for the highpass filter obtained by choosing \(-1 < a < 0\).

4.87 Sketch the magnitude and phase response of the multipath channel
\[ y(n) = x(n) + ax(n - M) \quad a > 0 \]
for \( a << 1 \).

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4.88 Determine the system functions and the pole-zero locations for the systems shown in Fig. P4.88(a) through (c), and indicate whether or not the systems are stable.

Figure P4.88

(a)

(b)

(c)

4.89 Determine and sketch the impulse response and the magnitude and phase responses of the FIR filter shown in Fig. P4.89 for \( b = 1 \) and \( b = -1 \).

Figure P4.89

4.90 Consider the system
\[ y(n) = x(n) + 0.95x(n - 6) \]
(a) Sketch its pole-zero pattern.
(b) Sketch its magnitude response using the pole-zero plot.
(c) Determine the system function of its causal inverse system.
(d) Sketch the magnitude response of the inverse system using the pole-zero plot.

4.91 Determine the impulse response and the difference equation for all possible systems specified by the system functions.
4.98 Let \( x(n) \) be a real-valued minimum-phase sequence. Modify \( x(n) \) to obtain another real-valued minimum-phase sequence \( y(n) \) such that \( y(0) = x(0) \) and \( y(n) = |x(n)| \).

4.99 The frequency response of a stable LTI system is known to be real and even. Is the inverse system stable?

4.100 Let \( h(n) \) be a real filter with nonzero linear or nonlinear phase response. Show that the following operations are equivalent to filtering the signal \( x(n) \) with a zero-phase filter.

(a) \( g(n) = h(n) \ast x(n) \)

(b) \( g(n) = h(n) \ast x(-n) \)

(Hint: Determine the frequency response of the composite system \( y(n) = H[s(n)] \).)

4.101 Check the validity of the following statements:

(a) The convolution of two minimum-phase sequences is always minimum-phase sequence.

(b) The sum of two minimum-phase sequences is always minimum phase.

4.102 Determine the minimum-phase system whose squared magnitude response is given by:

(a) \( |H(\omega)|^2 = \frac{5}{4} - \frac{10}{3} \cos \omega \)

(b) \( |H(\omega)|^2 = \frac{2(1 - a^2)}{(1 + a^2) - 2a \cos \omega} \quad |a| < 1 \)

4.103 Consider an FIR system with the following system function:

\( H(z) = (1 - 0.8e^{j\pi/3} z^{-1})(1 - 0.8e^{-j\pi/3} z^{-1})(1 - 1.5e^{j\pi/4} z^{-1})(1 - 1.5e^{-j\pi/4} z^{-1}) \)

(a) Determine all systems that have the same magnitude response. Which is the minimum-phase system?

(b) Determine the impulse response of all systems in part (a).

(c) Plot the partial energy

\[ E(n) = \sum_{k=0}^{n} h^2(k) \]

for every system and use it to identify the minimum- and maximum-phase systems.

4.104 The causal system

\( H(z) = \frac{1}{1 + \sum_{k=0}^{N} \alpha_k z^{-k}} \)

is known to be unstable.

We modify this system by changing its impulse response \( h(n) \) to

\( h'(n) = \lambda^n h(n) u(n) \)
(a) Show that by properly choosing $\lambda$ we can obtain a new stable system.

(b) What is the difference equation describing the new system?

Given a signal $x(n)$, we can create echoes and reverberations by delaying and scaling the signal as follows

$$y(n) = \sum_{k=-\infty}^{\infty} g_k x(n - kD)$$

where $D$ is positive integer and $g_k > g_{k-1} > 0$.

(a) Explain why the comb filter

$$H(z) = \frac{1}{1 - az^{-D}}$$

can be used as a reverberator (i.e., as a device to produce artificial reverberations).

(Hint: Determine and sketch its impulse response.)

(b) The all-pass comb filter

$$H(z) = \frac{z^{-D} - a}{1 - az^{-D}}$$

is used in practice to build digital reverberators by cascading three to five such filters and properly choosing the parameters $a$ and $D$. Compute and plot the impulse response of two such reverberators each obtained by cascading three sections with the following parameters.

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(c) The difference between echo and reverberation is that with pure echo there are clear repetitions of the signal, but with reverberations, there are not. How is this reflected in the shape of the impulse response of the reverberator? Which unit in part (b) is a better reverberator?

(d) If the delays $D_1$, $D_2$, $D_3$ in a certain unit are prime numbers, the impulse response of the unit is more “dense.” Explain why.

(e) Plot the phase response of units 1 and 2 and comment on them.

(f) Plot $h(n)$ for $D_1$, $D_2$, and $D_3$ being nonprime. What do you notice?


4.106* By trial-and-error design a third-order lowpass filter with cutoff frequency at $\omega_c = \pi/9$ radians/sample interval. Start your search with

(a) $z_1 = z_2 = z_3 = 0$, $p_1 = r$, $p_2 = re^{\pi j/2}$, $r = 0.8$

(b) $r = 0.9$, $z_1 = z_2 = z_3 = -1$

4.107* A speech signal with bandwidth $B = 10$ kHz is sampled at $F_s = 20$ kHz. Suppose that the signal is corrupted by four sinusoids with frequencies

$$F_1 = 10,000 \text{ Hz}, \quad F_2 = 7778 \text{ Hz}$$
$$F_3 = 8889 \text{ Hz}, \quad F_4 = 6667 \text{ Hz}$$

(a) Design a FIR filter that eliminates these frequency components.

(b) Choose the gain of the filter so that $|H(0)| = 1$ and then plot the log magnitude response and the phase response of the filter.

(c) Does the filter fulfill your objectives? Do you recommend the use of this filter in a practical application?

4.108* Compute and sketch the frequency response of a digital resonator with $\omega = \pi/6$ and $r = 0.6, 0.9, 0.99$. In each case, compute the bandwidth and the resonance frequency from the graph, and check if they are in agreement with the theoretical results.

4.109* The system function of a communication channel is given by

$$H(z) = (1 - 0.9e^{j0.4\pi z^{-1}})(1 - 0.9e^{-j0.4\pi z^{-1}})(1 - 1.5e^{j0.6\pi z^{-1}})(1 - 1.5e^{-j0.6\pi z^{-1}})$$

Determine the system function $H_c(z)$ of a causal and stable compensating system so that the cascade interconnection of the two systems has a flat magnitude response. Sketch the pole-zero plots and the magnitude and phase responses of all systems involved into the analysis process. [Hint: Use the decomposition $H(z) = H_{in}(z)H_{out}(z)$.]