Assignment 10 (Treatment Effects)  Consider the following DGP

\[
y_i = a + b d_i + e_i, \quad d_i = 1 \{d_i^* = \delta x_i + u_i\}
\]

\[
\begin{bmatrix}
e_i \\
u_i
\end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}\right)
\]

Q1. Compare the finite sample performance among propensity score weighted and matching estimators. (15 points)

(Hint: There are many estimators. You have to compare them one by one in terms of bias, variance etc.)

Q2. Now assume \( x_i \) is not observable. You have a proxy variable \( w \) for \( x \). To be specific, let

\[ w_i = \phi x_i + \varepsilon_i \]

where \( \varepsilon_i \sim iidN(0, \sigma^2) \) and \( \phi = 1 \). Repeat Q1 with the proxy variable \( w_i \). Here we assume that \( x_i \) is not observable. Analyze the impact of \( \sigma^2 \) on the estimation of treatment effects. (10 points)