For each problem, you need to choose a correct answer among 5 given answers.

1. Once a fire is reported to a fire insurance company, the company makes an initial estimate, $X$, of the amount it will pay to the claimant for the fire loss. When the claim is finally settled, the company pays an amount $Y$, to the claimant. The company determined that $X$ and $Y$ have the joint density function

$$f(x, y) = \frac{2}{x^2(x-1)}y^{-(2x-1)/(x-1)}I(x > 1)I(y > 1).$$

Given that the initial claim estimated by the company is 2, determine the probability that the final settlement amount is between 1 and 3.


2. An auto insurance policy will pay for damage to both the policyholder’s car and the other driver’s car in the event that the policyholder is responsible for an accident. The size of the payment for the damage to the policyholder’s car, $X$, has the marginal density function of 1 for $0 < x < 1$. Given $X = x$, the size of the payment for damage to the other driver’s car, $Y$, has the conditional density of 1 for $x < y < x + 1$. If the policyholder is responsible for an accident, what is the probability that the payment for damage to the other driver’s car will be greater than 0.5?


3. The future lifetimes (in months) of two components of a machine have the following joint density function:

$$f(x, y) = \frac{6}{125,000}(50 - x - y)I(0 < x < 50 - y < 50).$$

What is the probability that both components are still functioning 20 months from now?

[A] $\int_0^{20} \int_0^{20} (50 - x - y)dydx$

[B] $\int_{20}^{50} \int_{20}^{50-x} (50 - x - y)dydx$

[C] $\int_{20}^{50} \int_{20}^{50-x} (50 - x - y)dydx$

[D] $\int_{20}^{50} \int_{20}^{50-x} (50 - x - y)dydx$

[E] $\int_{20}^{50} \int_{20}^{50-x} (50 - x - y)dydx$
4. Let $X$ and $Y$ be continuous random variables with joint density function

$$f(x, y) = 15y I(x^2 \leq y \leq x).$$

Let $g$ be the marginal density function of $Y$. Which of the following represents $g$?

- [A] $g(y) = 15y I(0 \leq y \leq 1)$
- [B] $g(y) = (15y^2/2) I(x^2 \leq y \leq x)$
- [C] $g(y) = (15y^2/2) I(0 \leq y \leq 1)$
- [D] $g(y) = 15y^{3/2}(1 - y^{1/2}) I(x^2 \leq y \leq x)$
- [E] $g(y) = 15y^{3/2}(1 - y^{1/2}) I(0 \leq y \leq 1)$

5. An insurance company insures a large number of drivers. Let $X$ be the random variable representing the company’s losses under collision insurance, and let $Y$ represent the company’s losses under liability insurance. $X$ and $Y$ have the joint density function

$$f(x, y) = \frac{1}{4}(2x + 2 - y) I(0 < a < 1) I(0 < y < 2).$$

What is the probability that the total loss is at least 1?

- [A] .33
- [B] .38
- [C] .41
- [D] .71
- [E] .75

6. Let $X$ and $Y$ be continuous random variables with joint density function

$$f(x, y) = 24xy I(0 < x < 1) I(0 \leq y \leq 1 - x).$$

Calculate $P(Y < X | X = 1/3)$.

- [A] $\frac{1}{27}$
- [B] $\frac{2}{27}$
- [C] $\frac{1}{4}$
- [D] $\frac{1}{3}$
- [E] $\frac{4}{9}$

7. A device contains two components. The device fails if either component fails. The joint density function of the lifetimes of the components, measured in hours, is $f(s, t)$, where $0 < s < 1$ and $0 < t < 1$. What is the probability that the device fails during the first half hour of operation?

- [A] $\int_0^{0.5} \int_0^{0.5} f(s, t) ds dt$
- [B] $\int_0^{1} \int_0^{0.5} f(s, t) ds dt$
- [C] $\int_0^{1} \int_0^{0.5} f(s, t) ds dt$
- [D] $\int_0^{0.5} \int_0^{0.5} f(s, t) ds dt + \int_0^{1} \int_0^{0.5} f(s, t) ds dt$
- [E] $\int_0^{0.5} \int_0^{0.5} f(s, t) ds dt + \int_0^{0.5} \int_0^{0.5} f(s, t) ds dt$

8. A company offers a basic life insurance policy to its employees, as well as a supplemental life insurance policy. To purchase the supplemental policy, an employee must first purchase the basic policy. Let $X$ denote the proportion of employees who purchase the basic policy, and $Y$ the proportion of employees who purchase the supplemental policy. Let $X$ and $Y$ have the joint density $f(x, y) = 2(x + y) I(x + y > 0)$. Given that 10% of the employees buy the basic policy, what is the probability that fewer than 5% buy the supplemental policy?

- [A] 0.01
- [B] 0.013
- [C] 0.108
- [D] 0.417
- [E] 0.500
9. A company is reviewing tornado damage claims under a farm insurance policy. Let \( X \) be the portion of a claim representing damage to the house and let \( Y \) be the portion of a claim representing damage to the rest of the property. The joint density function of \( X \) and \( Y \) is
\[
f(x, y) = 6[1 - (x + y)]I(x > 0)I(y > 0)I(x + y < 1).
\]
determine the probability that the portion of a claim representing damage to the house is less than .2.

[A] 0.36  [B] 0.48  [C] 0.488  [D] 0.512  [E] 0.520

10. The distribution of Smith’s future lifetime is \( X \), an exponential random variable with mean \( \alpha \), and the distribution of Brown’s future lifetime is \( Y \), an exponential random variable with mean \( \beta \). Smith and Brown have future lifetimes that are independent of each other. Find the probability that Smith outlives Brown.

[A] \( \alpha/(\alpha + \beta) \)  [B] \( \beta/(\alpha + \beta) \)  [C] \( (\alpha - \beta)/\alpha \)  [D] \( (\beta - \alpha)/\beta \)  [E] \( \alpha/\beta \)