A Comparison of Buyer-Determined and Price-Based Multiattribute Mechanisms

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Reverse auctions are fast becoming the standard for many procurement activities. In the past, the majority of such auctions have been solely price based, but increasingly attributes other than price affect the auction outcome. Specifically, the buyer uses a scoring function to compare bids and the bid with the highest score wins. We investigate two mechanisms commonly used for procurement in business-to-business markets, in a setting in which buyers’ welfare is affected by exogenous nonprice attributes such as the quality, service, and past relationships. Under both mechanisms, bidders bid based on price, but in the “buyer-determined” mechanism, the buyer is free to select the bid that maximizes her surplus while in the “price-based” mechanism, the buyer commits to awarding the contract to the low price bidder. We find, both in theory and in the laboratory, that the “buyer-determined” mechanism increases the buyer’s welfare only as long as enough suppliers compete. If the number of suppliers is small and the correlation between cost and quality is low, the buyer is better off with the “price-based” mechanism. These findings are intended to help procurement managers make better decisions in designing procurement mechanisms for a variety of settings.

Key words: bidding; procurement; reverse auctions; multiattribute auctions; behavioral game theory; experimental economics

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1. Introduction

The purpose of this work is to better understand mechanisms commonly used for procurement and to help managers design better markets. We examine a setting in which multiple suppliers compete for a contract where attributes other than price contribute to the buyer’s welfare and some of those nonmonetary attributes are exogenous. We investigate the practical question regarding when it is to the buyer’s advantage to commit to awarding the contract to the low price supplier, as opposed to reserving the right to select any supplier. When procuring complex products, only a small number of potential suppliers may be qualified and it turns out that the number of bidders often determines whether the buyer benefits from committing to accepting the lowest bid. A number of studies have established that when some feature of a market mechanism affects the number of bidders, standard auction theory results change (e.g., see Engelbrecht-Wiggans 1993 for endogenous entry). In the context of procurement, Seshadri et al. (1991) show that the number of potential bidders influences whether multiple sourcing is profitable for the buyer.

We present a theoretical analysis of the problem as well as laboratory experiments designed to provide empirical validity to the theoretical predictions.

Options for dealing with multiple attributes fall into three categories: RFx, reverse auctions, and multiattribute auctions. RFx includes request for information (RFI), request for proposals (RFP), and request for quotes (RFQ). With an RFI, a buyer does not commit to selecting any supplier but simply requests information for the purpose of evaluating the possibility of dealing with a supplier; thus, an RFI is a completely nonbinding mechanism. In the RFP process, the buyer develops a detailed set of specifications, suppliers submit quotes that must meet those specifications, and the buyer commits to awarding the contract to the lowest cost supplier. In contrast, the RFP process, commonly used by the federal government, involves formally evaluating proposals along technical and cost dimensions, and awarding the contract to the supplier who achieves the best overall score.

Reverse auctions, as they are currently being used in practice, can be viewed as structured versions of the RFQ and RFP mechanisms. In standard binding
reverse auctions, which we call price-based (PB) mechanisms, the buyer prepares and disseminates a set of detailed specifications as in an RFQ, but suppliers respond through a live bidding event. Suppliers bid on price and at the end of the auction, just as in the RFQ process, the buyer commits to awarding the contract to the lowest bidder provided that bidder, can meet the specifications.

Another reverse auction mechanism, commonly offered by FreeMarkets (now part of Ariba) as well as used by such companies as Wyeth and DuPont, is the nonbinding or buyer-determined (BD) mechanism. This mechanism is, essentially, a structured version of an RFP: Suppliers respond through a live auction event and bid on price, but the buyer does not commit to awarding the contract to the lowest bidder. He instead reserves the right to select the winner based on a scoring rule that combines cost with a set of technical parameters. Nonbinding auctions are surprisingly common. Anderson and Frolich (2001, p. 60) report that "...clients do not normally make award decisions on bid day. In the days and weeks that follow the bidding event, buyers examine bid results, review supplier information (such as supplier capability, quality certifications, and manufacturing processes), and sometimes conduct a buyer audit before making a final decision. The client does not have to select the lowest bidder." Jap (2002, p. 510) reports that "...the vast majority of [FreeMarkets procurement] auctions used in the marketplace today do not determine a winner ... and the buyer may reserve the right to select a winner on any basis."

Mechanisms we commonly observe (RFx and reverse auctions) involve primarily bidding on price. In practice, some nonmonetary attributes are exogenous in the short run or close to being so, and others are not. Examples of exogenous attributes include geographical location, reputation, established relationships, brand name, access to specific technical expertise, and proprietary production processes. Other attributes can be changed at a cost. For example, lead time can be reduced by using alternative distribution channels and reliability can be improved by using more expensive materials or quality control processes. We analyze settings in which some of the nonmonetary attributes are exogenous.

In our baseline model, we study a setting in which suppliers bid on price and some of their nonmonetary attributes are exogenous. We compare a stylized version of the RFQ (that we call the PB mechanism)—a sealed-bid reverse auction in which the buyer commits to award the contract to the supplier who placed the lowest bid—to a stylized version of the RFP (that we call the BD mechanism)—a sealed-bid reverse auction in which the buyer is free to award the contract to any supplier.

The contribution of this paper includes the following three findings: (1) we show theoretically that as the exogenous nonmonetary component becomes a relatively more important part of the buyer surplus, price-based mechanisms can yield more expected surplus than buyer-determined mechanisms, especially for small numbers of bidders; (2) we test the theory in the laboratory and find that the actual behavior is close, on average, to theoretical benchmarks and, specifically, we find that the PB mechanism results in higher buyer surplus in auctions with two bidders but with four bidders the BD mechanism dominates; and (3) bidder experience decreases buyer surplus, especially for the BD mechanism.

2. Literature Review

Details of market mechanisms often have a considerable effect on buyer and seller profits, and the basic question of how to help the market designer (be it a buyer or a seller) to increase profits by making appropriate design choices is one that has received considerable attention in the marketing literature. Examples of analytical marketing studies on auctions include Sinha and Greenleaf (2000), Fay (2004), and Rothkopf (1991), to name a few.

Full-blown multiattribute reverse auctions allow bidders to place bids on multiple dimensions—not just price. Electricity reserve supply auctions (Bushnell and Oren 1994, Wilson 2002), highway construction works in the United States (Herbsman et al. 1995), and Department of Defense contracts (Fox 1974, Che 1993) all use such formats. In theory, such formats are likely to offer the most socially efficient mechanism for dealing with multiattribute products, but in practice some attributes which are valued by the buyer such as reputation and relationships are not easily quantifiable in a formal manner.

Che (1993) and Branco (1997) characterized full-blown multiattribute auctions with two dimensions—price and quality. Che (1993) was the first to show revenue equivalence between first-score and second-score reverse auctions. While Che studied endogenous nonmonetary attributes, the essence of that proof applies in the present setting as well. Che (1993) also characterized optimal scoring rules and found that optimal rules involve giving less weight to nonmonetary attributes relative to the buyer’s actual valuation of those attributes. The present setting deviates from Che (1993) in two ways: First, we focus on a comparison of price-based and buyer-determined formats. Second, we study a setting in which some of the nonmonetary attributes are exogenous. This latter exogeneity of nonmonetary attributes is key because when nonmonetary attributes are fully endogenous (that is, bidders choose their quality levels and cost is
increasing in quality), the price-based format would result in each bidder offering its minimum possible quality level.

Researchers have successfully used laboratory experiments to improve our understanding of markets, including the effect of the number of competitors on market outcomes (Amaldoss and Rapoport 2005). In the area of auctions, experiments provide a controlled way for testing theory and comparing market design mechanisms. We refer the interested reader to Kagel (1995) for a review. We are aware of only two laboratory studies of multiattribute auction mechanisms. In Bichler (2000), the bidding takes place in two dimensions in the context of financial derivatives. In Chen-Ritzo et al. (2005), bidding takes place in three dimensions (price, quality, and delivery time). In both studies, the multiattribute auction increased efficiency and buyer’s welfare.

3. Analytical Results

In our model, $N$ ($N \geq 2$) sellers or “bidders” compete to provide one unit to buyer. We assume bidders are heterogeneous in quality and cost and are pre-qualified and verified to meet the minimum quality requirements the buyer has specified. Bidder $i$ has a privately known, nonnegative cost $C_i$ of providing the unit. The buyer’s value for a unit depends on which seller provides the unit. That is, some sellers’ products suit the buyer better than others. We refer to the value the buyer places on these nonmonetary, sellerspecific, exogenously fixed attributes as the quality $Q_i$. We assume that both the buyer and bidder $i$ know $Q_i$ but that none of the other bidders do. In many situations, at least some component of quality satisfies these assumptions; we focus on those settings and study the impact of this type of quality on the procurement process.

For notational ease, let $C$ and $Q$ denote generic costs and qualities. Each bidder’s type $(C, Q)$ is a random vector. Although bidders are heterogeneous over $(C, Q)$, the random vector $(C, Q)$ for each bidder comes from a common distribution. We assume that the $(C, Q)$’s are identically distributed, independent, and nondegenerate. In general, the $C$’s are not independent of the $Q$’s.

The price-based (PB) mechanism is a standard onedimensional reverse auction. In the first-price based version, sellers submit price offers, the lowest offer wins, and the buyer pays the winner an amount equal to the winning offer. The second-price version is similar except that the buyer pays an amount equal to the lowest losing offer. Each version of such auctions has a unique symmetric risk-neutral bidding equilibrium, and the two versions generate the same expected surplus to the buyer at their respective equilibria.

At equilibrium in the second-price PB mechanism, bidders bid their actual cost. Therefore, the buyer pays $C_{(N-1)}$ for a unit worth $Q_i$, the buyer’s surplus is $Q_i - C_{(N-1)}$, where $i^*$ denotes the bidder $i$ with the smallest cost $C_i$, and the subscript in parentheses denotes the order statistic, so that a subscript $(K)$ generally denotes the $K$th largest of $N$ draws of a random variable. Note that $Q_{i^*} - C_{(N-1)} = (Q_{i^*} - C_{(N)}) + (C_{(N)} - C_{(N-1)}) = (Q_{i^*} - C_r) + (C_{(N)} - C_{(N-1)})$ and, therefore, the expected buyer surplus in the PB mechanism is, $\Pi_{PB} = E[Q - C | C = C_{(N)}] + E[C_{(N)} - C_{(N-1)}]$.

In the first-price buyer-determined (BD) mechanism, each seller $i$ bids a price $B(C_i, Q_i)$. The buyer selects the seller $i$ with the highest score $S(C_i, Q_i) \equiv Q_i - B(C_i, Q_i)$ and pays the winning seller $i$ an amount $B(C_i, Q_i) = Q_i - S(C_i, Q_i)$. In the corresponding second-price version, the winning bidder $i$ receives an amount equal to $Q_i - S(C, Q_i)$.

Since the buyer and seller $i$ both know the quality $Q_i$, we could think of each seller $i$ as bidding the score $S(C_i, Q_i)$ directly. Bidding the score makes the BD mechanism isomorphic to a regular (price-based forward) sealed-bid auction with independent, privately known valuations. Therefore, just like for the PB mechanism, the first- and the second-price versions of the BD mechanism each have a unique symmetric risk-neutral bidding equilibrium, and the two versions generate the same expected surplus to the buyer at their respective equilibria.

At equilibrium in the second-price BD mechanism, bidders bid the scores corresponding to their actual $C$’s and $Q$’s. Let $i^{**}$ denote the bidder $i$ with the largest score ($Q_i - C_i$). Then, the buyer pays $Q^{**} - (Q - C)_{(2)}$ for a unit worth $Q^{**}$ and the buyer’s surplus is $(Q - C)_{(2)}$.

Summarizing the above, we have the following lemma:

**Lemma.** Both the first- and second-price versions of each mechanism have a unique pure strategy symmetric risk-neutral bidding equilibrium. For each mechanism, the first- and second-price equilibria generate the same expected buyer surplus. Let $\Pi_{BD}$ and $\Pi_{PB}$ denote the expected buyer’s surplus under the BD and PD mechanisms, respectively. Then, $\Pi_{BD} = E[Q - C | C = C_{(N)}] + E[C_{(N)} - C_{(N-1)}]$ and $\Pi_{PB} = E[Q - C | C = C_{(N)}] + E[C_{(N)} - C_{(N-1)}]$.

In order to see how $\Pi_{BD}$ and $\Pi_{PB}$ compare, start by considering the case in which $C$ and $Q$ are perfectly negatively correlated. In this case, the lowest

1 The second-price sealed-bid mechanism is rarely used in procurement auctions in spite of its useful theoretical properties (see Rothkopf et al. 1990 for some reasons for why second-price auctions are rare); they will, however, be useful in our analysis of the buyer’s surplus.
cost corresponds with the highest quality minus cost. Therefore, the same supplier wins in both the PB and BD mechanisms. However, the winning price will differ across mechanisms. In particular, the BD mechanism considers the suppliers’ Q’s in deciding the winner. Therefore, the winning supplier can capture her incremental contribution to the quality. In contrast, the PB mechanism does not consider Q and, therefore, the winner can capture any of this incremental quality. Hence, when C and Q are perfectly negatively correlated, \( \Pi_{BD} < \Pi_{PB} \) for all finite \( N \).

Now consider the other extreme. To illustrate what happens, consider the special case where \( Q = C \) (and, therefore, C and Q are perfectly positively correlated). In this case, each supplier generates the same amount of surplus. In the BD mechanism, each supplier has the same \( (Q_i - C) \) and, therefore, the buyer captures the entire available surplus. In the PB mechanism, different suppliers have different costs and, therefore, the winning supplier captures some of the surplus. Hence, when \( C = Q \), \( \Pi_{BD} > \Pi_{PB} \) for all finite \( N \).

In order to generalize this result for small enough \( N \), we need to define an appropriate concept to describe the relationship between \( C \) and \( Q \). Specifically, given \( N = 2 \) suppliers and values \( C_1, C_2, Q_1, Q_2 \), there are two possible ways of assigning these values to bidders. In the ”positively related” (PR) case, one supplier has the values \( \{\max\{C_1, C_2\}, \max\{Q_1, Q_2\}\} \) and the other has the values \( \{\min\{C_1, C_2\}, \min\{Q_1, Q_2\}\} \), while in the ”negatively related” (NR) case, one supplier has the values \( \{\max\{C_1, C_2\}, \min\{Q_1, Q_2\}\} \) and the other has the values \( \{\min\{C_1, C_2\}, \max\{Q_1, Q_2\}\} \). Note that if the C’s and Q’s are independent, then the PR and NR cases are equally likely.

Now we can state the following result (proven in the appendix):

**Proposition 1.** If \( N \) is sufficiently small and either the NR outcomes are at least as likely as the PR outcomes or the magnitude by which the PR outcomes are more likely than the NR outcomes is sufficiently small, then \( \Pi_{BD} < \Pi_{PB} \). If \( N \) is sufficiently small and the PR outcomes are sufficiently more likely than the NR outcomes, then \( \Pi_{BD} > \Pi_{PB} \).

The relative likelihood of NR versus PR outcomes corresponds roughly to the correlation of \( C \) and \( Q \). However, positive versus negative correlation is not the deciding factor in which a mechanism generates the greater buyer surplus. Indeed, the proof of Proposition 1 establishes that \( \Pi_{BD} < \Pi_{PB} \) when \( C \) and \( Q \) are close enough to being independent. In such cases, the correlation will be close to zero but may have either sign. However, the proposition does have the following corollary:

**Corollary.** If \( N \) is sufficiently small and \( C \) and \( Q \) are either independent or sufficiently negatively correlated, then \( \Pi_{BD} < \Pi_{PB} \). If \( N \) is sufficiently small and \( C \) and \( Q \) are sufficiently positively correlated, then \( \Pi_{BD} > \Pi_{PB} \).

Next, consider increasing the number of suppliers \( N \). The ranking of the two mechanisms may change. The ranking for large enough \( N \) depends on the shape of the support of the random variable \( (Q, C) \). Figure 1 shows a generic support set \( \Omega \).

Let \( \mathcal{E} \) denote the subset of \( \Omega \) where \( C \) is minimized, and let \( \Omega - \mathcal{E} \) denote the subset of \( \Omega \) where \( Q - C \) is maximized; \( \mathcal{E} = \{X_1, X_2\} \) and \( \Omega - \mathcal{E} = \{X_3\} \) in Figure 1.

As \( N \) goes to infinity, the support set becomes densely covered with actual outcomes and \( C(N-1) \) will be arbitrarily close to the southern boundary of the support set (points \( X_1 \) and \( X_2 \) in Figure 1). Therefore, the buyer’s surplus converges to the surplus of some southernmost point. Similarly, the buyer’s surplus in the second-price BD mechanism equals \( (Q - C)_2 \) and this will be arbitrarily close to a southeasternmost point (point \( X_3 \) in Figure 1). Note that the further southeast the point, the larger the surplus. Unless all the southernmost points are also as far southeast as possible, the BD mechanism will yield a strictly larger buyer’s surplus in the limit as \( N \) goes to infinity.

We state this result formally (proof in the appendix):

**Proposition 2.**

(A) If \( \mathcal{E} \) is not a subset of \( \Omega - \mathcal{E} \), then \( \Pi_{BD} > \Pi_{PB} \) for all large enough \( N \).

(B) If \( \mathcal{E} \) is a subset of \( \Omega - \mathcal{E} \), then \( \Pi_{BD} - \Pi_{PB} \) converges to zero as \( N \) goes to infinity.

It is possible for \( \mathcal{E} \) to be a subset of \( \Omega - \mathcal{E} \). For example, when \( C \) and \( Q \) are perfectly negatively correlated and when \( Q = C \), then the support set is a line

\[ \Pi_{BD} = \frac{1}{2}(Q - C)_2 \]

\[ \Pi_{PB} = \frac{1}{2}(Q - C)_2 \]

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segment and $\mathcal{E}$ will be a subset of $\mathcal{O} - \mathcal{E}$. We already saw that $\Pi_{BD} > \Pi_{PB}$ for all $N$ when $C$ and $Q$ are perfectly negatively correlated, and that $\Pi_{BD} < \Pi_{PB}$ for all finite $N$ when $Q = C$. Proposition 2 allows us to add “but the gap shrinks to zero as $N$ goes to infinity” for both these examples.

A key insight from Proposition 2 is that in almost all cases, BD will yield higher buyer surplus than PB in large enough markets. Hence, for buyers facing numerous qualified sellers, the choice between PB and BD is simple. The sole exception to this rule occurs when $\mathcal{E}$ is a subset of $\mathcal{O} - \mathcal{E}$. What this represents is a case in which the lowest cost supplier is also necessarily the supplier with the largest (or tied for the largest) score ($Q_i - C_i$).

In general, however, $\mathcal{E}$ will not be a subset of $\mathcal{O} - \mathcal{E}$. To illustrate what happens in such cases, consider the following example which is also our laboratory setting. Assume $Q = C + \gamma X$, where $C \sim \text{Uniform}(0, 100)$, $X \sim \text{Uniform}(0, 1)$, and $\gamma$ is a constant. Note that $C$ and $Q$ are positively correlated, but the correlation shrinks from one to zero as $\gamma$ increases from zero to infinity. This relationship enables us to illustrate what happens as the correlation changes.

Figure 2 shows the support of $(C, Q)$ for the two levels of $\gamma$ that we will be using in our experiment: $\gamma = 100$ and $\gamma = 300$. It also shows the sets $\mathcal{E}$ and $\mathcal{O} - \mathcal{E}$ for this example.

In either case, outcomes are distributed uniformly over the support. Note that as $\gamma$ shrinks toward zero, the example converges to the $Q = C$ case and, therefore, $\Pi_{BD} > \Pi_{PB}$ for all finite $N$. In the other direction, as $\gamma$ goes to infinity, the support becomes arbitrarily close to being a rectangle; we approach the case of $C$ and $Q$ being independent and Proposition 1 implies that $\Pi_{BD} < \Pi_{PB}$ for $N = 2$. Therefore, at some finite $\gamma$ (it happens to be at $\gamma = 200$) the buyer’s surpluses from $N = 2$ suppliers switch from $\Pi_{BD} > \Pi_{PB}$ to $\Pi_{BD} < \Pi_{PB}$.

Now consider what happens as $N$ increases. Since $\mathcal{E}$ is not a subset of $\mathcal{O} - \mathcal{E}$, Proposition 2 implies that $\Pi_{BD} > \Pi_{PB}$ for large enough $N$. Therefore, if $\gamma$ is large enough, then $\Pi_{BD} < \Pi_{PB}$ for $N = 2$ but switches to $\Pi_{BD} > \Pi_{PB}$ as $N$ becomes large enough.

We can verify these claims by direct calculations. Specifically, in this example, $Q - C = \gamma X$, where $X \sim \text{Uniform}(0, 1)$. Therefore,

$$
\Pi_{BD} = E[(Q - C)_{(i)}] = \gamma X_{(i)} = \frac{N - 1}{N + 1} \quad (1)
$$

$$
\Pi_{PB} = E[Q - C \mid C = C_{(N)}] + E[C_{(N)} - C_{(N-1)}] = E[\gamma X] - E[C_{(N-1)} - C_{(N)}] = \frac{\gamma}{2} - \frac{100}{N + 1}. \quad (2)
$$

Equations (1) and (2) imply that $\Pi_{BD} - \Pi_{PB} = (\gamma(N - 3) + 200)/(2N + 1)$, which in turn implies that if $N = 2$, $\Pi_{BD} < \Pi_{PB}$ whenever (and only if) $\gamma > 200$; i.e., when $Q$ and $C$ are not too positively correlated. Furthermore, if $\gamma > 200$, then the buyer’s surpluses switch from $\Pi_{BD} < \Pi_{PB}$ to $\Pi_{BD} > \Pi_{PB}$ as the number of suppliers increases from $N = 2$ to $N > 2$. In short, the theory predicts a switch between $\Pi_{BD} < \Pi_{PB}$ and $\Pi_{BD} > \Pi_{PB}$ as $N$ increases and as $\gamma$ decreases. In the next section, we describe the experiment and the hypotheses that we use to test these two predictions.

4. Experimental Hypotheses and Design

Since in our laboratory experiment $Q = C + \gamma X$ in which $C \sim \text{Uniform}(0, 100)$ and $X \sim \text{Uniform}(0, 1)$, it is a straightforward result that in equilibrium the optimal bid function under the PB mechanism is

$$
B_{PB}(C_i, Q_i) = C_i + (100 - C_i)/N. \quad (3)
$$

The corresponding equilibrium bid function under the BD mechanism is

$$
B_{BD}(C_i, Q_i) = C_i + (Q_i - C_i)/N. \quad (4)
$$

The two bid functions are tractable and sufficiently similar to constitute a fair test of the two mechanisms. Additionally, we wanted a setting that is realistic in that $Q$ increases in $C$ but the two are not perfectly correlated.

Our model predicts that whether the buyer surplus is higher under PB or under BD depends on both $N$ and $\gamma$. Therefore, we select three combinations of $\gamma$ and $N$ in a way that given $\gamma$, $\Pi_{BD} < \Pi_{PB}$ when $N$ is small and $\Pi_{BD} > \Pi_{PB}$ when $N$ is large, and also given $N$, $\Pi_{BD} < \Pi_{PB}$ when $\gamma$ is large and $\Pi_{BD} > \Pi_{PB}$ when $\gamma$ is small. Given the above-mentioned uniform distributions for $C$ and $X$, $\Pi_{BD} < \Pi_{PB}$ if and only if $N = 2$ and $\gamma > 200$. Therefore, the three settings we
consider are:
1. \( N = 2 \) and \( \gamma = 100 \).
2. \( N = 2 \) and \( \gamma = 300 \).
3. \( N = 4 \) and \( \gamma = 300 \).³

We compute the theoretical benchmarks for the expected buyer surplus using Equations (1) and (2). We also numerically compute approximate standard deviations of those average buyer surplus levels. This gives us the following three research hypotheses dealing with the average buyer surplus levels:

**Hypothesis 1.** When \( \gamma = 100 \) and \( N = 2 \), the average buyer’s surplus in the BD mechanism will be larger than the average buyer’s surplus in the PB mechanism. In this case, \( \Pi_{BD} = 33\frac{1}{2} \) (standard deviation = 12) and \( \Pi_{PB} = 16\frac{3}{4} \) (standard deviation = 31).

**Hypothesis 2.** When \( \gamma = 300 \) and \( N = 2 \), the average buyer’s surplus in the BD mechanism will be smaller than the average buyer’s surplus in the PB mechanism. In this case, \( \Pi_{BD} = 100 \) (standard deviation = 35) and \( \Pi_{PB} = 116\frac{3}{4} \) (standard deviation = 89).

**Hypothesis 3.** The above relationship will reverse when \( \gamma = 300 \) and \( N = 4 \). In this case, \( \Pi_{BD} = 180 \) (standard deviation = 37) and \( \Pi_{PB} = 130 \) (standard deviation = 87).

Note that the theoretical predictions of the buyer surplus (and their standard deviations) for the realizations of the C’s and Q’s in our experiment differ slightly from the theoretical predictions in Hypotheses 1–3 because we computed the theoretical predictions for the experiment for the actual draws of C’s and Q’s used in the experiment. Also, note that the theoretical standard deviations under PB are more than twice those of BD although the draws of C and Q are the same in the two conditions and their covariance is identical. This is because the theoretical winning bid in the PB case does not take quality directly into account, whereas the theoretical winning bid in the BD case does. Hence, the variance in quality will increase the variance in buyer surplus in the PB case more than in the BD case.

The prior experience of the subjects is an important dimension for procurement auctions, in which professional bidders presumably are highly experienced with auctions. To increase subjects’ experience, we let subjects in some of the sessions, prior to the experiment, bid repeatedly against rational computerized bidders. There is a possible drawback to this method in that it artificially exposes subjects to rational bidding. Alternatively, one could let subjects participate in auctions against other subjects for many rounds and then invite them back at a later date. However, matching inexperienced subjects against other inexperienced subjects as a way to generate experience may introduce interaction effects and may not result in rapid learning, since the feedback subjects receive is likely to be noisy. In contrast, matching subjects against computerized bidders which efficiently maximize their own payoffs will result in better experimental control, cleaner feedback, and hence faster learning. We therefore compare inexperienced subjects with subjects that had prior experience with computerized bidders.

The experiment manipulates the \( N \) and \( \gamma \) combination (\( N = 2 \) and \( \gamma = 100 \); \( N = 2 \) and \( \gamma = 300 \); \( N = 4 \) and \( \gamma = 300 \)), the mechanism (BD for each combination and PB for the different levels of \( N \), since bidding under PB is independent of \( \gamma \)), and the prior experience of bidders. This yields a design with 10 treatments that we summarize in Table 1. Each subject participated in a single treatment in a between-subjects design.

Each treatment contained 2 independent cohorts of 8 participants for a total of 16 participants, per treatment and 160 participants in the experiments. We used the first-price sealed-bid mechanism in all ten treatments. Participants bid in 40 auctions and we randomized the matching for each auction. We also drew the quality and the cost randomly for each seller in each auction.

We programmed the laboratory software using the zTree system (Fischbacher 2007). At the end of each auction, we revealed the same information for the BD and PB conditions. This information included the bids and qualities of the other bidders and the winner in that period’s auction. The entire history of past winning prices and qualities in the session was also provided for explicit reference points (Dholakia and Simonson 2005).

The sole difference between the PB and BD conditions was in how the winner was determined. That is, in the PB condition, the lowest price bidder won whereas in the BD condition, the highest surplus (quality minus price) bidder won. Note, however, that in both conditions bidders submitted price (not score) bids. See the Technical Appendix at http://mktsci.pub.informs.org/ for specific instructions and screenshots.

In the experienced treatments, subjects came back following their participation in 200 rounds of a web-based auction against computerized rivals. The web-based auction was always of the same format (PB or BD) and had the same number of bidders as the corresponding follow-up treatment. We told participants in the web auction that they would be matched with computerized rivals programmed to

³ The size of \( \gamma \) affects the correlation between C and Q. In the \( \gamma = 300 \) condition, the average correlation coefficient between C and Q in our experiments was 0.34 and in the \( \gamma = 100 \) condition, it was 0.72.
maximize their own earnings against other similarly programmed rivals. We paid participants in the web-based auctions based on performance. Following the web-based auction, we asked the majority (approximately 90%) of the participants to come back and participate in an experienced session. The few subjects whom we did not ask back were ones who performed extremely poorly in the web experiments and did not show any improvement over time. For example, there was one person who lost money in the web experiment by always bidding below cost—this person was not asked to come back. The experienced laboratory subjects had an additional piece of information about their bids that inexperienced subjects did not have: the probability of a given bid winning in the web experiments.

5. Experimental Results

5.1. Buyer Surplus: Aggregate Comparison

We start by directly testing Hypothesis 1–3 about the buyer surplus rankings. We compare the average buyer surplus levels for the two mechanisms and the two auction group sizes in Table 2. Recall that each treatment consisted of two independent cohorts of 8 subjects who were randomly matched for 40 periods. We kept the randomly-generated C’s, Q’s, and matching constant across treatments. We show the

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<tr>
<th>Table 2</th>
<th>Actual and Predicted Buyer Surplus Averaged Over All Periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experience</td>
<td>Yes</td>
</tr>
<tr>
<td>Setting</td>
<td>$N = 2/y = 100$</td>
</tr>
<tr>
<td>Mechanism</td>
<td>BD</td>
</tr>
<tr>
<td>Actual</td>
<td>[49.00]</td>
</tr>
<tr>
<td>Theoretical</td>
<td>45.93</td>
</tr>
<tr>
<td></td>
<td>(12.32)</td>
</tr>
<tr>
<td>Proportion of efficient allocations</td>
<td>BD</td>
</tr>
<tr>
<td>Actual</td>
<td>0.89</td>
</tr>
<tr>
<td>Theoretical</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Notes. Median buyer surplus levels over all periods are in square brackets. Standard deviations are in parentheses.
actual median (in square brackets) and average buyer surplus in each treatment, with standard deviations in parentheses, as well as the (theoretical) expected buyer surplus levels and their standard deviations for the generated profiles. In the bottom four rows of Table 2, we report the actual and theoretical percentages of efficient allocations—outcomes in which the bidder with the largest score \((Q_i - C_i)\) wins the auction.

We start by comparing the average levels of buyer surplus across mechanisms directly by using the cohort as the relevant unit of analysis. The data are consistent with the prediction that BD will produce higher average buyer surplus than PB when \(\gamma = 100\) and \(N = 2\) but lower average buyer surplus than PB when \(\gamma = 300\) and \(N = 2\) (Hypotheses 1 and 2). The data also confirm the prediction that BD will produce higher average buyer surplus than PB when \(\gamma = 300\) and \(N = 4\) but lower average buyer surplus than PB when \(\gamma = 300\) and \(N = 2\) (Hypotheses 2 and 3). Hereafter, we will use the term BD and PB surplus for short when referring to the average buyer’s surplus in the BD and PB mechanisms, respectively.

For \(\gamma = 100\) and \(N = 2\), the average PB surplus is significantly lower than the average BD surplus (\(p\) value = 0.025 for experienced treatments and 0.032 for inexperienced treatments). For \(\gamma = 300\) and \(N = 2\), PB surplus is higher than the BD surplus and the difference is statistically significant in experienced treatments and weakly significant in inexperienced treatments (\(p\) value = 0.038 for experienced treatments and 0.099 for inexperienced treatments). For \(\gamma = 300\) and \(N = 4\), the average PB surplus is significantly lower than the average BD surplus (\(p\) value = 0.025 for experienced treatments and 0.019 for inexperienced treatments).

When we examine the effect of the number of bidders on the buyer surplus in the \(\gamma = 300\) condition, we find that under PB the buyer surplus levels are not significantly different with two bidders than with four bidders (\(p\) values are 0.259 for experienced and 0.253 for inexperienced). The lack of statistically significant difference is not particularly surprising given that theoretical predictions are quite close. The number of bidders does have a significant effect under BD, where the buyer surplus is significantly higher with four than with two bidders (\(p\) values are 0.010 for experienced and 0.023 for inexperienced).

Bidder experience has some effect on the average buyer surplus. The statistically significant differences at the 5% level are in the \(\gamma = 100\) and \(N = 2\) condition (\(p\) value = 0.034 for PB and 0.046 in BD), as well as in the \(\gamma = 300\) and \(N = 4\) BD condition (\(p\) value = 0.038). Experience slightly reduces noise in several of the treatments (as can be seen from lower standard deviations).

As noted in §4, since bids under PB do not incorporate quality, we expect standard deviations of the buyer surplus to be higher under PB than under BD and they are (\(p\) values < 0.05). The actual standard deviations are quite close to their theoretical benchmarks. The actual efficiency achieved in PB is significantly lower than that in BD in all treatments (\(p\) values < 0.05). Generally, the levels of efficiency are slightly lower than the theory predicts and are not significantly affected by experience.

Each cohort includes data for 80 (when \(N = 4\)) and 160 (when \(N = 2\)) individual auctions. Since each cohort includes the same 8 subjects bidding in 40 auctions, the auctions within a cohort are not independent. In order to compare buyer surplus using the individual auction as the unit of analysis (instead of a cohort), we need to account for the fact that the same subjects compete in the different auctions. We do this using a random effects model of individual bidding behavior, described in the following section.

### 5.2. Buyer Surplus: Bid Analysis

The terms \(B_{it}^{PB} - C_{it}\) and \(B_{it}^{BD} - C_{it}\) denote the markups over cost of individual bidder \(i\)'s bid in period \(t\) for the PB and BD treatments, respectively. To analyze individual bidding behavior, we estimate the following models, corresponding to Equations (3) and (4):

\[
B_{it}^{PB} - C_{it} = \alpha + \beta(100 - C_{it}) + \eta_i + \varepsilon_{it}. \tag{5}
\]

\[
B_{it}^{BD} - C_{it} = \alpha + \beta(Q_{it} - C_{it}) + \eta_i + \varepsilon_{it}. \tag{6}
\]

Note that there are two error components in the model—one that is independent across all observations \(\varepsilon_{it}\) and one that is participant specific \(\eta_i\). Both error terms have a mean of zero and a positive standard deviation. This treatment of the individual effect is known as the random effects model.

The data exhibit essentially no bids below cost.\(^4\) While this is reasonable, since bids below cost would result in negative profit for the bidder in case of winning, it presents a restriction on the error terms. Specifically, symmetric error terms would result in some bids below cost, especially for high cost bidders under PB and low quality-minus-cost bidders under BD. Hence, we had to restrict the error terms to control for this censoring. This required a Tobit approach. Using Tobit, we accounted for the censoring at one token above cost, which is the minimum positive profit. Approximately 10% of all bids were at exactly one token above cost and only a negligible number of bids (less than 1%) were at or below cost.

We summarize coefficient estimates for Equations (5) and (6) in Table 3 along with their theoretical

\(^4\) Seven out of 10 treatments had 2 or fewer bids below cost out of 640 bids per treatment. Fewer than 1% of bids were exactly at cost.
predictions. Note that this analysis uses each bid as the relevant unit of observation but accounts for the dependence across observations for the same subject.

Using the coefficients we estimated from Equations (5) and (6), we compute buyer surplus levels for PB and BD mechanisms in each of the four conditions and summarize the results in Table 3 in the two rows labeled “Buyer Surplus.” Note that the direction of surplus differences is consistent with our research hypothesis in all five conditions. The estimated buyer surplus levels in Table 3 provide further support for all our hypotheses: In the $\gamma = 300$ condition, the PB mechanism is better for the buyer when the number of bidders is two but the BD mechanism becomes better when the number of bidders is four. In the $N = 2$ condition, the PB mechanism is better for the buyer when $\gamma = 100$ and this is reversed when $\gamma = 300$.

To get a sense about the individual bidding behavior, we plot in Figure 3 the markups over cost ($B - C$) in all treatments. In the BD treatments, the markups are shown as a function of $Q - C$ and in PB treatments, as a function of $100 - C$. Two qualitative patterns emerge from this figure. First, we see that at the low levels of ($Q - C$) in BD, and ($100 - C$) in PB, the bids are visibly censored at costs. This censoring makes scatter plots look slightly curved at the lower end of the support.

Second, the difference between experienced and inexperienced bidders noted above can also be observed in the scatter plots. The pattern of the difference is informative. Qualitatively, we observe more noise in the inexperienced data as well as a substantial amount of data in the lower-right portion of the plots, corresponding to insufficient markup. In general, inexperienced bidders bid lower than experienced bidders and this underbidding is especially severe at high levels of $Q - C$ (in BD) and $100 - C$ (in PB). While underbidding is less severe for experienced bidders, it is still present. This tendency of bidders to underbid (overbid in forward auctions) as well as the curvature of the bidding function, partially accounted for by censoring, are open issues and under debate in the experimental auction literature (Chakravarti et al. 2002).

### 6. Conclusion

The main contribution of this work is the comparison of two commonly used procurement formats—the “buyer determined” and “price-based” mechanisms—in settings in which the buyer derives value from some exogenous nonmonetary attributes of the suppliers. Procurement processes often include buyer-determined auctions, also referred to as nonbinding auctions. They involve price competition as in traditional one-dimensional price-based auctions but, unlike price-based auctions, the buyer is not obligated to choose the lowest price bid.

In our discussions with practitioners, we learned that purchasing managers feel that buyer-determined mechanisms give them the best of both worlds: the intense price competition of standard price-based auctions as well as the ability to account for subjective

<table>
<thead>
<tr>
<th>Setting</th>
<th>BD</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inexperienced</td>
<td>Experienced</td>
</tr>
<tr>
<td>$N = 2$ and $\gamma = 100$</td>
<td>0.555</td>
<td>0.502</td>
</tr>
<tr>
<td>$\beta$</td>
<td>(0.006)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-7.424</td>
<td>-11.434</td>
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<tr>
<td>Buyer surplus</td>
<td>37.95</td>
<td>45.60</td>
</tr>
<tr>
<td>$N = 2$ and $\gamma = 300$</td>
<td>0.537</td>
<td>0.540</td>
</tr>
<tr>
<td>$\beta$</td>
<td>(0.005)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-19.551</td>
<td>-28.284</td>
</tr>
<tr>
<td>Buyer surplus</td>
<td>114.75</td>
<td>122.74</td>
</tr>
<tr>
<td>$N = 4$ and $\gamma = 300$</td>
<td>0.090</td>
<td>0.277</td>
</tr>
<tr>
<td>$\beta$</td>
<td>(0.010)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-1.708</td>
<td>-20.287</td>
</tr>
<tr>
<td>Buyer surplus</td>
<td>225.26</td>
<td>197.90</td>
</tr>
</tbody>
</table>

*PB $N = 2$ sessions with $\gamma = 100$ have the same estimates as $\gamma = 300$, since $\gamma$ does not enter the PB bidder’s decision. Standard errors are in parentheses.*
difficult-to-quantify variables as in the traditional nonauction approaches to procurement. As we show in this paper, however, there is a trade-off between these two objectives. The buyer-determined mechanism does indeed result in some price competition, but in a setting with some exogenous quality, rational experienced suppliers will incorporate their non-price attributes into their bids and this can result in higher bids, and lower cost reductions relative to the price-based format. The additional value a buyer can expect from being able to choose a supplier other than the lowest-bid supplier can in many cases more than compensate for the higher price, especially when there are many bidders or when the correlation between the value this supplier provides and his cost is high. However, as the exogenous component of the supplier’s value becomes a more important part of the buyer surplus, in settings with few bidders and a high degree of independence between supplier’s value and cost, the buyer could benefit more from the increased price competition of a traditional price-based mechanism.

In the laboratory, in an environment with positively correlated cost and quality, we see that the buyer-determined format dominates the price-based auction in terms of buyer surplus when the number of
bidders is four; this relationship is reversed when the number of bidders is two, just as our theory suggests. However, the surplus from the BD auction is higher than the theory predicts whereas the surplus from the PB auction is only marginally higher.

We conjecture that the excess surplus from BD mechanisms could be due to systematic errors by inexperienced bidders who fail to sufficiently adjust their bid based on high quality. We tested this conjecture with experimental manipulations by first giving subjects experience against computerized opponents and then having these experienced subjects bid against one another. The evidence suggests that experienced subjects bidding against other experienced subjects bid closer to theoretical predictions than inexperienced subjects bidding against inexperienced opponents.

The study of experience is particularly relevant in light of the fact that B2B settings are generally repeated. The implication is that as sellers accumulate experience over time, they should learn to bid more effectively. Our theoretical framework is in equilibrium, meaning sellers are assumed to behave optimally. However, the experimental framework is repeated and we studied how sellers behaved in response to experience. The repeated nature of the interaction also presents opportunities for future research on issues such as reputation building (e.g., Bolton et al. 2004).

While experienced bidders bid much closer to theoretical predictions, some deviations remain. Further explanations for deviations from optimal bidding behavior are outside the scope of this work. We refer interested readers to Kagel (1995) for a review of how bidding behavior in sealed-bid first-price (forward) auctions differs from theoretical predictions, as well as to several more recent articles that explain some aspects of behavior using direction learning theory (Neugebauer and Selten 2006), impulse balance equilibrium (Ockenfels and Selten 2005), hierarchical models of others (Gneezy 2005), spiteful bidding (Morgan et al. 2003), and sensitivity to regret (Engelbrecht-Wiggans and Katok 2007).

Our work offers several managerial implications. First, purchasing managers should carefully think about the procurement format and map the relevant variables such as the number of bidders and the distribution of costs and qualities to the choice of procurement format. We do not expect that the buyer will have the sellers’ quality and cost distributions precisely mapped. However, it is reasonable to assume that the buyer knows approximately how many bidders to expect. If there are very few bidders, the revenue-maximizing (cost-minimizing) buyer is likely to prefer a price-based format.

It is also reasonable that the buyer knows something about the relationship between cost and quality. In some industries (for example, when quality control is costly), the cost is highly positively correlated with quality. In other industries, quality and cost may not be correlated or may even be negatively correlated (due to outdated technology, for example). If the buyer deems the correlation between cost and quality to be low or negative, the benefit of the buyer-determined format is smaller and the price-based format may be preferred.

There are several important auction benefits (for both buyers and sellers) not directly related to revenue maximization (Shugan 2005), among which is accelerated transaction time. Buyer-determined auctions may add a significant time component to the transaction, as necessitated by post-auction evaluations by the buyer. Practitioners should evaluate this additional time component when making the format decision.

Reverse auctions are often criticized because they may have a negative effect on long-term relationships and suppliers apparently feel that reverse auctions degrade their relationship premiums (Emiliani and Stec 2001; Jap 2002, 2003). However, BD auctions of the type described here are ideally suited to provide a premium for incumbent suppliers who have valuable relationships with the buyer, as long as these relationships can be translated into higher quality offerings. In light of the results of our work, it is a reasonable speculation that some of this disgruntlement, especially by incumbent suppliers, may be due to misalignment between the auction format and the perceived auction format. We leave this as a topic for future research but note that if a high quality bidder participating in a buyer-determined format erroneously believes he is participating in a price based format, he will indeed make suboptimal price concessions. In a buyer-determined auction, however, the rational informed seller should and would charge a premium for a long-held relationship as long as that relationship carries value to the buyer. With a carefully communicated procurement format that clearly specifies relationship as a value-generating asset, neither party should expect relationships to be harmed.

Acknowledgments
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Appendix. Proofs of Propositions
Proof of Proposition 1. Since the smallest $N$ can be is 2, it is sufficient to prove the claim for $N = 2$. Consider any values $C_1$, $C_2$, $Q_1$, and $Q_2$. Figure A.1 shows all the possible
cases: $|C_1 - C_2| \geq |Q_1 - Q_2|$ in cases 1 and 2, while $|C_1 - C_2| < |Q_1 - Q_2|$ in cases 1’ and 2’. Cases 1 and 1’ show the PR outcome, while cases 2 and 2’ show the NR outcomes.

As noted in deriving the lemma, the buyer’s surplus in the second-price BD mechanism equals $(Q - C_{(2)})$. One of the diagonal lines in each plot shows all the points $(Q, C)$ with the BD mechanism surplus. Similarly, the buyer’s surplus in the second-price PB mechanism equals $Q_i - C_{(N-1)} = Q_i - C_{(1)}$. The other diagonal line in each plot shows all the points $(Q, C)$ with PB surplus. The buyer’s surplus increases as one goes further southeast; the $\delta$’s show the gap between the surpluses of the two mechanisms in each case. From the geometry of the plots, note that $\delta_1 = \delta_2$ and $\delta_1' < \delta_2$.

Now consider what happens if $C$ and $Q$ are independent. This implies that the PR and NR cases occur with equal probability. The BD mechanism yields $\delta_1 = \delta_2$ more surplus to the buyer than the PB mechanism does in case 1, while the PB mechanism yields $\delta_2 = \delta_1'$ more surplus to the buyer than the BD mechanism does in case 2. Since these two cases occur with equal probability, we have the following result: for any values $C_1$, $C_2$, $Q_1$, and $Q_2$ such that $|C_1 - C_2| \geq |Q_1 - Q_2|$, the two mechanisms yield the same expected buyer’s surplus.

We use the same argument for cases 1’ and 2’. Since $\delta_1' < \delta_2'$ and since cases 1’ and 2’ are equally likely if $C$ and $Q$ are independent, the PB mechanism yields the greater expected buyer’s surplus if $|C_1 - C_2| < |Q_1 - Q_2|$. Since $C$ and $Q$ are nondegenerate, $|C_1 - C_2| < |Q_1 - Q_2|$ with positive probability. Thus, in expectation, $\Pi_{BD} < \Pi_{PB}$ when $C$ and $Q$ are independent.

If $C$ and $Q$ are not independent, this result continues to hold unless the PR outcomes become sufficiently more likely than the NR outcomes. To see this, note that relaxing the independence of $C$ and $Q$ means that the PR and NR outcomes are no longer equally likely. If case 1’ is more likely than case 2’, then the surplus difference $\delta_1'$ occurs more often than the surplus difference $\delta_2$. When case 1’ is sufficiently more likely so that probability of the PR outcome times $\delta_1'$ is bigger than probability of the NR outcome times $\delta_2$, the BD mechanism will generate in expectation higher buyer surplus than the PB mechanism.

Proof of Corollary. The proof of Proposition 1 establishes that $\Pi_{BD} < \Pi_{PB}$ whenever $C$ and $Q$ are independent. Next, in the limit of perfectly negatively correlated $C$ and $Q$,
all outcomes will be NR and $\Pi_{BD} < \Pi_{PB}$. Since the expectations vary continuously, it must be the case that just short of the limit, $\Pi_{BD} < \Pi_{PB}$ still holds. Similarly, in the limit of perfectly positively correlated $C$ and $Q$, all outcomes will be PR and $\Pi_{BD} > \Pi_{PB}$. Since the expectations vary continuously, it must be the case that just short of the limit, $\Pi_{BD} > \Pi_{PB}$ still holds. □

Proofs of Proposition 2. Let $C^*$ denote the minimum value of $C$ in the support set $\Omega$ and let $(Q - C)^*$ denote the maximum value of $Q - C$ in $\Omega$, $C = C^*$ for all point in $E$, and $Q - C = (Q - C)^*$ for all points in $Q - E$. As $N$ goes to infinity, the support set becomes densely covered with actual outcomes and, therefore, $\Pi_{BD} = E[(Q - C)_{(d_i)}]$ converges to $(Q - C)^*$. Also, as $N$ goes to infinity, $C_{(N)}$ and $C_{(N-1)}$ both converge to $C^*$ and, therefore, $\Pi_{PB} = E[(Q - C) | C = C_{(N)}] + E[C_{(N)} - C_{(N-1)}]$ converges to $E[(Q - C) | C = C^*]$.  

(A) If $E$ is not a subset of $Q - E$, then for all but at most one point, $Q - C < (Q - C)^*$. This implies that $E[(Q - C) | C = C^*] < (Q - C)^*$ and, therefore, $\Pi_{BD} > \Pi_{PB}$ for all large enough $N$.

(B) If $E$ is a subset of $Q - E$, then $E$ consists of a single point, and $Q - C = (Q - C)^*$ for that point. Therefore, $\Pi_{BD} - \Pi_{PB}$ converges to zero as $N$ goes to infinity. □

References


