1. Consider a Binomial random variable \( X \sim Binom(\theta, n) \). Consider a Bayes approach with \( Beta(a, b) \) prior. Consider a squared loss function and find the Bayes estimator of \( g(\theta) = \theta(1 - \theta) \), that is, consider the case of the variance being the estimand.

2. For the same setting as in Problem 1, compare the Bayes estimator with the UMVU estimator.

3. Let \( X_1, \ldots, X_n \) be iid according to the Poisson distribution \( P(\lambda) \), and consider a Bayes approach with \( \Lambda \) having a gamma distribution \( Gamma(g, 1/\alpha) \). [Here \( g \) is the shape parameter and \( 1/\alpha \) is the scale parameter, that is \( p_\lambda(\lambda) = \alpha^g \lambda^{g-1} e^{-\lambda}/\Gamma(g) \).

(a) For squared error loss, find the Bayes estimator.

(b) What happens to the Bayes estimator as: (i) \( n \to \infty \); (ii) \( \alpha \to \infty \), \( g \to 0 \), or both?

4. Let \( X_1, \ldots, X_n \) be iid according to \( N(0, \sigma^2) \). Set \( \tau = 1/2\sigma^2 \). As conjugate prior for \( \tau \), take the gamma density \( Gamma(g, 1/\alpha) \). Find the Bayes estimator of \( \sigma^2 \) for the squared loss and the scale-equivariant loss \((\delta - \sigma^2)/\sigma^4 \).

Hint: Note that
\[
E(\tau) = g/\alpha; \quad E(\tau^2) = g(g+1)/\alpha^2; \\
E(1/\tau) = \alpha/(g-1); \quad E(1/\tau^2) = \frac{\alpha}{(g-1)(g-2)}.
\]

5. Let \( X \) and \( Y \) be independently distributed according to distributions \( P_\xi \) and \( P_\eta \), respectively. Suppose that \( \xi \) and \( \eta \) are real-valued and independent according to some prior distributions \( \Lambda \) and \( \Lambda' \). If, with squared error loss, \( \delta_\Lambda \) is the Bayes estimator of \( \xi \) on the basis of \( X \), and \( \delta'_{\Lambda'} \) is the Bayes estimator of \( \eta \) on the basis of \( Y \),

(a) show that \( \delta'_{\Lambda'} - \delta_\Lambda \) is the Bayes estimator of \( \eta - \xi \) on the basis of \( (X, Y) \).

(b) if \( \eta > 0 \) and \( \delta'_{\Lambda'} \) is the Bayes estimator of \( 1/\eta \) on the basis of \( Y \), show that \( \delta_\Lambda \delta'_{\Lambda'} \) is the Bayes estimator of \( \xi/\eta \) on the basis of \( (X, Y) \).