Transient DC Circuits

- Lab #4 examines inductors and capacitors and their influence on DC circuits.
- As R is the symbol for a resistor, C and L are the symbols for capacitors and inductors.
- Capacitors and inductors are the other two passive circuits components.
- In a circuit with capacitors and inductors (and normally, also resistors), turning a DC power source on or off causes a brief, non-linear behavior of current in the circuit.
- Such circuits (usually referred to as RL, RC, or RLC circuits) are of great interest in electrical engineering, as is their transient behavior.
The Capacitor

- A capacitor consists of two conducting surfaces separated by a dielectric, or insulator.
- A capacitor stores electric charge when current flows due to an applied voltage, just as a water tank stores water.
- The capacitor develops an equal and opposite voltage as it collects charge.
- When the voltage on the capacitor = the applied voltage, current flow ceases.
- Charge cannot cross the dielectric barrier of a capacitor.
- Voltage cannot appear instantaneously across a capacitor.
The Inductor

• The inductor has the property of **electrical inertia**.
• Physical inertia is the property of mass that resists a change in motion (acceleration). If at rest, an object resists moving; if moving, it resists a change in speed.
• **Similarly, an inductor resists a change in current.** If no current flows, it resists the start of current. If current is flowing, it resists a change in current.
• Just as a voltage cannot instantaneously appear across a capacitor, current cannot flow instantaneously in an inductor.

A massive truck would have high resistance to rapid acceleration or braking.

DC voltage source \( i(t) \)
Exponential Behavior

- Exponential behavior is mathematical behavior such that one of the variables is an exponent.
- Some functions have an exponential behavior that involves $e$, the base of natural logarithms.
- Some exponential behavior is asymptotic; it approaches a value but never reaches it. Such a behavior is exhibited in the equation to the right.
- DC transient circuit behavior is characterized by this mathematical description.

Plot of $1 - e^{-x}$
Behavior of an RC Circuit

- Asymptotic, transient behavior is exhibited in an RC circuit.
- When the switch is closed, current flows into the capacitor.
- Current flow ceases when charge collected on the capacitor produces a voltage equal and opposite to $V$.
- An equation describing the behavior is shown; it is both exponential and asymptotic.

\[ v_c(t) = V(1 - e^{-(t/RC)}) \]
The Time Constant $\tau$

- In the equation shown, as time passes, $v_c(t) \to V$, as the value of $e^{-t/RC} \to 0$.
- In the equation, the value $RC$ is called $\tau$.
- Clearly, as $\tau$ grows smaller, transient behavior disappears much faster.
- Since $\tau$ determines how quickly the transient response of the circuit dies, it is called the time constant.
- Note: For $R = 1000 \, \Omega$, $C = 0.05 \, \mu F$, then $\tau \approx 0.00005$ sec. Transient effects last a very short time.

$$v_c(t) = V(1 - e^{-t/\tau})$$

The time constant in an RC circuit is sometimes referred to as “the RC time constant.”
We also see **asymptotic**, **transient** behavior in an RL circuit.

When the switch is closed, current flow is inhibited as the inductor develops an **opposite voltage to the one applied**.

Current slowly begins to flow, as the inductor voltage falls toward 0.

As the transient effect dies, current flow approaches $V/R$.

An equation describing the behavior is shown.

$$v_L(t) = Ve^{-(t/(L/R))} = Ve^{-(R/L)t}$$
The time constant $\tau$ in an RL circuit is defined as $\tau = \frac{L}{R}$.

In the equation shown, as time passes, $v_L(t) \to 0$, as the value of $e^{-t/L/R} = e^{- (R/L)t} \to 0$.

As $\tau$ grows smaller, transient behavior disappears much faster, as in the $RC$ case.

$$v_L(t) = Ve^{-\left(\frac{t}{L/R}\right)} = Ve^{-(R/L)t}$$

The time constant in an RL circuit is often referred to as “the RL time constant.”
Odd Behavior of an RLC Circuit

- A circuit with $R$, $L$, and $C$ can exhibit oscillatory behavior if the components are chosen properly.
- For many values of $R$-$L$-$C$, there will be no oscillation.
- The expression that describes this behavior is shown at right.
- The parameter $\omega_d$ is the radian frequency ($\omega_d = 2\pi f$, $f$ the frequency in Hz), which depends on the values of $R$ and $C$.
- $\alpha$ is the damping factor, which determines the rate at which the oscillation dies out.
Behavioral Parameters in the RLC Circuit

• In the formula for $v_C(t)$, the radian frequency of oscillation, $\omega$, depends on $R$, $L$, and $C$.

• Note that in general, the smaller $L$ and $C$, the higher frequency the oscillation. Also, if $R$ is too large the quantity under the square root is negative, which means there is no oscillation.

• Note that $\alpha$ is very similar to $\tau$. In fact the value of $\alpha$ is exactly $\frac{1}{2}$ the value of $\tau$ for an $RL$ circuit.

$$v_C(t) = V (1 - [\cos \omega_d t] e^{-\alpha t})$$

$$\alpha = \frac{R}{2L}$$

$$\omega_d = \sqrt{(1 / LC) - \left(\frac{R}{2L}\right)^2}$$
Using the Signal Generator as a “DC Power Source”

• For our RC transient circuit, as mentioned on a previous slide, \( \tau = RC \approx 1000 \times 0.05 \times 10^{-6} = 0.00005 \) seconds, or 50\( \mu \)sec. Then 10 \( \tau = \frac{1}{2} \) msec.

• That is a very short time.

• We will need to use the oscilloscope to observe transient behavior.

• It is not very convenient to try to rapidly turn the DC power supply on and off to evoke the transient signals we want to watch.

• Instead, why not use the signal generator square wave pattern as a “rapidly switching DC power source?”

• One hitch: the normal square wave pattern is equally above and below 0V. We need a varying voltage level from 0 to a positive voltage (say 5V).

• Solution: The signal generator will let us “dial in” a DC level to algebraically add to the AC voltage. Thus, dial in +2.5 V to a 5 V p-p AC signal to get a voltage that varies 0-5 VDC.
Adding a DC Level to an AC Signal

- **Dialing in an offset**: Press the “offset” soft button (A) and use the dial (B) to add in the desired DC level.
Review of the Oscilloscope

- Important controls: Cursor on (A), cursor control (B, C), autoscale (D), manual sweep (E), trigger (F), manual sensitivity (G).
Oscilloscope (2)

• We will be using the oscilloscope to view transient signals as shown.

• Note that the oscilloscope must be switched to “DC coupling” to register the DC signal value; otherwise it is stripped away and ignored.

• Use controls mentioned on the previous slide to get the right voltage sensitivity and time base to view the transient signals as shown.