1. Using our laws for sets (associative, distributive,...), verify that 

\[(A \cap B) \cup (A \cap B^c) = A.\]

[Note that this is Exercise 2.4.a, p.30]

2. Prove that 

\[P(A \cup B) \leq P(A) + P(B).\]

3. Let \(A\) and \(B\) be two mutually exclusive events. Let \(C\) be any event. What assumption do we need (or no additional assumption) for the validity of

\[P(C) = P(A)P(C|A) + P(B)P(C|B)\]

4. Urn 1 contains two white chips and one red chip. Urn 2 has one white chip and two red chips. One chip is drawn at random from urn 1 and transferred into urn 2. Then one chip is drawn from urn 2. Suppose that a red chip is selected from urn 2. What is the probability that the transferred chip was white?

5. A family has a little boy A and two dogs D and C. None of them is fond of the mailman. Given that they are outside, dogs D and C have a 30\% and 40\% chance, respectively, of biting the mailman. Boy A, if he is outside, has a 15\% chance of doing the same thing. Suppose that only one of three
is outside when the mailman comes.
a). If D is outside 50% of time, C is 20% of time, and A is 30% of time, what is the probability that the mailman will be bitten?
b). Under these assumptions, if the mailman is bitten, what are the chances that boy A did it?

6. Let $E$ and $F$ are two events and $P(E|F) = .6$, $P(F) = .1$. What is the probability $P(E^c|F)$?

7. Let $P(A) = .5$, $P(B) = .3$, and $P(A \cap B) = .7$. Are A and B independent or an extra information is needed to resolve this issue? (please explain your answer)

8. The probability mass function of a random variable $X$ is given by $p(i) = c\lambda^i / i!$, $i = 0, 1, 2, \ldots$, where $\lambda$ is some given positive constant, and it is up to you to find $c$. What is the value of the cumulative distribution function at point 1.5?

9. Suppose that $X$ is a continuous random variable whose probability density function is given by

$$f(x) = C(4x - 2x^2), \quad 0 < x < 2; \quad f(x) = 0 \text{ otherwise.}$$

(a). Find $P(X > 1)$?
(b). Find the corresponding cumulative distribution function.

10. The distribution function $F(x)$ is 0 for $x < -1$, $(x + 5)/10$ for $-1 \leq x < 3$ and 1 for $x \geq 3$. Find:
(a) $P(-1 \leq X < 2)$.
(b) Probability density $f(x)$ at point $x = 0$. 