1. Suppose that $X$ has the binomial distribution $Binom(n, p)$ and we wish to estimate $p$ with squared error loss. Let $\delta^*(X) = X/n$ with probability $1 - \epsilon$ and $\delta^*(X) = 1/2$ with probability $\epsilon$. Determine the risk function of $\delta^*$ and show that for $\epsilon = 1/(n + 1)$, its risk is constant and less than $\sup_p E(p - X/n)^2$.

2. The minimax estimator for the above-formulated problem 1 is

$$
\delta = \frac{X}{n} \frac{n^{1/2}}{1 + n^{1/2}} + \frac{1}{2(1 + n^{1/2})}.
$$

Find its bias and discuss its direction.

3. Let $X_i, i = 1, \ldots, n$ be iid with unknown distribution $F$. Show that

$$
\delta := \frac{\sum_{i=1}^n I(X_i \leq 0)}{n^{1/2}} \frac{n^{1/2}}{1 + n^{1/2}} + \frac{1}{2(1 + n^{1/2})}
$$

is minimax for estimating $F(0) = P(X_i \leq 0)$ with squared error loss.

4. Let $X_1, \ldots, X_m$ and $Y_1, \ldots, Y_n$ be independently distributed as $N(\xi, \sigma^2)$ and $N(\eta, \tau^2)$, respectively, and consider the problem of estimating $\theta = \eta - \xi$ with squared error loss.

   a) If $\sigma$ and $\tau$ are given, prove that $\bar{Y} - \bar{X}$ is minimax.

   b) If $\sigma$ and $\tau$ are restricted by $\sigma^2 \leq A < \infty$ and $\tau^2 \leq B < \infty$, respectively ($A$ and $B$ are known) then $\bar{Y} - \bar{X}$ continues to be minimax.