Chapter 7

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Chapter 7

7.2-1

(a) The Fourier spectrum of triangular pulse \( p_1(t) = \Delta (t/T_b) \) is

\[
\frac{T_b}{2} \Delta (t/T_b) = \Pi \left( \frac{t}{0.5T_b} \right) \ast \Pi \left( \frac{t}{0.5T_b} \right)
\]

\[
\frac{T_b}{2} \mathcal{F}\{\Delta (t/T_b)\} = \mathcal{F}\left\{ \Pi \left( \frac{t}{0.5T_b} \right) \ast \Pi \left( \frac{t}{0.5T_b} \right) \right\}
\]

\[
= \mathcal{F}\left\{ \Pi \left( \frac{t}{0.5T_b} \right) \right\} \mathcal{F}\left\{ \Pi \left( \frac{t}{0.5T_b} \right) \right\}
\]

\[
= \frac{T_b^2}{4} \text{sinc}^2 \left( \frac{\pi fT_b}{2} \right)
\]

\[
P(f) = \frac{T_b}{2} \text{sinc}^2 \left( \frac{\pi fT_b}{2} \right)
\]

For polar signaling [Eq. (7.13)],

\[
S_y(f) = \frac{|P(f)|^2}{T_b} R_0
\]

and \( R_0 = 1 \)

Therefore,

\[
S_y(f) = \frac{T_b^2}{4} \text{sinc}^4 \left( \frac{\pi fT_b}{2} \right) = \frac{T_b}{4} \text{sinc}^4 \left( \frac{\pi fT_b}{2} \right)
\]

For on-off signaling [Eq. (7.20)],

\[
S_y(f) = \frac{|P(f)|^2}{4T_b} \left[ 1 + \frac{1}{T_b} \sum_{n=-\infty}^{n=\infty} \delta \left( f - \frac{n}{T_b} \right) \right]
\]

\[
S_y(f) = \frac{T_b^2}{4} \text{sinc}^4 \left( \frac{\pi fT_b}{2} \right) \left[ 1 + \frac{1}{T_b} \sum_{n=-\infty}^{n=\infty} \delta \left( f - \frac{n}{T_b} \right) \right]
\]

\[
= \frac{T_b \text{sinc}^4 \left( \frac{\pi fT_b}{2} \right)}{16} \left[ 1 + \frac{1}{T_b} \sum_{n=-\infty}^{n=\infty} \delta \left( f - \frac{n}{T_b} \right) \right]
\]

For bipolar signaling [Eq. (7.21a)],

\[
S_y(f) = \frac{|P(f)|^2}{T_b} \text{sinc}^2 (\pi fT_b)
\]

\[
S_y(f) = \frac{T_b^2}{4} \text{sinc}^4 \left( \frac{\pi fT_b}{2} \right) \text{sinc}^2 (\pi fT_b)
\]

\[
= \frac{T_b \text{sinc}^4 \left( \frac{\pi fT_b}{2} \right)}{4} \text{sinc}^2 (\pi fT_b)
\]

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(b) Bandwidth: For the three cases, bandwidth is about $1.5/T_b$ Hz. See Fig. S7.2-1.

2.2-2

(a) The Fourier spectrum of triangular pulse $p_2(t) = \Pi(t/0.5T_b)$ is

$$\mathcal{F}(p_2(t)) = \frac{T_b}{2} \text{sinc} \left( \frac{\pi f T_b}{2} \right)$$

$$P(f) = \frac{T_b}{2} \text{sinc} \left( \frac{\pi f T_b}{2} \right)$$

$$|P(f)|^2 = \frac{T_b^2}{4} \text{sinc}^2 \left( \frac{\pi f T_b}{2} \right)$$

for polar signaling [Eq. (7.13)],

$$S_y(f) = \frac{|P(f)|^2}{T_b} R_0$$

and $R_0 = 1$

Therefore,

$$S_y(f) = \frac{T_b^2}{4} \text{sinc}^2 \left( \frac{\pi f T_b}{2} \right) = \frac{T_b}{4} \text{sinc}^2 \left( \frac{\pi f T_b}{2} \right)$$

For on-off signaling [Eq. (7.20)],

$$S_y(f) = \frac{|P(f)|^2}{4T_b} \left[ 1 + \frac{1}{T_b} \sum_{n=-\infty}^{n=\infty} \delta \left( f - \frac{n}{T_b} \right) \right]$$
\[ P(f) = \frac{T_b}{2} \left[ \frac{\text{sinc} \left( \frac{2\pi f T_b + \pi}{2} \right) + \text{sinc} \left( \frac{2\pi f T_b - \pi}{2} \right)}{\text{sinc} \left( \frac{\pi f T_b + \pi}{2} \right) + \text{sinc} \left( \frac{\pi f T_b - \pi}{2} \right)} \right] \]

\[ = \frac{T_b}{2} \left[ \frac{\cos \left( \frac{\pi f T_b}{2} \right) - \cos \left( \frac{\pi f T_b}{2} \right)}{\left( \frac{\pi f T_b}{2} \right)^2 - \left( \frac{\pi}{2} \right)^2} \right] \]

\[ = \frac{T_b}{2} \left[ \frac{\cos \left( \frac{\pi f T_b}{2} \right)}{\left( \frac{\pi f T_b}{2} \right)^2 - \left( \frac{\pi}{2} \right)^2} \right] \]

\[ = \frac{T_b}{2\pi} \left[ \frac{\cos \left( \frac{\pi f T_b}{2} \right)}{(1/4) - (f T_b)^2} \right] \]

(a) For polar signaling [Eq. (7.13)],

\[ S_p(f) = \frac{|P(f)|^2}{T_b} R_0 \quad R_0 = 1 \]

Therefore,

\[ S_p(f) = \frac{T_b^2}{4} \frac{[\text{sinc} \left( \frac{\pi f T_b}{2} \right) + \text{sinc} \left( \frac{\pi f T_b - \pi}{2} \right)]^2}{T_b} \]

\[ = T_b \frac{[\text{sinc} \left( \frac{\pi f T_b}{2} \right) + \text{sinc} \left( \frac{\pi f T_b - \pi}{2} \right)]^2}{4} \]

For on-off signaling [Eq. (7.20)],

\[ S_o(f) = \frac{|P(f)|^2}{4 T_b} \left[ 1 + \frac{1}{T_b} \sum_{n=-\infty}^{n=\infty} \delta \left( f - \frac{n}{T_b} \right) \right] \]

\[ S_o(f) = \frac{T_b}{16} \frac{[\text{sinc} \left( \frac{\pi f T_b}{2} \right) + \text{sinc} \left( \frac{\pi f T_b - \pi}{2} \right)]^2}{1 + \frac{1}{T_b} \sum_{n=-\infty}^{n=\infty} \delta \left( f - \frac{n}{T_b} \right)} \]

For bipolar signaling Eq. (7.21a), \( S_b(f) = \frac{|P(f)|^2}{T_b} \sin^2(\pi f T_b) \)

\[ S_b(f) = \frac{T_b}{4} \frac{[\text{sinc} \left( \frac{\pi f T_b}{2} \right) + \text{sinc} \left( \frac{\pi f T_b - \pi}{2} \right)]^2}{\sin^2(\pi f T_b)} \]

(b) Bandwidth: For three cases, bandwidth is about \( 1.5/T_b \) Hz or 1.5 Hz for \( T_b = 1 \). See Fig. S7.2-3b

\[ 7.3-4 \]

\[(a) \quad \text{The pulse waveform is } p(t) = \Pi \left( \frac{t + \frac{T_b}{4}}{2} \right) - \Pi \left( \frac{t - \frac{T_b}{4}}{2} \right) \]

See Figure S7.2-4a for the modulated data

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Fig. S7.2-3b

Fig. S7.2-4a
\[ P(f) = \frac{T_b}{2} \text{sinc} \left( \frac{\pi f T_b}{2} \right) \exp \left( j \frac{2\pi f T_b}{4} \right) - \frac{T_b}{2} \text{sinc} \left( \frac{\pi f T_b}{2} \right) \exp \left( -j \frac{2\pi f T_b}{4} \right) \]

\[ = jT_b \text{sinc} \left( \frac{\pi f T_b}{2} \right) \sin \left( \frac{\pi f T_b}{2} \right) \]

1 and 0 are assumed to be equally likely, we have

\[ S_y(f) = \frac{|P(f)|^2}{T_b} \]

\[ S_y(f) = \frac{T_b^2 \text{sinc}^2 \left( \frac{\pi f T_b}{2} \right) \sin^2 \left( \frac{\pi f T_b}{2} \right)}{T_b} \]

\[ = T_b \text{sinc}^2 \left( \frac{\pi f T_b}{2} \right) \sin^2 \left( \frac{\pi f T_b}{2} \right) \]

(first null) bandwidth is \( \frac{4\pi}{T_b} \text{ rad/s} \) or \( 2/T_b = R_b \text{Hz} \).

Fig. S7.2-4b

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3) The Fourier transform of the pulse is \( P(f) \).

For the binary signal using differential signaling, all the bits are equally likely. Thus, we have

\[ R_0 = \lim_{N \to \infty} \frac{1}{N} \left[ \frac{N}{2} (1)^2 + \frac{N}{2} (-1)^2 \right] = 0 \]

To compute \( R_1 \) the four possible sequences 00, 11, 01, 10 are equally likely, so

\[ R_1 = \lim_{N \to \infty} \frac{1}{N} \left[ \frac{N}{2} (1) + \frac{N}{2} (-1) \right] = 0 \]

Similarly, \( R_n = 0 \), \( n > 0 \). Therefore

\[ S_y(f) = \frac{|P(f)|^2}{T_b} R_0 = \frac{T_b}{4} \left[ \text{sinc} \left( \frac{2\pi f T_b}{2} \right) + \text{sinc} \left( \frac{2\pi f T_b}{2} \right) \right]^2 \]

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(a) Random binary sequence is: 01010011011010... See Figure 7.2-6a.

(b) To compute $R_0$, we observe that, on the average, half of the pulses have $a_k = 0$ and remaining half have $a_k = 1$ or $-1$. Hence

$$R_0 = \lim_{N \to \infty} \frac{1}{N} \left[ \frac{N}{2} (\pm 1)^2 + \frac{N}{2} (0)^2 \right] = \frac{1}{2}$$

To determine $R_1$, we need to find different cases of $a_k a_{k+1}$. There are four possible equally likely sequences with two bits: 00, 01, 10, 11. This means on average $3N/4$ combinations have $a_k a_{k+1} = 0$ and the remaining $N/4$ combinations have nonzero $a_k a_{k+1}$. Because of the duobinary rule, the bit sequence 11 can only be encoded by 2 consecutive pulses of the same polarity (either both positive or both negative). This means $a_k a_{k+1} = 1$ for 11. Therefore

$$R_1 = \lim_{N \to \infty} \frac{1}{N} \left[ \frac{N}{4} (1) + \frac{3N}{4} (0) \right] = \frac{1}{4}$$

To compute $R_2$ in a similar way, we need to observe $a_k a_{k+2}$. We need to observe all possible combinations of three bits. There are 8 combinations, 111, 101, 000, 001, 010, 011, 100, 110. The last six combinations have either first bit or last bit 0. Using the duobinary rule, we encode the first combination by three pulses of the same polarity, leading to $a_k a_{k+2} = 1$. For the second combination, we encode with the first and the last bits of opposite polarities, leading to $a_k a_{k+2} = -1$.

Therefore, given $N$ combinations, on the average, $a_k a_{k+2}$ equals $-1$, 1 and 0 for $N/8$, $N/8$, and $3N/4$ combinations, respectively.

Hence

$$R_2 = \lim_{N \to \infty} \frac{1}{N} \left[ \frac{N}{8} (1) + \frac{N}{8} (-1) + \frac{3N}{4} (0) \right] = 0$$

In a similar way, we find $R_n = 0$, $n > 2$.

7.3-1 Bit rate equals to the pulse rate $R_b = 5 \times 10^6$ bit/s and $r = 0.25$ are given. Minimum required bandwidth with rolloff $r$ is

$$B_T = \frac{(1 + r) R_b}{2}$$

$$B_T = \frac{(1 + 0.25) 5 \times 10^6}{2} = 3.125 \times 10^6 \text{ Hz}$$

7.3-2 The pulse rate $R_b = 13.0248 \times 10^6$ bit/s and $r = 0.2$ are given. Minimum required bandwidth with roll-off $r$ is $B_T = \frac{(1 + r) R_b}{2}$:

$$B_T = \frac{(1 + 0.2) 13.0248 \times 10^6}{2} = 7.8149 \times 10^6 \text{ Hz}$$
\[
P_r = \begin{pmatrix}
0.17 & 0 & 0 & 0 & 0 \\
0.6 & 0.17 & 0 & 0 & 0 \\
1 & 0.6 & 0.17 & 0 & 0 \\
-0.1 & 1 & 0.6 & 0.17 & 0 \\
-0.2 & -0.1 & 1 & 0.6 & 0 \\
0 & -0.2 & -0.1 & 1 & 0 \\
0 & 0 & 0 & -0.2 & -0.1 \\
0 & 0 & 0 & 0 & -0.2 \\
\end{pmatrix}, \quad p_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}
\]

\[c = P_r^tp_0 = [0.1632, -0.5689, 1.0150, -0.1212, 0.1837]\]

MSE of 5-tap MMSE = 0.0023

MSE of 5-tap ZF = 0.0024

5-tap MMSE and ZF equalizers in this problem are better than better than the 3-tap equalizers.

6.1 \[p(t) = \Pi\left(\frac{t}{3T_b/4}\right)\]
See Figures S7.6-1a, S7.6-1b, S7.6-1c, and S7.6-1d for the eye diagrams.