Excess Volatility of Realized Excess Profit from Currency Speculation in a Two-Country General Equilibrium Model

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Abstract
This paper examines whether the Lucas model can explain stylized facts in foreign exchange markets, by employing Monte Carlo studies. It is assumed that changes in the logarithms of endowments and of money supplies follow a multivariate Markov switching process. From the results of the Monte Carlo studies, with plausible values of the preference parameters, the excess volatility of the realized excess profit from currency speculation, the strong autocorrelation of the forward premium in the sample can be found in the model for four exchange rates. However, the implied covariance between the forward premium and depreciation rates is positive.

1. Introduction
Since the abandonment of the Bretton Woods arrangement, a number of empirical studies have established stylized facts in foreign exchange markets. Among the stylized facts, the excess volatility of realized excess profit (EP) from currency speculation and the negative covariance between the forward premium (FP) and the depreciation rate (DR) have only occasionally been found in models. Excellent surveys have been produced by Hodrick (1987) and by Froot and Frankel (1989).

The trend of recent research in this field has changed. Researchers now examine the plausibility of preference parameters with which a theoretical economy can explain stylized facts in foreign exchange markets. This method is called "calibration."

Backus et al. (1993) were the first to employ a calibration method in this field. They examined whether the theoretical risk premium and the first autocorrelation of theoretical forward premium, which are derived from partial equilibrium conditions, can match those in the sample at the admissible range of preference parameters. Their empirical results are unsatisfactory; a two-country, one-good partial equilibrium model cannot explain the excess volatility of the risk premium, the strong autocorrelation of FP, and the negative covariance between FP and DR. Furthermore, their study contains several problems. First, the study’s use of a one-good model provides no incentive for the two countries to trade. Second, the reduced forms of forward premium and the risk premium are derived from partial equilibrium conditions; thus, the model has the weakness of ignoring the general equilibrium conditions. Third, the reduced forms of forward premium and the risk premium in a theoretical economy are a function of exogenous processes, which are assumed to be governed by a simple Markov process.

In this context, Bekaert’s (1994) work differs from that of Backus et al. (1993). Bekaert assumes that exogenous processes are governed by a vector autoregressive (VAR) process and uses a two-good and general equilibrium version of Svensson’s (1985) model with a cash-in-advance constraint. Bekaert employs an estimation tech-

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nique which minimizes the weighted quadratic distance between sample moments and theoretical moments and tests the over-identifying restrictions. However, Bekaert fails to match the volatility of EP from currency speculation and the negative covariance between FP and the DR from the sample with a model. He argues that the failure is due to the assumption of the purchasing power parity in Svensson's (1985) model.

The main purpose of this paper is similar to that of previous empirical studies that examine whether a simple, general-equilibrium, Lucas (1982) two-country, two-good model can explain stylized facts in foreign exchange markets. However, this paper differs from previous, related studies in several respects. First, the common calibration method usually ignores one or two sources of uncertainty, the first of which is the assumption that estimates of sample moments, $Z_T$, are true parameters $Z$. The other comes from the assumption that estimates of technology parameters in an exogenous stochastic process, $\theta$, are equal to the true technology parameters, $\theta$. Backus et al. (1993) ignore both sources of uncertainty, and Bekaert (1994) ignores the second source of uncertainty. The estimation results that ignore the second source of uncertainty are valid only when $\theta = \theta$. Furthermore, the test of the over-identifying restrictions is based on asymptotic theory. Thus, here I employ Monte Carlo studies to obtain the empirical distributions of a small theoretical sample vector $H_T$ and to compare a vector of the sample moments $Z_T$ with $H_T$.

Second, previous empirical work did not consider the fact that exogenous variables can have a structural change or a regime shift. To allow for a structural change or a regime shift of the exogenous stochastic process, the law of motion for the forcing process in this paper is estimated from a multivariate Markov switching process. Endowments in two countries have the same Markov state variable while two money supplies are governed by an independent univariate Markov switching process.


The paper is organized as follows. The stylized facts in foreign exchange markets are reviewed in section 2. Section 3 presents a version of Lucas's two-country model. Section 4 presents a Markov switching process and derives the final reduced forms of the forward and spot exchange rates from general equilibrium conditions. It goes on to explain the strategy of choosing preference parameters, and then shows the procedures of the Monte Carlo studies and how to perform univariate and multivariate tests. Section 5 provides the estimation results of technology and preference parameters, and shows the estimation results of univariate and multivariate tests using the Monte Carlo studies. From the Monte Carlo studies, the model explains the excess volatility of realized excess profit in foreign exchange markets and the strong autocorrelation of FP for the cases of the US dollar per Canadian dollar ($/C\$', the US dollar per UK pound ($/¥$), the US dollar per Deutsche mark ($/DM$) and the US dollar per Japanese yen ($/¥$). Furthermore, the model successfully matches the negative covariance between FP and the DR at the 5% significance level, but the implied covariance by the model is close to zero. Section 6, which includes a brief conclusion, provides some plausible reasons for the success in matching the high volatility of EP from the sample with a model and the failure to derive the negative covariance between FP and DR from the model.

2. Stylized Facts in the Foreign Exchange Market

The unbiasedness hypothesis in the foreign exchange market is

$$H_0: F_i = E_i(S_{it})$$

(1)
where \( F_t \) is the three-month forward exchange rate observed at time \( t \) for both delivery and payment taking place after three months, and \( S_{t+1} \) is the spot exchange rate at \( t+1 \). The unbiasedness hypothesis has been of interest since the mid-1970s because spot rates have fluctuated since the abandonment of the Bretton Woods arrangement. Numerous studies using various equivalent specifications have rejected the unbiasedness hypothesis. See Boothe and Longworth (1986), Hodrick (1987), and Froot and Frankel (1989) for surveys of these studies.

To test the unbiasedness hypothesis, the following two complementary data regressions with nonoverlapping data are often used:

\[
(F_t - S_{t+1})/S_t = a_1 + b_1(F_t - S_t)/S_t + u_{t+1},
\]

(2)

\[
(S_{t+1} - S_t)/S_t = -a_t + (1 - b_t)(F_t - S_t)/S_t - u_{t+1},
\]

(3)

where the variables on the left-hand sides in (2) and (3) are the realized excess profit rate from currency speculation (EP) and the rate of depreciation (DR), respectively. The independent variable in equation (2) is the forward premium. The unbiasedness hypothesis implies that \( a_t \) and \( b_t \) are zero. According to the empirical results of Fama (1984), Backus et al. (1993), and Bekaert and Hodrick (1993), \( b_t \) is greater than one.

Two opposite interpretations have been proposed regarding the evidence against the unbiasedness hypothesis. The first interpretation is based on market inefficiency. Bilson (1981) rejected the unbiasedness hypothesis because the foreign exchange market is possibly inefficient and there is no risk premium. Froot and Frankel (1989) and Froot and Thaler (1990) developed the hypothesis that market forecasts of future spot exchange rates are not efficient.

The other interpretation comes from international finance models developed by Hodrick (1981), Hodrick and Srivastava (1984), and Mark (1985), which are based on the pricing of forward exchange contracts. According to these models, the forward exchange rate may not equal the expectation of the future spot exchange rate because of the existence of the risk premium. Empirical researchers such as Fama (1984), Mark (1985), Hodrick and Srivastava (1986), Liu and Maddala (1992), and Bekaert and Hodrick (1993) provide evidence for the existence of the risk premium. Therefore, they reject the unbiasedness hypothesis while maintaining the hypothesis of market efficiency.

The other stylized facts concerning foreign exchange rates can be established from the summary statistics of the forward and the spot exchange rates. Table 1 shows the ordinary least squares (OLS) coefficient \( b_t \) in equation (2), means, variances, and autocorrelations of EP, DR, and FP. There are 72 quarterly observations covering the period from 1974Q1 to 1991Q4 taken from the Harris Bank Foreign Exchange Weekly Review. Four exchange rates were investigated: $/C$, $/E$, $/DM$, and $/¥$. According to the statistics in Table 1, the stylized facts in the foreign exchange market can be summarized as follows:

1. The unconditional means of EP and FP are close to zero. From the zero-mean of FP, the evidence against the unbiasedness hypothesis implies that the risk premium must vary over time.
2. The variance of EP is larger than that of DR. This means that the spot rate is a slightly better predictor of the future spot rate than the forward rate in terms of the standard deviation of the forecast errors.
3. EP and DR show very little autocorrelation.
4. The first autocorrelation of FP exceeds 0.45. This might be caused by the serial correlation in the risk premium and the expected rate of depreciation.
Table 1. Summary Statistics

<table>
<thead>
<tr>
<th>Statistics</th>
<th>$/C$</th>
<th>$/$</th>
<th>$/DM$</th>
<th>$$/yen$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(S_n - S_r)/S_r$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.00228</td>
<td>-0.00181</td>
<td>0.00827</td>
<td>0.01291</td>
</tr>
<tr>
<td>Variance</td>
<td>0.00042</td>
<td>0.00321</td>
<td>0.00375</td>
<td>0.00362</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.11453</td>
<td>0.21722</td>
<td>0.16664</td>
<td>0.21979</td>
</tr>
<tr>
<td>AR(2)</td>
<td>0.00072</td>
<td>-0.11964</td>
<td>-0.07152</td>
<td>0.02615</td>
</tr>
<tr>
<td>AR(3)</td>
<td>0.17878</td>
<td>0.14327</td>
<td>0.12214</td>
<td>0.04886</td>
</tr>
<tr>
<td>$(F_r - S_m)/S_r$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.00097</td>
<td>-0.00509</td>
<td>0.00088</td>
<td>-0.00711</td>
</tr>
<tr>
<td>Variance</td>
<td>0.00045</td>
<td>0.00344</td>
<td>0.00386</td>
<td>0.00393</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.18843</td>
<td>0.26359</td>
<td>0.20521</td>
<td>0.25450</td>
</tr>
<tr>
<td>AR(2)</td>
<td>0.05487</td>
<td>-0.09183</td>
<td>-0.05061</td>
<td>0.05980</td>
</tr>
<tr>
<td>AR(3)</td>
<td>0.21595</td>
<td>0.14160</td>
<td>0.13377</td>
<td>0.06737</td>
</tr>
<tr>
<td>$(F_r - S_r)/S_r$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.00325</td>
<td>-0.00689</td>
<td>0.00915</td>
<td>0.00580</td>
</tr>
<tr>
<td>Variance</td>
<td>0.00002</td>
<td>0.00006</td>
<td>0.00003</td>
<td>0.00016</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.70936</td>
<td>0.71657</td>
<td>0.45659</td>
<td>0.59940</td>
</tr>
<tr>
<td>AR(2)</td>
<td>0.63612</td>
<td>0.53267</td>
<td>0.33823</td>
<td>0.52804</td>
</tr>
<tr>
<td>AR(3)</td>
<td>0.52994</td>
<td>0.43880</td>
<td>0.47331</td>
<td>0.49081</td>
</tr>
<tr>
<td>$b_i$</td>
<td>1.66689</td>
<td>2.53954</td>
<td>2.69026</td>
<td>1.51610</td>
</tr>
<tr>
<td>(0.61453)</td>
<td>(0.87232)</td>
<td>(0.52410)</td>
<td>(0.57600)</td>
<td></td>
</tr>
</tbody>
</table>

Standard errors are in parentheses.

(5) The variance of FP is smaller than that of EP or DR.
(6) The estimate of $b_i$ is significantly greater than one. This implies that the covariance between FP and DR is negative.\(^4\)

In order to examine whether a theoretical economy can explain the above stylized facts, the following seven moments are considered:\(^5\)

$$Z_t = [E(EP), \text{var}(EP), \rho_1, b_i, E(FP), \text{var}(FP), \rho_2]',$$

where $E(\cdot)$ and $\text{var}(\cdot)$ refer to mean and variance, and $\rho_1$ and $\rho_2$ are the first autocorrelations of EP and FP, respectively.

3. Lucas's Two-Country Model

This section presents a two-country general equilibrium model based on Lucas (1982). There are two countries, labeled 0 and 1, with identical, constant populations. Representative economic agents in both countries are identical and agents are risk-adverse with identical preferences. Each citizen of country 0 is endowed each period with $\xi$ units of a freely transportable, nonstorable consumption good, $X$. Each citizen of country 1 is endowed each period with $\eta$ units of a different consumption good, $Y$. That is, each country produces one good but consumes both goods $X$ and $Y$. The endowments, $\xi$ and $\eta$, are stochastic, following a Markov process. Each agent in country $i$ maximizes
\[ E_0 \sum_{t=0}^{\infty} S^t U_t(X_t, Y_t), \]  

(5)

where \( \delta > 0 \) and \( i = 0 \) or \( 1 \), and \( X_t \) and \( Y_t \) are consumption in country \( i \) in period \( t \) of the goods \( X \) and \( Y \), respectively.

Following Giovannini (1989), Hodrick (1989), and Abel (1990), utility is assumed to be separable in the two goods:

\[ U = X_0^{1-\alpha}/(1-\alpha) + Y_0^{-\beta}/(1-\beta), \]  

(6)

where \( 0 \leq \alpha < \infty, \ 0 \leq \beta < \infty \) are the preference parameters.

Agents are risk-averse, so in the face of stochastic endowments they use available security markets to pool their risk. In the perfect pooling equilibrium, a single representative is consuming half of the endowments in each country. The perfect pooling equilibrium condition is

\[ X_{0t} = X_{1t} = x = 0.5 \xi, \quad Y_{0t} = Y_{1t} = y = 0.5 \eta, \]  

(7)

In the equilibrium, therefore, the relative price of \( Y \) in terms of \( X \), expressed as \( P \), is given by the ratio of the marginal utility of \( Y \) to the marginal utility of \( X \):

\[ P = U_Y(x_t, y_t)/U_X(x_t, y_t), \]  

(8)

where \( U_X \) and \( U_Y \) are the marginal utility of consuming \( X \) and \( Y \), respectively.

Money is introduced into this framework with a cash-in-advance constraint. Thus, the representative agent’s financial constraints are

\[ P_{0t} \xi = M_t, \quad P_{1t} \eta = N_t, \]  

(9)

where \( P_{0t} \) and \( P_{1t} \) are the domestic and foreign prices, and \( M_t \) and \( N_t \) are the money supply in the domestic and foreign countries, respectively.

Given the relative price in equation (8) and the nominal prices in (9), the equilibrium exchange rate is given by

\[ S_t = (M_t/N_t)(\eta_t/\xi_t)P_t, \]  

(10)

The expression for the forward and the spot exchange rates can be stated in terms of the state variables. The prices of the domestic and foreign currencies at \( t \) of one unit of the domestic and foreign currencies delivered at \( t+1 \) can be expressed as

\[ (1 + \delta_t) E_t [\delta U_X(x_{t+1}, y_{t+1})/P_{0t+1}] = U_X(x_t, y_t)/P_{0t}, \]  

(11)

\[ (1 + \delta_t) E_t [\delta U_Y(x_{t+1}, y_{t+1})/P_{1t+1}] = U_Y(x_t, y_t)/P_{1t}, \]  

(12)

where \( \delta_t \) and \( \delta_t \) are the interest rates in the domestic and foreign countries, respectively. From the covered interest rate parity, the forward exchange rate can be expressed as

\[ F_t = (1 + \delta_t) S_t / (1 + \delta_t). \]  

(13)

Combining the perfect pooling equilibrium condition and the cash-in-advance constraint, FP and EP can be written as follows:

\[ (F_t - S_t)/S_t = E_t [(\eta_t/\eta_t)^{-\beta}(N_t/N_{t+1})]/[E_t[(\xi_t/\xi_t)^{1-\alpha}(M_t/M_{t+1})] - 1, \]  

(14)

\[ (S_{t+1} - S_t)/S_t = [(\eta_t/\eta_t)^{-\beta}(N_t/N_{t+1})]/[E_t[(\xi_t/\xi_t)^{1-\alpha}(M_t/M_{t+1})] - 1. \]  

(15)

According to equations (14) and (15), the theoretical forward and spot exchange rates can be expressed in terms of the endowments and money supply.
4. Methodology and Specification

Markov Switching Process and Solution

In Lucas’s two-country model, the outputs and money supply in both countries are assumed to be stochastic and governed by a Markov process. Without a specific form of a Markov process it is extremely difficult to obtain, for example, the OLS coefficient in equation (2) in terms of the outputs and money supply in both countries. Hence, we assume that the changes in the logarithms of outputs are governed by a bivariate Markov switching process and that the changes in the logarithms of the two money supplies are governed by an independent univariate Markov switching process:

\[
\begin{align*}
\ln(\xi_{t+1}) &= \ln(\xi_t) + a_0 + a_1 I_{t+1} + \epsilon_{\xi t+1}, \\
\ln(\eta_{t+1}) &= \ln(\eta_t) + b_0 + b_1 I_{t+1} + \epsilon_{\eta t+1}, \\
\ln(M_{t+1}) &= \ln(M_t) + c_0 + c_1 J_{t+1} + \epsilon_{M t+1}, \\
\ln(N_{t+1}) &= \ln(N_t) + d_0 + d_1 K_{t+1} + \epsilon_{N t+1}, \\
\sum_e &= \begin{pmatrix} \sigma^2_1 & \sigma_{12} & 0 & 0 \\
\sigma_{12} & \sigma^2_2 & 0 & 0 \\
0 & 0 & \sigma^2_i & 0 \\
0 & 0 & 0 & \sigma^2_2 \\
\end{pmatrix}
\end{align*}
\]  

(16)

where \( \{\epsilon\} \) is a sequence of independent and identically distributed normal errors with a mean of zero and a variance–covariance matrix \( \Sigma_e \). \( \{I_t\}, \{J_t\}, \) and \( \{K_t\} \) are sequences of independent Markov random variables that have two states (0 or 1) with the following transition probabilities:

\[
\begin{align*}
\Pr[I_{t+1} = 0 | I_t = 0] &= q_i, & \Pr[I_{t+1} = 1 | I_t = 0] &= 1 - q_i, \\
\Pr[I_{t+1} = 0 | I_t = 1] &= 1 - p_i, & \Pr[I_{t+1} = 1 | I_t = 1] &= p_i, \\
\Pr[J_{t+1} = 0 | J_t = 0] &= q_j, & \Pr[J_{t+1} = 1 | J_t = 0] &= 1 - q_j, \\
\Pr[J_{t+1} = 0 | J_t = 1] &= 1 - p_j, & \Pr[J_{t+1} = 1 | J_t = 1] &= p_j, \\
\Pr[K_{t+1} = 0 | K_t = 0] &= q_k, & \Pr[K_{t+1} = 1 | K_t = 0] &= 1 - q_k, \\
\Pr[K_{t+1} = 0 | K_t = 1] &= 1 - p_k, & \Pr[K_{t+1} = 1 | K_t = 1] &= p_k.
\end{align*}
\]  

(17)

This assumption is supported by Stockman (1990), Phillips (1991), and Della (1986, 1987). Stockman (1990) argues that outputs are positively correlated across countries, and that Solow residuals are also positively correlated across countries. Phillips (1991) uses a bivariate Markov switching process to demonstrate that worldwide shocks dominate any transmission of the business cycle. Empirical results by Della (1986, 1987) for the US, Japan, Germany, and the UK for 1960–80 suggest that common shocks rather than trade links are responsible for comovements in aggregate economic activity, investment, and trade, which are persistent over time and positively correlated across countries.

The outputs and the money supply are in a high-growth state when \( \{I_t\} = \{J_t\} = \{K_t\} = 0 \), and in a low-growth state when \( \{I_t\} = \{J_t\} = \{K_t\} = 1 \). The probability of a high-growth state in the outputs (money supply of domestic and foreign countries) in the next period, given that the outputs (money supply of domestic and foreign countries) are in a high-growth state, is \( q_i \) (\( q_j \) and \( q_k \), respectively); while the probability of a low-growth state in the outputs (money supply of domestic and foreign countries) in the
next period, given that both the outputs (money supply of domestic and foreign countries) are in a low-growth state, are \( p_i, (p_i, and p_k, respectively). Thus, we impose the restriction that all coefficients \( a_i, b_i, c_i, d_i \) on the state variables are negative. The unconditional probabilities that the outputs and the two money supplies have a high-growth state are \( (1 - p_i)(2 - p_i - q_i), (1 - p_i)(2 - p_i - q_i), \) and \( (1 - p_i)(2 - p_k - q_k), \) respectively. The model requires an estimation of the 19-dimensional vector of the parameters, \( \theta = (a_0, b_0, c_0, d_0, a_1, b_1, c_1, d_1, \sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, p_0, q_0, p_1, q_1, p_k, q_k). \)

From equation (16), we know the stochastic process of endowments and money supply in two countries. Substitution of (16) into (14) and (15) yields explicit solutions for FP and EP. FP and EP can be expressed as

\[
\frac{(F_i - S_{in})}{S_i} = \nu \cdot \Sigma \cdot \psi(I_i) \cdot \psi(J_i) \cdot \psi(K_i) - \nu \cdot \exp(A) \cdot \exp(B),
\]

\[
\frac{(F_i - S_i)}{S_i} = \nu \cdot \Sigma \cdot \psi(I_i) \cdot \psi(J_i) \cdot \psi(K_i) - 1,
\]

where \( \nu = \exp[(1 - \beta)h_i - d_i - (1 - \alpha)a_i + c_i], \)

\( \Sigma = \exp\left[\frac{1}{2} \left(1 - \beta\right)^2 \sigma_i^2 + \sigma_i^2 - (1 - \alpha)^2 \sigma_i^2 - \sigma_i^2\right], \)

\( \psi(I_i) = \left(Q_1/Q_2\right)^{1-i} \cdot \left(Q_1/Q_3\right)^{i}, \)

\( \psi(J_i) = \{q_j + (1 - q_j) \cdot \exp(-c_j)\}^{1-i} \cdot \{1 - p_j + p_j \cdot \exp(-c_j)\}^{i}, \)

\( \psi(K_i) = \{q_k + (1 - q_k) \cdot \exp(-d_k)\}^{1-k} \cdot \{1 - p_k + p_k \cdot \exp(-d_k)\}^{k}, \)

\( Q_1 = q_i + (1 - q_i) \cdot \exp[(1 - \beta)h_i], \quad Q_2 = q_i + (1 - q_i) \cdot \exp[(1 - \alpha)a_i], \)

\( Q_3 = 1 - p_i + p_i \cdot \exp[(1 - \beta)h_i], \quad Q_4 = 1 - p_i + p_i \cdot \exp[(1 - \alpha)a_i], \)

\( A = (1 - \beta)h_i - (1 - \alpha)a_i + c_i - d_i, \)

\( B = (1 - \beta) \sigma_i^2 - (1 - \alpha) \sigma_i^2 + \sigma_i^2 - \sigma_i^2. \)

FP is a function of the state variables, \( \{I_i, J_i, K_i\} \), the technology parameters, \( \theta \), and the preference parameters, \( \alpha \) and \( \beta \).

**Choosing Preference Parameters**

With given estimates \( \hat{\theta} \) in equation (16) and given some specific values of preference parameters \( \{\alpha, \beta\} \) we can obtain the implied moments in the theoretical economy, \( H_T \), which are a function of the preference and technology parameters:

\[
H_T = g(\hat{\theta}, \alpha, \beta).
\]

The final step of the common calibration method employed by Mehra and Prescott (1985) and Backus et al. (1993) is to compare the theoretical moments, \( H_T \), with the sample moments, \( Z_T \), in the estimates of \( Z \) in the sample, with the given preference parameters.

Cecchetti et al. (1993) argue that the common calibration method ignores two sources of uncertainty. The first uncertainty comes from the use of the sample moments vector, \( Z_T \), rather than the unknown population vector \( \bar{Z} \):

\[
\sqrt{T}(Z_T - \bar{Z}) \sim N(0, \Sigma_Z).
\]

The second source of uncertainty arises from the fact that the implied moments vector, \( H_T \), is stochastic and dependent on the parameter vector, \( \theta \), whose estimate is also subject to sampling error.
\[ \sqrt{T} (\hat{\theta} - \theta) \sim N(0, \Sigma_\theta). \]  

(22)

For example, Backus et al. (1993) assume that the estimates of the vector of parameters are the same as the true values of the vector of parameters: \( \hat{\theta} = \theta \). Under this assumption, they choose the preference parameters intuitively and attempt to match the risk premium in the model to that of the sample. They also ignore the uncertainty of the estimates of the risk premium in the sample. In other words, they set \( Z_T = Z \).

This approach makes it difficult for a researcher to find plausible values for the preference parameters by which the model can explain some characteristics in the sample.

We account for the uncertainty that the estimated sample moments are governed by normal distribution asymptotically, while we ignore the uncertainty that the technology parameters, \( \theta \), are asymptotically normally distributed. Thus, we set \( \theta = \hat{\theta} \), or \( H = H_T \). If the model is capable of explaining stylized facts in the sample and the specification for the stochastic process of endowments and money supplies is correct, then the estimates for the vector of seven moments in the sample, \( Z_T \), must equal those in the model, \( H_T \). Then we can obtain the estimates for the vector of the preference parameters by letting \( Z_T = H_T \). In other words, by minimizing the quadratic distance between \( Z_T \) and \( H_T \), we can obtain the best candidates of the preference parameters while accounting for \( Z \)'s uncertainty:

\[ \min_{\alpha, \beta} [Z_T - H_T(\alpha, \beta)]' \hat{\Sigma}_T [Z_T - H_T(\alpha, \beta)]. \]  

(23)

Since \( \theta \)'s uncertainty in equation (22) is ignored, the variance–covariance matrix, which measures the variation of the distance between \( Z_T \) and \( H_T \) or the spectral density at zero frequency, can be expressed as follows:

\[ \hat{\Sigma}_T = \hat{\Omega}_0 + \sum_{j=1}^{m} \omega_{jm} [\hat{\Omega}_j + \hat{\Omega}_j']. \]  

(24)

where \( \hat{\Omega}_0 = E[(1/T) \Sigma_{_{mn}}'[Z_T - E(Z_T)][Z_T - E(Z_T)]] \), \( \hat{\Omega}_j = E[(1/T) \Sigma_{_{mn}}'[Z_T - E(Z_T)][Z_T - E(Z_T)]] \), and \( \omega_{jm} = (1 + m - j)/(1 + m) \) and \( m \) is the Newey and West (1987) lag. Then the value of the objective function times the number of observations follows asymptotically \( \chi^2 \)-distribution with \( k - n \) degrees of freedom, where \( k \) is the dimensionality of vector \( H \) and \( n \) is the number of the preference parameters. Thus, in this case, the assumption \( H = Z \) can be tested. The null hypothesis is \( H_0: H = Z \) and tested by using a two-tailed test.

**Monte Carlo Experiments**

We have explained how to obtain the best candidates for the preference parameters and how to test the null hypothesis \( H_0: H = Z \). This test is valid only when the true technology parameters \( \theta \) are equal to the estimates \( \hat{\theta} \). Since the test is based on the asymptotic theory, we do not know the small-sample properties of FP and EP in the model. In order to answer the question, Monte Carlo studies are performed to obtain the empirical distributions of the small-sample vector \( H_T \) and to compare the sample moments \( Z_T \) with the empirical distributions of \( H_T \).

The Monte Carlo studies for the univariate case in this paper have the following steps. First, the artificial FP and EP are generated with given \( \hat{\theta}, \hat{\alpha}, \) and \( \hat{\beta} \). In this step, we assume that \( \hat{\theta} = \theta \). Next, the empirical distribution of each theoretical moment is constructed. Finally, the sample value is compared with the empirical distribution of
each theoretical moment. Thus, we can obtain the small-sample properties of each theoretical moment.

Even though each moment in the model can be matched with that in the sample from the univariate test, it is difficult to judge only by the univariate results whether the model can explain some stylized facts in the sample or not. Hence, we perform the multivariate test also. The multidimensional test is based on the summary measure, which can be defined as

$$
\zeta = [Z_T - E(H_n)]\Sigma^{-1}[Z_T - E(H_n)],
$$

(25)

where $$\Sigma = (1/N)\Sigma_{t=0}^{N-1} [H_t - E(H_t)][H_t - E(H_t)]'$$, $$Z_T$$ is a column vector of estimates for the sample moments, $$H_n$$ is a column vector of theoretical moments from the Monte Carlo studies, and $$N$$ is the number of replications in the Monte Carlo studies.

The asymptotic distribution of $$\zeta$$ is a $$\chi^2$$-distribution with $$k$$ degrees of freedom, where $$k$$ is the dimensionality of the vector $$Z_T$$. In a small sample, we cannot be sure that $$\zeta$$ is distributed as a $$\chi^2$$-distribution. Thus, we construct the sampling distribution of $$\zeta$$ using Monte Carlo techniques.

5. Estimation and Monte Carlo Studies

Estimation Results of Technology and Preference Parameters

The maximum-likelihood estimation results of the Markov switching model and some summary statistics are reported in Table 2. We used quarterly per capita real GNP (GDP) and quasi-money taken from the OECD's Main Economic Indicators database.7

Except the estimate of $$\sigma_\omega$$, all estimates are statistically significant at the 5% level. When the economy is in a boom one quarter, the estimated probability that it continues in a boom next quarter is $$q_t$$. This is estimated to be 0.898, 0.907, 0.944, and 0.957 for the cases of $$/C$$, $$$/F$$, $$$/DM$$ and $$$/Y$$, respectively. Despite the assumption that the outputs in two countries have the same Markov state variable $$\{I_t\}$$, the output growth in the US for Canada is slightly different from that for the UK or Germany both in boom and recession periods. Table 2 also reports the unconditional probability of observing boom states. Pr $$[I_t = 0]$$ are 0.629, 0.641, and 0.663 for the cases of $$$/C$$, $$$/F$$, and $$}$/DM$$, respectively, while it is 0.877 for $$}$/Y$$.

Table 3 shows the estimation results of preference parameters. Case I in Table 3 gives the estimation results when the weighted quadratic distance is minimized between all seven moments in a theoretical economy and those in a sample, while case II shows estimates of the preference parameters when only the variances of FP and EP are used. The first and second columns show the estimates for $$\alpha$$ and $$\beta$$ that are the constant relative risk-aversion parameters in the consumption of the domestic and foreign goods, respectively. The third column is the $$\chi^2$$-statistics under the null hypothesis that the vector of sample moments is equal to the theoretical vector of moments: $$Z = H$$. The fourth column gives the $$p$$-values of the $$\chi^2$$-statistics.8

In case II, the estimates for $$\alpha$$ and $$\beta$$ lie between 0 and 3, with none being significantly different from zero. Furthermore, the $$\chi^2$$-statistics are very large so that the over-identifying restrictions cannot be accepted. The Newey–West lag is 11. When smaller or larger lags are considered, the results do not significantly change. From the estimation results in case I, it is hard to find a set of preference parameters with which seven moments in a theoretical economy are close to those in the sample. Thus we consider only two moments, such as the variances of FP and EP.
Table 2. MLE of Multivariate Markov Switching Process

**Endowments**

\[
\begin{align*}
\ln(\xi_{t+1}) &= \ln(\xi_t) + a_0 + a_t I_{t+1} + \varepsilon_{1t+1}^1 \\
\ln(\eta_{t+1}) &= \ln(\eta_t) + b_0 + b_t I_{t+1} + \varepsilon_{2t+1}^2 \\
\begin{pmatrix} \varepsilon^1 & \varepsilon^2 \end{pmatrix} &\sim N(0, \Sigma), \Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix} \\
Pr[I_{t+1} = 0 | I_t = 0] &= q_t, \quad Pr[I_{t+1} = 1 | I_t = 0] = 1 - q_t \\
Pr[I_{t+1} = 0 | I_t = 1] &= 1 - p_t, \quad Pr[I_{t+1} = 1 | I_t = 1] = p_t
\end{align*}
\]

| \( \theta \) | $/CS$ | $$/E$$ | $$/DM$$ | $$/V$$ |
|---|---|---|---|
| \( a_0 \) | 0.790 (0.169) | 0.765 (0.177) | 0.795 (0.158) | 0.507 (0.137) |
| \( a_1 \) | -1.547 (0.280) | -1.544 (0.294) | -1.489 (0.318) | -2.031 (0.389) |
| \( b_0 \) | 0.688 (0.193) | 0.757 (0.263) | 0.919 (0.184) | 0.853 (0.136) |
| \( b_1 \) | -1.226 (0.332) | -0.804 (0.495) | -1.140 (0.334) | -1.361 (0.467) |
| \( q \) | 0.898 (0.056) | 0.907 (0.052) | 0.944 (0.047) | 0.957 (0.033) |
| \( p \) | 0.827 (0.123) | 0.834 (0.121) | 0.890 (0.010) | 0.694 (0.183) |
| \( \sigma_1 \) | 0.906 (0.090) | 0.913 (0.092) | 0.975 (0.101) | 0.952 (0.089) |
| \( \sigma_2 \) | 1.149 (0.103) | 1.549 (0.135) | 1.115 (0.105) | 1.040 (0.090) |
| \( \sigma_{12} \) | 0.342 (0.148) | 0.117 (0.202) | -0.033 (0.153) | 0.104 (0.124) |
| \( \Pr [I_t = 0] \) | 0.629 | 0.641 | 0.663 | 0.877 |

**Money supplies**

\[
\begin{align*}
\ln(M_{t+1}) &= \ln(M_t) + c_0 + c_t J_{t+1} + \varepsilon_{3t+1}^3 \\
\ln(N_{t+1}) &= \ln(N_t) + d_0 + d_t K_{t+1} + \varepsilon_{4t+1}^4 \\
\begin{pmatrix} \varepsilon^3 & \varepsilon^4 \end{pmatrix} &\sim N(0, \Sigma), \Sigma = \begin{pmatrix} \sigma_1^3 & 0 \\ 0 & \sigma_2^4 \end{pmatrix} \\
Pr[J_{t+1} = 0 | J_t = 0] &= q_J, \quad Pr[J_{t+1} = 1 | J_t = 0] = 1 - q_J \\
Pr[J_{t+1} = 0 | J_t = 1] &= 1 - p_J, \quad Pr[J_{t+1} = 1 | J_t = 1] = p_J \\
Pr[K_{t+1} = 0 | K_t = 0] &= q_K, \quad Pr[K_{t+1} = 1 | K_t = 0] = 1 - q_K \\
Pr[K_{t+1} = 0 | K_t = 1] &= 1 - p_K, \quad Pr[K_{t+1} = 1 | K_t = 1] = p_K
\end{align*}
\]

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>US</th>
<th>Canada</th>
<th>UK</th>
<th>Germany</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_0 )</td>
<td>2.192 (0.094)</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>( c_1 )</td>
<td>-1.511 (0.176)</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>( d_0 )</td>
<td>—</td>
<td>3.579 (0.250)</td>
<td>3.405 (0.135)</td>
<td>1.856 (0.193)</td>
<td>2.275 (0.078)</td>
</tr>
<tr>
<td>( d_1 )</td>
<td>—</td>
<td>-2.312 (0.323)</td>
<td>-1.814 (0.613)</td>
<td>-1.992 (0.878)</td>
<td>-2.750 (0.337)</td>
</tr>
<tr>
<td>( q_J )</td>
<td>0.987 (0.015)</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>( p_J )</td>
<td>0.979 (0.028)</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>( q_K )</td>
<td>—</td>
<td>0.982 (0.022)</td>
<td>0.983 (0.021)</td>
<td>0.979 (0.030)</td>
<td>0.971 (0.020)</td>
</tr>
<tr>
<td>( p_K )</td>
<td>—</td>
<td>0.986 (0.017)</td>
<td>0.929 (0.128)</td>
<td>0.870 (0.224)</td>
<td>0.639 (0.273)</td>
</tr>
<tr>
<td>( \sigma_1 )</td>
<td>0.659 (0.056)</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>( \sigma_2 )</td>
<td>—</td>
<td>1.329 (0.114)</td>
<td>0.988 (0.085)</td>
<td>1.227 (0.114)</td>
<td>0.637 (0.054)</td>
</tr>
<tr>
<td>( \Pr [J_t = 0] )</td>
<td>0.618</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>( \Pr [K_t = 0] )</td>
<td>—</td>
<td>0.438</td>
<td>0.807</td>
<td>0.861</td>
<td>0.926</td>
</tr>
</tbody>
</table>

Standard errors are in parentheses.

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Table 3. Estimation Results of Preference Parameters

\[ U = \frac{X_a^{1-\alpha} + Y_a^{1-\beta}}{1-\alpha + 1-\beta} \]

<table>
<thead>
<tr>
<th>Cases</th>
<th>(\alpha)</th>
<th>(\beta)</th>
<th>(\chi^2)-stat</th>
<th>(p)-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$/C$</td>
<td>0.238389 (1.606956)</td>
<td>1.48E-10 (0.025706)</td>
<td>9816</td>
<td>0.00</td>
</tr>
<tr>
<td>$/E$</td>
<td>0.556286 (5.474868)</td>
<td>3.10E-11 (0.883665)</td>
<td>90.07</td>
<td>0.00</td>
</tr>
<tr>
<td>$/D$</td>
<td>0.425769 (3.963660)</td>
<td>5.44E-11 (0.061155)</td>
<td>521.0</td>
<td>0.00</td>
</tr>
<tr>
<td>$/Y$</td>
<td>2.716827 (1.862638)</td>
<td>1.193489 (0.883467)</td>
<td>120.7</td>
<td>0.00</td>
</tr>
<tr>
<td>Case II:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$/C$</td>
<td>1.012932 (9.581942)</td>
<td>1.032472 (9.909386)</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>$/E$</td>
<td>2.866000 (1.977784)</td>
<td>4.597013 (1.592791)</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>$/D$</td>
<td>4.232747 (1.260651)</td>
<td>5.232932 (1.586298)</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>$/Y$</td>
<td>6.069471 (3.006136)</td>
<td>5.256795 (6.595351)</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

Notes: Case I reports the estimation result when all seven moments are used. They are the means, variances, the first autocorrelations of FP and EP and \(b_t\) in equation (1). Case II shows the estimation result when variances of FP and EP are used. The number of Newey-West (1987) lags is 11 and the estimates and standard errors of preference parameters \((\alpha, \beta)\) are not very dependent upon the number of Newey-West lags. The standard errors are in parentheses and the \(\chi^2\)-statistic has five degrees of freedom.

In case II, the estimates of preference parameters seem to differ from those in case I. The magnitude of estimates becomes larger so that the estimates for \(\alpha\) and \(\beta\) lay between one and six. Some estimates become statistically significant at the 5% level.

Only for $/D$ in case II, the estimates of both \(\alpha\) and \(\beta\) are significant at the 5% level. However, the bad estimation results do not at all imply that a theoretical economy cannot explain stylized facts in foreign exchange markets. We can obtain implied moments in the theoretical economy with the estimates of the preference parameters in Table 3, which are reported in Table 4.

Results of Monte Carlo Studies

From Table 3, we obtained two possible candidates of the preference parameters. We now examine whether the theoretical and sample moments come from the same underlying population by employing Monte Carlo studies.

Table 4 shows the results. The first column is the sample moments. The second and third columns are the implied moments in cases I and II, respectively. The values in the fourth and fifth columns are the medians of the empirical distributions of the Monte Carlo studies for cases I and II, respectively. The number in parentheses is the probability value. If its value is less than 0.025 or greater than 0.975, the theoretical economy cannot explain a stylized fact in the sample at the 5% significance level.

The implied values of the variance of EP in case I seem to be far from the sample values because the sample values are out of the empirical distributions except for $/C$. However, with case II, the results are dramatically changed. For all exchange rates except for $/C$, the variances of EP in the sample lie inside the empirical distributions even at the 20% level. Surprisingly, the implied values of the variance of EP in case II
### Table 4. Univariate Test

<table>
<thead>
<tr>
<th>Moments</th>
<th>Sample</th>
<th>Case I</th>
<th>Case II</th>
<th>Case I</th>
<th>Case II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$/CS</td>
<td>$E(\text{EP})$</td>
<td>-0.00097</td>
<td>-0.00007</td>
<td>-0.00047</td>
<td>-0.00002 (0.674)</td>
</tr>
<tr>
<td></td>
<td>$V(\text{EP})$</td>
<td>0.00045</td>
<td>0.00036</td>
<td>0.00023</td>
<td>0.00033 (0.027)</td>
</tr>
<tr>
<td></td>
<td>$AR_{\text{EP}}(1)$</td>
<td>0.18843</td>
<td>-1.0E-06</td>
<td>5.2E-07</td>
<td>-0.01638 (0.047)</td>
</tr>
<tr>
<td></td>
<td>$b_{1}$</td>
<td>1.66689</td>
<td>-0.00003</td>
<td>-1.2E-05</td>
<td>0.04323 (0.053)</td>
</tr>
<tr>
<td></td>
<td>$E(\text{FP})$</td>
<td>-0.00325</td>
<td>-0.06618</td>
<td>-0.00695</td>
<td>-0.00554 (0.409)</td>
</tr>
<tr>
<td></td>
<td>$V(\text{FP})$</td>
<td>1.6E-05</td>
<td>0.00017</td>
<td>1.7E-04</td>
<td>6.8E-05 (0.845)</td>
</tr>
<tr>
<td></td>
<td>$AR_{\text{FP}}(1)$</td>
<td>0.70936</td>
<td>0.96793</td>
<td>0.96797</td>
<td>0.90451 (0.842)</td>
</tr>
<tr>
<td>$/£</td>
<td>$E(\text{EP})$</td>
<td>-0.00509</td>
<td>-5.6E-05</td>
<td>-0.0025</td>
<td>6.3E-06 (0.986)</td>
</tr>
<tr>
<td></td>
<td>$V(\text{EP})$</td>
<td>0.00344</td>
<td>0.00039</td>
<td>0.00323</td>
<td>0.00037 (0.000)</td>
</tr>
<tr>
<td></td>
<td>$AR_{\text{EP}}(1)$</td>
<td>0.26359</td>
<td>7.6E-07</td>
<td>7.5E-07</td>
<td>-0.01500 (0.012)</td>
</tr>
<tr>
<td></td>
<td>$b_{1}$</td>
<td>2.53954</td>
<td>-6.3E-06</td>
<td>-0.00019</td>
<td>0.04474 (0.280)</td>
</tr>
<tr>
<td></td>
<td>$E(\text{FP})$</td>
<td>-0.00689</td>
<td>-0.01036</td>
<td>-0.02547</td>
<td>-0.00959 (0.372)</td>
</tr>
<tr>
<td></td>
<td>$V(\text{FP})$</td>
<td>5.8E-05</td>
<td>9.2E-05</td>
<td>8.9E-05</td>
<td>5.0E-05 (0.372)</td>
</tr>
<tr>
<td></td>
<td>$AR_{\text{FP}}(1)$</td>
<td>0.71657</td>
<td>0.94088</td>
<td>0.94126</td>
<td>0.86567 (0.821)</td>
</tr>
<tr>
<td>$/DM</td>
<td>$E(\text{EP})$</td>
<td>0.00088</td>
<td>-8.1E-05</td>
<td>-0.00107</td>
<td>-6.4E-06 (0.351)</td>
</tr>
<tr>
<td></td>
<td>$V(\text{EP})$</td>
<td>0.00386</td>
<td>0.00038</td>
<td>0.00345</td>
<td>0.00037 (0.000)</td>
</tr>
<tr>
<td></td>
<td>$AR_{\text{EP}}(1)$</td>
<td>0.20521</td>
<td>2.7E-06</td>
<td>4.5E-06</td>
<td>-0.01420 (0.037)</td>
</tr>
<tr>
<td></td>
<td>$b_{1}$</td>
<td>2.69026</td>
<td>-1.7E-05</td>
<td>-0.00101</td>
<td>0.05600 (0.014)</td>
</tr>
<tr>
<td></td>
<td>$E(\text{FP})$</td>
<td>0.00915</td>
<td>0.00042</td>
<td>0.01194</td>
<td>0.00529 (0.203)</td>
</tr>
<tr>
<td></td>
<td>$V(\text{FP})$</td>
<td>2.6E-05</td>
<td>8.7E-05</td>
<td>8.3E-05</td>
<td>5.0E-05 (0.761)</td>
</tr>
<tr>
<td></td>
<td>$AR_{\text{FP}}(1)$</td>
<td>0.45659</td>
<td>0.91681</td>
<td>0.91779</td>
<td>0.83134 (0.940)</td>
</tr>
<tr>
<td>$/¥</td>
<td>$E(\text{EP})$</td>
<td>-0.00711</td>
<td>-0.00038</td>
<td>-0.00239</td>
<td>-0.00040 (0.982)</td>
</tr>
<tr>
<td></td>
<td>$V(\text{EP})$</td>
<td>0.00393</td>
<td>0.00045</td>
<td>0.00394</td>
<td>0.00044 (0.000)</td>
</tr>
<tr>
<td></td>
<td>$AR_{\text{EP}}(1)$</td>
<td>0.25450</td>
<td>-0.00077</td>
<td>-0.00044</td>
<td>-0.01419 (0.014)</td>
</tr>
<tr>
<td></td>
<td>$b_{1}$</td>
<td>1.51610</td>
<td>0.00332</td>
<td>0.01238</td>
<td>0.06252 (0.004)</td>
</tr>
<tr>
<td></td>
<td>$E(\text{FP})$</td>
<td>0.00580</td>
<td>-0.00151</td>
<td>-0.02073</td>
<td>-0.00711 (0.025)</td>
</tr>
<tr>
<td></td>
<td>$V(\text{FP})$</td>
<td>0.00016</td>
<td>0.00012</td>
<td>0.00016</td>
<td>8.2E-05 (0.038)</td>
</tr>
<tr>
<td></td>
<td>$AR_{\text{FP}}(1)$</td>
<td>0.59940</td>
<td>0.77913</td>
<td>0.74257</td>
<td>0.68293 (0.732)</td>
</tr>
</tbody>
</table>

Notes: The p-values are in parentheses. If its value is less than 0.025 or greater than 0.975, then the null hypothesis that a sample moment is equal to a median value from the Monte Carlo study is rejected at the 5% significance level.

are extremely close to those in the sample. Thus, this may indicate that the theoretical economy can reliably explain the variability of EP. It is natural because the preference parameters in case II are chosen to match the variances of EP and FP.

The theoretical economy can sufficiently explain the unconditional means and the first autocorrelations of EP in the sample with different sets of preference parameters. For all exchange rates, the unconditional means and the first autocorrelation of EP in the sample fall inside the empirical distributions.

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With the estimates of the preference parameters for case I, the unconditional means of the theoretical FP are quite close to the sample values. On the contrary, the means of FP in the sample are outside the empirical distributions for case II except for $$/C$$. However, the first autocorrelations of the theoretical FP are not significantly different from those in the sample even at the 10% level for all cases.

The most interesting stylized fact in the foreign exchange market is that the OLS coefficient $b_1$ is greater than one, which implies the negative covariance between FP and DR. Surprisingly, the $b_1$ in the sample lay inside the empirical distributions at the 5% significance level for all cases and all exchange rates. However, for all cases, the implied values of the OLS coefficient $b_1$ are close to zero and the median values from the empirical distributions are far from one. Therefore, these results may be interpreted as evidence that the theoretical economy shows the potential to explain the negative covariance between FP and DR in the sample. The evidence against the unbiasedness hypothesis is not overwhelmed by the model because the model shows a very large variation.

Table 5 provides the results of various multidimensional tests. Four groups of multivariate tests are performed; each group consists of two multivariate tests with different preference parameters. Case A tests whether the model can explain the joint behavior of all seven moments in the sample. Cases B and C present the $\zeta$-statistics when we use the means, the variances, and the first autocorrelations of EP and FP, respectively. Case D tests whether the model can explain the joint behavior of three sample moments (the variance of EP, $b_1$, and the first autocorrelation of FP), which are the most interesting moments in this field.

When we consider all seven moments, except for the $$/C$$ in case I and the $$/£$$ in case II, the joint statistics are high enough to reject the null hypothesis that all seven moments in the model are equal to those in the sample. According to the results of case B, the model can explain the behavior of EP, which can be divided by the risk premium and the rational-expectation error. From the results of case C, the model seems to explain the joint behavior of FP better for case I than case II. According to the results of case D, we may conclude that the variance of EP, $b_1$, and the first autocorrelation of FP can be jointly found in the model.

<table>
<thead>
<tr>
<th>Moments</th>
<th>Parameters</th>
<th>$$/C$$</th>
<th>$$/£$$</th>
<th>$$/DM$$</th>
<th>$$/¥$$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case A</td>
<td>Case I</td>
<td>9.35 (0.23)</td>
<td>2.162 (0.00)</td>
<td>2.589 (0.00)</td>
<td>2.020 (0.00)</td>
</tr>
<tr>
<td></td>
<td>Case II</td>
<td>40.77 (0.00)</td>
<td>15.74 (0.03)</td>
<td>18.83 (0.01)</td>
<td>34.20 (0.00)</td>
</tr>
<tr>
<td>Case B</td>
<td>Case I</td>
<td>7.56 (0.06)</td>
<td>2.135 (0.00)</td>
<td>2.539 (0.00)</td>
<td>1.787 (0.00)</td>
</tr>
<tr>
<td></td>
<td>Case II</td>
<td>37.53 (0.00)</td>
<td>6.18 (0.10)</td>
<td>3.84 (0.28)</td>
<td>6.64 (0.08)</td>
</tr>
<tr>
<td>Case C</td>
<td>Case I</td>
<td>1.18 (0.76)</td>
<td>0.57 (0.90)</td>
<td>3.83 (0.28)</td>
<td>8.69 (0.03)</td>
</tr>
<tr>
<td></td>
<td>Case II</td>
<td>1.24 (0.74)</td>
<td>10.07 (0.02)</td>
<td>15.28 (0.02)</td>
<td>28.53 (0.00)</td>
</tr>
<tr>
<td>Case D</td>
<td>Case I</td>
<td>4.83 (0.18)</td>
<td>2.135 (0.00)</td>
<td>2.528 (0.00)</td>
<td>1.794 (0.00)</td>
</tr>
<tr>
<td></td>
<td>Case II</td>
<td>35.25 (0.00)</td>
<td>0.34 (0.95)</td>
<td>1.26 (0.73)</td>
<td>0.86 (0.83)</td>
</tr>
</tbody>
</table>

Notes: Case A is the $\zeta$-statistic when all seven moments are used. Cases B and C are the $\zeta$-statistics when the means, variances, and the first autocorrelations of EP and FP, respectively, are used. Case D is the $\zeta$-statistic when the variance of EP, $b_1$, and the first autocorrelation of EP is used. The asymptotic distribution of $\zeta$-statistic in case A is a $\chi^2$-distribution with seven degrees of freedom, while those in cases B, C, and D are with three degrees of freedom. The probability values are in parentheses. If its value is less than 0.025 or greater than 0.975, the null hypothesis is rejected at the 5% significance level.
6. The Feature of the Theoretical Economy and Conclusion

This paper demonstrates that the simple general equilibrium model, where an economic agent has time-separable utility and the changes in the logarithms of money supply and endowments follow a multivariate Markov switching process, shows the potential to explain some stylized facts in foreign exchange markets. This simple theoretical model can explain some behaviors of EP and FP in the foreign exchange market.

We considered four exchange rates—$/C$, $$/E$, $$/DM$, and $$/Y—with seven moments; the means, the variances, and the first autocorrelation of EP and of FP, and the OLS coefficient $b_1$. When outputs are governed by a bivariate Markov switching process with the common Markov state variable while two money supplies are governed by an independent univariate Markov switching process, we find that the values of the preference parameters lie between zero and six.

With the estimates of the preference parameters, we are able to match some sample moments with a model individually. Specifically, the excess volatility of EP in the sample can be explained very well in the model. Other stylized facts, such as the strong autocorrelation and the small variability of FP, can be explained theoretically and quantitatively by the model. According to the Monte Carlo studies, the OLS coefficients, $b_1$, in the sample lie inside the empirical distribution for all cases, but the implied and the median values of $b_1$ in the model are close to zero. Thus, it is difficult to say that the negative covariance between FP and DR is perfectly explained by the model. The above results, however, are surprising compared with the disappointing results of Backus et al. (1993) and Bekker (1994).

There are two natural questions which we must answer. How did we succeed in matching the variability of EP with that of the sample? Even though we succeeded in matching the negative covariance between FP and DR with that of the sample, why did we fail to derive the implied negative covariance from the model?

The possible reasons for our success are as follows. First, a Markov switching process seems to reflect very well on the behavior of exogenous processes. Backus et al. assume that the national income and money supply in two countries are governed by a Markov process. The assumption that the national income is governed by a Markov process or by a random walk may not be acceptable. Bekker assumes that exogenous processes are governed by a VAR; however, a VAR specification may not reflect on structural changes or regime shifts. Thus, our specification may reflect much more correctly on the behavior of exogenous processes.

Second, with a Markov switching process, our utility specification, which is commonly used in international finance, seems to help us obtain some successful results. By the multivariate Markov switching model and the time-separable utility specification, the characteristics of FP and EP in the theoretical economy are almost built. As we find in Table 3, the strong autocorrelation of FP is generally found in the model and it does not seem to be dependent on the preference parameters. This may occur because the theoretical FP is a function of the Markov state variables. If the transition probabilities, $p$ and $q$, are close to one, then the model may generate a similar artificial FP at every occurrence. Therefore, the first autocorrelation of the theoretical FP must be high and its variance must be small. Furthermore, FP has only three sources of variability, such as the Markov state variables, $[I, J, K]$; while EP has four sources of variability, such as the unexpected random errors, $e$, and the Markov state variables. Thus, it may be natural for EP to show a much higher variability than FP.
Third, the strategy of choosing the pairs of preference parameters is more carefully devised. To match the variability of EP, we used the preference parameters that were obtained by minimizing the quadratic distance between the variances of FP and EP in the sample and those in the theoretical economy.

Some plausible reasons for why our model fails to explain all the joint behaviors of FP and EP and to derive the implied negative covariance between FP and DR may include the following. There are at least three possible reasons. The first two reasons come from a feature of Lucas’s model. In order to derive the final reduced form of the forward and spot exchange rates in equations (14) and (15), we assume the perfect pooling equilibrium. However, that condition causes the perfect correlation of consumption between two countries, which may not be realistic at all. Furthermore, it implies that there is a perfect capital mobility between two countries according to the definition by Obstfeld (1994). The perfect-capital-mobility assumption may not be in accordance with the negative correlation between FP and DR. The relationship between the perfect consumption correlation and the uncovered interest parity has yet to be studied. However, both the perfect consumption correlation and the uncovered interest parity are possible definitions of the perfect capital mobility. Thus, there is some possibility that the perfect consumption correlation causes the positive correlation between the interest differential and DR. If the above conjecture is reasonable, the perfect pooling equilibrium condition results in the non-negative correlation between FP and DR.

As Hodrick (1987, p. 10) and Svensson (1985) argue, the exchange determination rule is much too simple. Since the spot exchange rates in the Lucas model depend only on time t information, there is no forward-looking component of the exchange rates.

The final reason comes from the specification of exogenous processes. A Markov switching process is certainly a good stochastic process that reflects the behaviors of endowments and money supplies. However, the strong assumption that three Markov state variables are independent is not realistic.

Therefore, in order to derive the negative covariance between FP and DR from the theoretical economy, a well-known puzzle in international finance, further in-depth research in this field is necessary.

References


Notes


2. According to Mehra and Prescott (1985), the admissible range of preference parameters is between 0 and 10.

3. For the DM, there are 64 observations covering the period from 1974Q1 to 1988Q4. Since the Berlin Wall collapsed in 1989, we did not use the forward and spot exchange rates after 1989.

4. Since the estimate of \( b \) is significantly different from zero, this stylized fact seems to indicate that FP gives information of forecasting the future spot rate. Bekaert and Hodrick (1992) find that FP has power for forecasting EP. They show that a 1% increase in FP is associated with a 6–8% decrease in EP at the one-month horizon.

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5. Except the unconditional means, they are not moments. However, it is hard to find any appropriate name for them. Thus, we will call them "moments" in the sense that they are a function of moments of (between) FP and EP.
6. Since the estimates for the preference parameters were obtained by minimizing the quadratic distance between the moments in the sample and those in the model, the above approach can be regarded as a variation of the generalized method of moments (GMM).
7. To calculate per capital real GNP (GDP), I used consumer price indices and population data taken from the OECD's Main Economic Indicators.
8. In case II, there is no $\chi^2$-statistic and $p$-value because the system is just identified.
9. Since we assume that the covered interest parity holds exactly in equation (13), the negative covariance between FP and DR implies the rejection of the uncovered interest parity.