Chapter 3: National Income: Where it Comes From and Where it Goes

Outline of model
- A closed economy, market-clearing model
- Supply side
  - factor markets (supply, demand, price)
  - determination of output/income
- Demand side
  - determinants of C, I, and G
- Equilibrium
  - goods market
  - loanable funds market

Factors of production

\[ K = \text{capital: } \text{tools, machines, and structures used in production} \]
\[ L = \text{labour: the physical and mental efforts of workers} \]

The production function: \( Y = F(K, L) \)
- Shows how much output (\( Y \)) the economy can produce from \( K \) units of capital and \( L \) units of labor
- Reflects the economy’s level of technology
- Exhibits constant returns to scale

Returns to scale: A review

Initially \( Y_1 = F(K_1, L_1) \)
Scale all inputs by the same factor \( z \):
\[ K_2 = zK_1 \quad \text{and} \quad L_2 = zL_1 \]
(e.g., if \( z = 1.2 \), then all inputs are increased by 20%)

What happens to output, \( Y_2 = F(K_2, L_2) \)?
- If constant returns to scale, \( Y_2 = zY_1 \)
- If increasing returns to scale, \( Y_2 > zY_1 \)
- If decreasing returns to scale, \( Y_2 < zY_1 \)

Returns to scale: Example 1

\[
F(K, L) = \sqrt{KL}
\]
\[
F(zK, zL) = \sqrt{(zK)(zL)} = z\sqrt{KL}
\]
\[
F(zK, zL) = z\sqrt{KL}
\]
\[
F(zK, zL) = zF(K, L) \quad \text{constant returns to scale for any } z > 0
\]
Returns to scale: Example 2

\[ F(K, L) = \sqrt{K} + \sqrt{L} \]
\[ F(zK, zL) = \sqrt{z^2} \cdot \sqrt{K} + \sqrt{z^2} \cdot \sqrt{L} \]
\[ = z \cdot \sqrt{K} + z \cdot \sqrt{L} \]
\[ = z \cdot (\sqrt{K} + \sqrt{L}) \]
\[ = \sqrt{z} \cdot F(K, L) \]

decreasing returns to scale for any \(z > 1\)

NOW YOU TRY:
Returns to Scale

- Determine whether each of these production functions has constant, decreasing, or increasing returns to scale:
  - (a) \( F(K, L) = \frac{K^2}{L} \)
  - (b) \( F(K, L) = K + L \)

NOW YOU TRY:
Answers, part (a)

\[ F(K, L) = \frac{K^2}{L} \]
\[ F(zK, zL) = \frac{(zK)^2}{zL} = \frac{z^2 K^2}{zL} = z \cdot \frac{K^2}{L} \]
\[ = z \cdot F(K, L) \]
constant returns to scale for any \(z > 0\)

NOW YOU TRY:
Answers, part (b)

\[ F(K, L) = K + L \]
\[ F(zK, zL) = zK + zL \]
\[ = z(K + L) \]
\[ = z \cdot F(K, L) \]
constant returns to scale for any \(z > 0\)

Assumptions

1. Technology is fixed.
2. The economy’s supplies of capital and labor are fixed at
   \[ K = \underline{K} \quad \text{and} \quad L = \underline{L} \]
Determining GDP

Output is determined by the fixed factor supplies and the fixed state of technology:

\[ \bar{Y} = F(\bar{K}, \bar{L}) \]

The distribution of national income

- determined by factor prices, the prices per unit firms pay for the factors of production
  - wage = price of \( L \)
  - rental rate = price of \( K \)

Notation

- \( W \) = nominal wage
- \( R \) = nominal rental rate
- \( P \) = price of output
- \( W/P \) = real wage (measured in units of output)
- \( R/P \) = real rental rate

How factor prices are determined

- Factor prices are determined by supply and demand in factor markets.
- Recall: Supply of each factor is fixed.
- What about demand?

Demand for labor

- Assume markets are competitive: each firm takes \( W, R, \) and \( P \) as given.
- Basic idea: A firm hires each unit of labor if the cost does not exceed the benefit.
  - cost = real wage
  - benefit = marginal product of labor

Marginal product of labor (\( MPL \))

- definition: The extra output the firm can produce using an additional unit of labor (holding other inputs fixed):
  \[ MPL = F(K, L+1) - F(K, L) \]
NOW YOU TRY: Compute & graph MPL

a. Determine MPL at each value of L.

b. Graph the production function.

c. Graph the MPL curve with MPL on the vertical axis and L on the horizontal axis.

<table>
<thead>
<tr>
<th>L</th>
<th>Y</th>
<th>MPL</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>n.a.</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>?</td>
</tr>
<tr>
<td>2</td>
<td>19</td>
<td>?</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>34</td>
<td>?</td>
</tr>
<tr>
<td>5</td>
<td>40</td>
<td>?</td>
</tr>
<tr>
<td>6</td>
<td>45</td>
<td>?</td>
</tr>
<tr>
<td>7</td>
<td>49</td>
<td>?</td>
</tr>
<tr>
<td>8</td>
<td>52</td>
<td>?</td>
</tr>
<tr>
<td>9</td>
<td>54</td>
<td>?</td>
</tr>
<tr>
<td>10</td>
<td>55</td>
<td>?</td>
</tr>
</tbody>
</table>

NOW YOU TRY: Answers

Diminishing marginal returns

- As a factor input is increased, its marginal product falls (other things equal).
- Intuition: Suppose \( L \) while holding \( K \) fixed
  - fewer machines per worker
  - lower worker productivity

NOW YOU TRY: Identifying Diminishing Marginal Returns

- Which of these production functions have diminishing marginal returns to labor?
  - a) \( F(K, L) = 2K + 15L \)
  - b) \( F(K, L) = \sqrt{KL} \)
  - c) \( F(K, L) = 2\sqrt{K} + 15\sqrt{L} \)

NOW YOU TRY: MPL and labor demand

Suppose \( W/P = 6 \).

- If \( L = 3 \), should firm hire more or less labor? Why?
- If \( L = 7 \), should firm hire more or less labor? Why?
MPL and the demand for labor

Each firm hires labor up to the point where \( MPL = \frac{W}{P} \).

The equilibrium real wage

The real wage adjusts to equate labor demand with supply.

Determining the rental rate

- We have just seen that \( MPL = \frac{W}{P} \).
- The same logic shows that \( MPK = \frac{R}{P} \):
  - diminishing returns to capital: \( MPK \downarrow \) as \( K \uparrow \)
  - The \( MPK \) curve is the firm's demand curve for renting capital.
- Firms maximize profits by choosing \( K \) such that \( MPK = \frac{R}{P} \).

The equilibrium real rental rate

The real rental rate adjusts to equate demand for capital with supply.

The Neoclassical Theory of Distribution

- states that each factor input is paid its marginal product
- a good starting point for thinking about income distribution

How income is distributed to L and K

total labor income = \( \frac{W}{P} \times L = MPL \times L \)

total capital income = \( \frac{R}{P} \times K = MPK \times K \)

If production function has constant returns to scale, then

\[ \bar{Y} = MPL \times L + MPK \times K \]

national income

labor income

capital income
The ratio of labor income to total income in the U.S., 1960-2007

Labor's share of total income is approximately constant over time. (Thus, capital's share is, too.)

The Cobb-Douglas Production Function

- Each factor's marginal product is proportional to its average product:

  \[ MPK = \alpha AK^{\alpha-1}L^{1-\alpha} = \frac{\alpha Y}{K} \]

  \[ MPL = (1-\alpha)AK^\alpha L^{1-\alpha} = \frac{(1-\alpha)Y}{L} \]

- The Cobb-Douglas production function has constant factor shares:

  \[ \alpha = \text{capital's share of total income:} \]

  \[ \text{capital income} = MPK \times K = \alpha Y \]

  \[ \text{labor income} = MPL \times L = (1-\alpha)Y \]

- The Cobb-Douglas production function is:

  \[ Y = AK^\alpha L^{1-\alpha} \]

  where \( A \) represents the level of technology.

Labor productivity and wages

- Theory: wages depend on labor productivity

- U.S. data:

<table>
<thead>
<tr>
<th>period</th>
<th>productivity growth</th>
<th>real wage growth</th>
</tr>
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<tbody>
<tr>
<td>1959-2007</td>
<td>2.1%</td>
<td>2.0%</td>
</tr>
<tr>
<td>1959-1973</td>
<td>2.8%</td>
<td>2.8%</td>
</tr>
<tr>
<td>1973-1995</td>
<td>1.4%</td>
<td>1.2%</td>
</tr>
<tr>
<td>1995-2007</td>
<td>2.5%</td>
<td>2.4%</td>
</tr>
</tbody>
</table>

Outline of model

A closed economy, market-clearing model

Supply side

DONE factor markets (supply, demand, price)

DONE determination of output/income

Demand side

Next \( \rightarrow \) determinants of \( C, I, \) and \( G \)

Equilibrium

- goods market
- loanable funds market

Demand for goods & services

Components of aggregate demand:

\[ C = \text{consumer demand for g & s} \]

\[ I = \text{demand for investment goods} \]

\[ G = \text{government demand for g & s} \]

(closed economy: no \( NX \))
Consumption, $C$
- **def**: Disposable income is total income minus total taxes: $Y - T$.
- Consumption function: $C = C(Y - T)$
  - Shows that $\frac{\Delta (Y - T)}{\Delta Y} = \frac{\Delta C}{\Delta Y}$
- **def**: Marginal propensity to consume (MPC) is the change in $C$ when disposable income increases by one dollar.

Investment, $I$
- The investment function is $I = I(r)$, where $r$ denotes the real interest rate, the nominal interest rate corrected for inflation.
- The real interest rate is
  - the cost of borrowing
  - the opportunity cost of using one’s own funds to finance investment spending
- So, $\uparrow r \Rightarrow \downarrow I$

Government spending, $G$
- $G$ = govt spending on goods and services.
- $G$ excludes transfer payments (e.g., social security benefits, unemployment insurance benefits).
- Assume government spending and total taxes are exogenous:
  - $G = G$ and $T = T$

The consumption function
- Graph: $C = C(Y - T)$
  - The slope of the consumption function is the MPC.

The investment function
- Graph: $I = I(r)$
  - Spending on investment goods depends negatively on the real interest rate.

The market for goods & services
- Aggregate demand: $C(\bar{Y} - \bar{T}) + I(r) + \bar{G}$
- Aggregate supply: $\bar{Y} = F(K, L)$
- Equilibrium: $\bar{Y} = C(\bar{Y} - \bar{T}) + I(r) + \bar{G}$
  - The real interest rate adjusts to equate demand with supply.
The loanable funds market

- A simple supply-demand model of the financial system.
- One asset: “loanable funds”
  - demand for funds: investment
  - supply of funds: saving
  - “price” of funds: real interest rate

Demand for funds: Investment

The demand for loanable funds...

- comes from investment:
  Firms borrow to finance spending on plant & equipment, new office buildings, etc.
  Consumers borrow to buy new houses.
- depends negatively on $r$
  the “price” of loanable funds
  (cost of borrowing).

Loanable funds demand curve

The investment curve is also the demand curve for loanable funds.

Supply of funds: Saving

The supply of loanable funds comes from saving:

- Households use their saving to make bank deposits, purchase bonds and other assets. These funds become available to firms to borrow to finance investment spending.
- The government may also contribute to saving if it does not spend all the tax revenue it receives.

Types of saving

<table>
<thead>
<tr>
<th>Type</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>private saving</td>
<td>$(Y - T) - C$</td>
</tr>
<tr>
<td>public saving</td>
<td>$T - G$</td>
</tr>
<tr>
<td>national saving</td>
<td>$S = (Y - T) - C + T - G$</td>
</tr>
</tbody>
</table>

Notation: $\Delta$ = change in a variable

- For any variable $X$, $\Delta X$ = “the change in $X$$$
  $\Delta$ is the Greek (uppercase) letter $Delta$$

Examples:

- If $\Delta L = 1$ and $\Delta K = 0$, then $\Delta Y = MPL$.
- More generally, if $\Delta K = 0$, then $MPL = \frac{\Delta Y}{\Delta L}$
- $\Delta (Y - T) = \Delta Y - \Delta T$, so
  $\Delta C = MPC \times (\Delta Y - \Delta T)$
  $\Delta C = MPC \Delta Y - MPC \Delta T$
NOW YOU TRY:

**Calculate the change in saving**

Suppose \( MPC = 0.8 \) and \( MPL = 20 \).

For each of the following, compute \( \Delta S \):

a. \( \Delta G = 100 \)

b. \( \Delta T = 100 \)

c. \( \Delta Y = 100 \)

d. \( \Delta L = 10 \)

**Answers**

\[
\Delta S = \Delta Y - \Delta C - \Delta G = \Delta Y - 0.8(\Delta Y - \Delta T) - \Delta G \\
= 0.2\Delta Y + 0.8\Delta T - \Delta G
\]

a. \( \Delta S = -100 \)

b. \( \Delta S = 0.8 \times 100 = 80 \)

c. \( \Delta S = 0.2 \times 100 = 20 \)

d. \( \Delta Y = MPL \times \Delta L = 20 \times 10 = 200, \Delta S = 0.2 \times \Delta Y = 0.2 \times 200 = 40 \).

**Budget surpluses and deficits**

- If \( T > G \), budget surplus \( = (T - G) \) = public saving.
- If \( T < G \), budget deficit \( = (G - T) \) and public saving is negative.
- If \( T = G \), “balanced budget,” public saving = 0.
- The U.S. government finances its deficit by issuing Treasury bonds – i.e., borrowing.

**U.S. Federal Government Surplus/Deficit, 1940-2007**

**Fact:** In the early 1990s, about 18 cents of every tax dollar went to pay interest on the debt. (In 2007, it was about 10 cents)

**Loanable funds supply curve**

National saving does not depend on \( r \), so the supply curve is vertical.
Loanable funds market equilibrium

\[ r = \frac{S - Y + C(\bar{Y} - T) - G}{Y} \]

The special role of \( r \)

\( r \) adjusts to equilibrate the goods market and the loanable funds market simultaneously:

If L.F. market in equilibrium, then

\[ Y - C - G = I \]

Add \((C+G)\) to both sides to get

\[ Y = C + I + G \] (goods market eq’m)

Thus,

Eq’m in L.F. market \( \iff \) Eq’m in goods market

Digression: Mastering models

To master a model, be sure to know:

1. Which of its variables are endogenous and which are exogenous.
2. For each curve in the diagram, know:
   a. definition
   b. intuition for slope
   c. all the things that can shift the curve
3. Use the model to analyze the effects of each item in 2c.

Mastering the loanable funds model

Things that shift the saving curve

- public saving
  - fiscal policy: changes in \( G \) or \( T \)
- private saving
  - preferences
  - tax laws that affect saving
    - 401(k)
    - IRA
  - replace income tax with consumption tax

CASE STUDY: The Reagan deficits

Reagan policies during early 1980s:

- increases in defense spending: \( \Delta G > 0 \)
- big tax cuts: \( \Delta T < 0 \)
- Both policies reduce national saving:

\[ \bar{S} = \bar{Y} - C(\bar{Y} - T) - \bar{G} \]

\[ \uparrow G \iff \downarrow \bar{S} \]

\[ \downarrow T \iff \uparrow C \iff \downarrow \bar{S} \]

CASE STUDY: The Reagan deficits

1. The increase in the deficit reduces saving...
2. ...which causes the real interest rate to rise...
3. ...which reduces the level of investment.
Are the data consistent with these results?

<table>
<thead>
<tr>
<th>variable</th>
<th>1970s</th>
<th>1980s</th>
</tr>
</thead>
<tbody>
<tr>
<td>T - G</td>
<td>-2.2</td>
<td>-3.9</td>
</tr>
<tr>
<td>S</td>
<td>19.6</td>
<td>17.4</td>
</tr>
<tr>
<td>r</td>
<td>1.1</td>
<td>6.3</td>
</tr>
<tr>
<td>I</td>
<td>19.9</td>
<td>19.4</td>
</tr>
</tbody>
</table>

T - G, S, and I are expressed as a percent of GDP. All figures are averages over the decade shown.

NOW YOU TRY: The effects of saving incentives

- Draw the diagram for the loanable funds model.
- Suppose the tax laws are altered to provide more incentives for private saving. (Assume that total tax revenue T does not change)
- What happens to the interest rate and investment?

Mastering the loanable funds model, continued

Things that shift the investment curve:
- some technological innovations
- to take advantage some innovations, firms must buy new investment goods
- tax laws that affect investment
  - e.g., investment tax credit

An increase in investment demand

...raises the interest rate.

But the equilibrium level of investment cannot increase because the supply of loanable funds is fixed.

Saving and the interest rate

- Why might saving depend on \( r \) ?
- How would the results of an increase in investment demand be different?
  - Would \( r \) rise as much?
  - Would the equilibrium value of \( I \) change?

An increase in investment demand when saving depends on \( r \)

An increase in investment demand raises \( r \), which induces an increase in the quantity of saving, which allows \( I \) to increase.
**FYI: Markets, Intermediaries, the 2008 Crisis**

- In the real world, firms have several options for raising funds they need for investment, including:
  - borrow from banks
  - sell bonds to savers
  - sell shares of stock (ownership) to savers
- The financial system includes:
  - bond and stock markets, where savers directly provide funds to firms for investment
  - financial intermediaries, e.g. banks, insurance companies, mutual funds, where savers indirectly provide funds to firms for investment

- Intermediaries can help move funds to their most productive uses.
- But when intermediaries are involved, savers usually do not know what investments their funds are financing.
- Intermediaries were at the heart of the financial crisis of 2008….

**A few details on the financial crisis:**

- July '06 to Dec '08: house prices fell 27%
- Jan '08 to Dec '08: 2.3 million foreclosures
- Many banks, financial institutions holding mortgages or mortgage-backed securities driven to near bankruptcy
- Congress authorized $700 billion to help shore up financial institutions

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**Chapter Summary**

- **Total output is determined by:**
  - the economy’s quantities of capital and labor
  - the level of technology
  - Competitive firms hire each factor until its marginal product equals its price.
  - If the production function has constant returns to scale, then labor income plus capital income equals total income (output).

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**Chapter Summary**

- **A closed economy’s output is used for:**
  - consumption
  - investment
  - government spending
- The real interest rate adjusts to equate the demand for and supply of:
  - goods and services
  - loanable funds

- A decrease in national saving causes the interest rate to rise and investment to fall.
- An increase in investment demand causes the interest rate to rise, but does not affect the equilibrium level of investment if the supply of loanable funds is fixed.