Sample Test for the Exam #2.

Part I: Dummy variables Suppose that you observe a large cross sectional data for individual earning \((y_i)\). You can also access to individual characterisitcs such as gender, education level etc. Let

\[
\begin{align*}
d_i &= \begin{cases} 
1 & \text{if male} \\
0 & \text{if female}
\end{cases} \quad \text{gender dummy} \\
s_i &= \begin{cases} 
1 & \text{if skilled} \\
0 & \text{if unskilled}
\end{cases} \quad \text{skill dummy} \\
f_i &= \begin{cases} 
1 & \text{if white} \\
0 & \text{if non-white}
\end{cases} \quad \text{race dummy}
\end{align*}
\]

Q1: You want to test the following hypothesis: Among skilled workers, there is no gender wage difference. However among unskilled workers, there is gender wage difference. Set up a regression to test this hypothesis. Explain how to test

Q2: Suppose that you run \(y_i = \alpha + \beta f_i + u_i\) and find that \(\hat{\beta}\) is significant. However when you run \(y_i = \alpha + \beta f_i + cs_i + e_i\) then \(\hat{\beta}\) becomes not significant. Interpret your result.

Q3. You run \(y_i = \alpha + \beta_1 d_i + \beta_2 s_i + \beta_3 f_i + \gamma_1 d_i s_i + \gamma_2 s_i f_i + \gamma_3 d_i f_i + \epsilon_i\)

Interprete the meaning of

(i) provide economic interpret for each coefficient.
(ii) \(\beta_1 = \gamma_1 = \gamma_2 = 0\) but the others are not zero.
(iii) \(\beta_1 = \beta_3 = \gamma_1 = \gamma_3 = 0\) but the others are not zero.

Part II Trend Regression Suppose that the true model is

\(y_t = b\sqrt{t} + e_t\)

but you run \(y_t = b_2 t + u_t\)

Q1: what is \(u_t\)?
Q2: Is \(\hat{b}_2\) consistent?

Part III. Solve 5.4 in the lecture note.
Part IV. Solve the old midterm Q4.

Part V. Serial correlation  Suppose that the true data generating process is given by

\begin{align*}
y_t &= \beta x_t + u_t, \quad u_t = \rho u_{t-1} + e_t, \quad e_t \sim iid(0,1). \\
x_t &= \phi x_{t-1} + \varepsilon_t, \quad E(e_s\varepsilon_t) = 0 \text{ for all } s \text{ and } t.
\end{align*}

Q1: Derive the limiting distribution of \( \beta \) when \( \rho = \phi = 0 \).
Q2: Derive the limiting distribution of \( \beta \) when \( \phi = 0 \) but \( 0 < \rho < 1 \).
Q3: Derive the limiting distribution of \( \beta \) when \( 0 < \rho < 1 \) and \( 0 < \phi < 1 \).