Test Adequacy Measurement & Enhancement

W. Eric Wong
Department of Computer Science
The University of Texas at Dallas
ewong@utdallas.edu
http://www.utdallas.edu/~ewong

Speaker Biographical Sketch

• Professor & Director of International Outreach
  Department of Computer Science
  University of Texas at Dallas

• Guest Researcher
  Computer Security Division
  National Institute of Standards and Technology (NIST)

• Vice President, IEEE Reliability Society

• Secretary, ACM SIGAPP (Special Interest Group on Applied Computing)

• Principal Investigator, NSF TUES (Transforming Undergraduate Education in Science, Technology, Engineering and Mathematics) Project
  – Incorporating Software Testing into Multiple Computer Science and Software Engineering Undergraduate Courses

• Founder & Steering Committee co-Chair for the SERE conference
  (IEEE International Conference on Software Security and Reliability)
  (http://paris.utdallas.edu/sere13)
Outline

• Test Adequacy Measurement
• Test Set Enhancement

What Is Test Adequacy?
What Is Adequacy?

- Consider a program $P$ written to meet a set of functional requirements $R$. We notate such a $P$ and $R$ as $(P, R)$
- Let $R$ contain $n$ requirements labeled $R_1, R_2, \ldots, R_n$
- Suppose now that a set $T$ containing $k$ test cases has been constructed to test $P$ to determine whether or not it meets all the requirements in $R$
  - Assume also $P$ has been executed against each test case in $T$ and has produced correct behavior
- We now ask: Is $T$ good enough?
  - This question can be stated differently as:
    - Has $P$ been tested thoroughly?
    - or as: Is $T$ adequate?

Measurement of Adequacy

- In the context of software testing, the terms “thorough,” “good enough,” and “adequate,” used in the questions above, have the same meaning.
- Adequacy is measured for a given test set designed to test $P$ to determine whether or not $P$ meets its requirements.
- This measurement is done against a given criterion $C$.
  - A test set is considered adequate with respect to a criterion $C$ when it satisfies $C$.
  - The determination of whether or not a test set $T$ for program $P$ satisfies a criterion $C$ depends on the criterion itself and is explained later.
**Black-box and White-box Criteria**

- For each adequacy criterion \( C \), we derive a finite set known as the **coverage domain** and denoted as \( C_e \).
- A criterion \( C \) is a **white-box** test adequacy criterion if the corresponding coverage domain \( C_e \) depends solely on the **program P under test**.
- A criterion \( C \) is a **black-box** test adequacy criterion if the corresponding coverage domain \( C_e \) depends solely on the **requirements R** for the program \( P \) under test.

**Coverage**

- We want to measure the adequacy of \( T \). Given that \( C_e \) has \( n \geq 0 \) elements, we say that \( T \) covers \( C_e \) if for each element \( e' \) in \( C_e \), there is at least one test case in \( T \) that tests \( e' \).
  - \( T \) is considered **adequate with respect to C** if it covers all elements in the coverage domain.
  - \( T \) is considered **inadequate with respect to C** if it covers \( k \) elements of \( C_e \) where \( k < n \).
- The fraction \( k/n \) is a measure of the extent to which \( T \) is adequate with respect to \( C \). This fraction is also known as the **coverage of T with respect to C, P, and R**.
Example I (1)

- Program `sumProduct` must meet the following requirements
  - \( R_1 \) Input two integers, say \( x \) and \( y \), from the standard input device
  - \( R_{2.1} \) Find and print to the standard output device the sum of \( x \) and \( y \) if \( x < y \)
  - \( R_{2.2} \) Find and print to the standard output device the product of \( x \) and \( y \) if \( x \geq y \)

Example I (2)

- Suppose now that the test adequacy criterion \( C \) is specified as

  A test \( T \) for program \( (P, R) \) is considered adequate if for each requirement \( r \) in \( R \) there is at least one test case in \( T \) that tests the correctness of \( P \) with respect to \( r \).

- In this case the coverage domain \( C_e = \{R_1, R_{2.1}, R_{2.2}\} \)

- \( T = \{r: x = 2, y = 3 \} \) (which has \( x < y \)) covers \( R_1 \) and \( R_{2.1} \) but not \( R_{2.2} \)
  - \( T \) is not adequate with respect to \( C \)
  - The coverage of \( T \) with respect to \( C, P, \) and \( R \) is 0.66
Example II (1)

• Consider the following criterion

A test $T$ for program $(P, R)$ is considered adequate if each path in $P$ is traversed at least once.

• Assume that $P$ has exactly two paths, one corresponding to condition $x < y$ and the other to $x \geq y$.
  – We refer to these as $p_1$ and $p_2$, respectively.
  – For the given adequacy criterion $C$ we have the coverage domain $C_e = \{p_1, p_2\}$.

• We assume that $P$ has exactly two paths. This assumption is based on the knowledge of the requirements. However, when the coverage domain contains elements from the code, such elements should be derived from the program directly and not by an examination of its requirements.

Example II (2)

• To measure the adequacy of $T$ of sumProduct against $C$, we execute $P$ against each test case in $T$.

• As $T = \{<x = 2, y = 3>\}$ contains only one test for which $x < y$, only the path $p_1$ is executed.
  – The coverage of $T$ with respect to $C$, $P$, and $R$ is 0.5.
  – $T$ is not adequate with respect to $C$.
  – We can also say that $p_1$ is tested and $p_2$ is not tested.
The following program is incorrect as per the requirements of sumProduct:

- Using the path-based coverage criterion $C$, we have the coverage domain $C_e=\{p_1\}$
- This path traverses all the statements
- $T=\{ \langle x = 2, y = 3 \rangle \}$ is adequate with respect to $C$ but does not reveal the bug.

```plaintext
sumProduct-1
1 begin
2 int x, y;
3 input (x, y);
4 sum=x+y;
5 output (sum);
6 end
```

**Lessons Learned**

- An adequate test set might not reveal even the most obvious bug in a program.
- However, this does not diminish in any way the need for the measurement of test adequacy as increasing coverage might reveal a bug!
While a test set adequate with respect to some criterion does not guarantee a bug-free program, an inadequate test set is a cause for worry. Inadequacy with respect to any criterion often implies deficiency.

Identification of this deficiency helps in the enhancement of the inadequate test set.

Enhancement is likely to test the program in ways it has not been tested before such as testing an untested portion, or testing the features in a sequence different from the one used previously.

Testing the program differently than before raises the possibility of discovering any hidden bugs.
Example IV (1)

- The following program is correct as per the requirements of sumProduct
  - It has two paths denoted by $p_1$ and $p_2$
  - $T = \{ \langle x = 2, y = 3 \rangle \}$ is inadequate with respect to the path-based coverage criterion $C$

```plaintext
sumProduct-2
1 begin
2 int x, y;
3 input (x, y);
4 if (x < y)
5 then
6 output(x+y);
7 else
8 output(x*y);
9 end
```

Example IV (2)

- For sumProduct-2, to make $T$ adequate with respect to the path coverage criterion we need to add a test case that covers $p_2$
  - One test case that does so is $\{ \langle x = 3, y = 1 \rangle \}$
  - Adding this test case to $T$ and denoting the expanded test set by $T'$ we have $T'$
    $=\{ \langle x = 3, y = 4 \rangle, \langle x = 3, y = 1 \rangle \}$

- Executing sumProduct-2 against the two test cases in $T'$ will have both $p_1$ and $p_2$ traversed
  - $T'$ is adequate with respect to the path coverage criterion
Consider a program intended to compute $x^y$ given integers $x$ and $y$. For $y < 0$ the program skips the computation and outputs a suitable error message.

```c
begin
int x, y;
int product, count;
input (x, y);
if(y<0) {
  product=1; count=y;
  while(count>0) {
    product=product*x;
    count=count-1;
  }
  output(product);
} else
  output ("Input does not match its specification");
end
```

Suppose that a test set $T$ is considered adequate if it tests the program on the previous slide for at least one zero and one non-zero value of each of the two inputs $x$ and $y$.

The coverage domain for $C$ can be determined without any inspection of the program.
- $C_x = \{x = 0, y = 0, x \neq 0, y \neq 0\}
- We can derive an adequate test set for the program by an examination of $C_x$.
- $T = \{t_1: <x = 0, y = 1>, t_2: <x = 1, y = 0>\}$ is adequate with respect to $C$. 

Example VI (Path Coverage) (1)

- Criterion $C$ of the previous example is a **black-box coverage criterion** as it does not require an examination of the program under test for the measurement of adequacy.

- Let us now consider the **path coverage criterion**.

- An examination of the exponentiation program reveals that it has an indeterminate number of paths due to the `while` loop.
  - The number of paths depends on the value of $y$ and hence that of count.

Example VI (Path Coverage) (2)

- Given that $y$ is any non-negative integer, the number of paths can be arbitrarily large. This simple analysis of paths in the exponentiation program reveals that for the path coverage criterion we cannot determine the coverage domain.

- The usual approach in such cases is to simplify $C$ and reformulate it as follows:

  A test $T$ is considered adequate if it tests all paths. In case the program contains a loop, then it is adequate to traverse the loop body zero times and once. (boundary-interior)
Example VI (Path Coverage) (3)

- The coverage domain of the modified path coverage criterion is \( \{ p_1, p_2, p_3 \} \)
  - \( p_1: 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 7 \rightarrow 9 \) (red path)
  - \( p_2: 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 5 \rightarrow 7 \rightarrow 9 \) (blue path)
  - \( p_3: 1 \rightarrow 2 \rightarrow 3 \rightarrow 8 \rightarrow 9 \) (green path)

- Let \( T = \{ t_1: < x = 0, y = 1 >, \ t_2: < x = 1, y = 0 >, \ t_3: < x = 5, y = -1 > \} \)
  - As \( T \) does not contain any test with \( y < 0 \),
    \( p_3 \) (the green path) remains uncovered.
  - The coverage is \( \frac{2}{3} = 0.66 \)

Example VI (Path Coverage) (4)

- Any test case with \( y < 0 \) will cause \( p_3 \) to be traversed.

- Let \( t_3 \) be \( < x = 5, y = -1 > \)
  - Test case \( t_3 \) covers path \( p_3 \) and \( P \) behaves correctly.
  - After adding \( t_3 \) to \( T \), we have covered all feasible elements of the coverage domain.
  - The enhanced test set is \( T = \{ t_1: < x = 0, y = 1 >, t_2: < x = 1, y = 0 >, t_3: < x = 5, y = -1 > \} \)
Infeasibility and Test Adequacy

- An element of the coverage domain is infeasible if it cannot be covered by any test in the input domain of the program under test.

- There does not exist an algorithm that would analyze a given program and determine if a given element in the coverage domain is infeasible or not. Thus it is usually the tester who determines whether or not an element of the coverage domain is infeasible.

Demonstrating Feasibility

- Feasibility can be demonstrated by executing the program under test against a test case and showing that indeed the element under consideration is covered.

- Infeasibility cannot be demonstrated by program execution against a finite number of test cases.
  - In some cases simple arguments can be constructed to show that a given element is infeasible.
  - For more complex programs the problem of determining infeasibility could be difficult. Thus, an attempt to enhance a test set by executing a test \( t \) aimed at covering element \( e \) of program \( P \), might fail.
**Adequacy and Infeasibility**

- In the presence of one or more infeasible elements in the coverage domain, a test is considered adequate when all feasible elements in the domain have been covered.

- While programmers might not be concerned with infeasible elements, testers attempting to obtain code coverage are.

- Prior to test enhancement, a tester usually does not know which elements of a coverage domain are infeasible.
  - Unfortunately, it is only during an attempt to construct a test case to cover an element that one might realize the infeasibility of an element.

**Fault Detection and Test Enhancement**

- The purpose of test enhancement is to determine test cases that test the untested parts of a program.
  - Even the most carefully designed tests based exclusively on requirements can be enhanced.

- The *more complex* the set of requirements, the *more likely* it is that a test set designed using requirements is *inadequate* with respect to *even the simplest* of various test adequacy criteria.
Example VII (1)

A program to meet the following requirements is to be developed.

R1: Upon start the program offers the following three options to the user:
   – Compute \( x^y \) for integers \( x \) and \( y \geq 0 \).
   – Compute the factorial of integer \( x \geq 0 \).
   – Exit.

R1.1: If the “Compute \( x^y \)” option is selected then the user is asked to supply the values of \( x \) and \( y \), \( x^y \) is computed and displayed. The user may now select any of the three options once again.

R1.2: If the “Compute factorial \( x \)” option is selected then the user is asked to supply the value of \( x \) and factorial of \( x \) is computed and displayed. The user may now select any of the three options once again.

R1.3: If the “Exit” option is selected the program displays a goodbye message and exits.

Example VII (2)

Consider this program written to meet the above requirements.

```
begin
  set y;  // set y to 1
  set product, request;
  while request = 1 do
    case request
      when request = 'compute' then
        request = input (request);
        if x > 0 then
          product = x * product;
        else
          product = 1;
        end if
      when request = 'factorial' then
        request = input (request);
        if x > 0 then
          product = 1;
        else
          product = 1;
        end if
      when request = 'exit' then
        request = input (request);
      end case
    end while
end
```
**Example VII (3)**

- Suppose now that the following test set has been developed to test whether or not our program meets its requirements.

- \( T = \{ \langle \text{request} = 1, x = 2, y = 3 \rangle, \langle \text{request} = 2, x = 4 \rangle, \langle \text{request} = 3 \rangle \} \)

- For the first two requests (*exponential* followed by *factorial*), the program correctly outputs 8 and 24. The program exits when executed against the third request. This program’s behavior is correct and hence one might conclude that the program is correct.

- Is this conclusion correct?

---

**Example VII (4)**

- Let us now evaluate \( T \) against the path coverage criterion.

- Construct the control flow graph of the example program and identify the paths not covered by \( T \).

- The coverage domain consists of all paths that traverse *each of the three loops* zero and once in the same or different executions of the program.
Consider the path \( p \) that begins execution at line 1, reaches the outermost while at line 10, then the first if at line 12, followed by the statements that compute the factorial starting at line 20, and then the code to compute the exponential starting at line 13.

\( p \) is traversed when the program is launched and the first input request is to compute the factorial of a number, followed by a request to compute the exponential. It is easy to verify that the sequence of requests in \( T \) (on slide 34) does not exercise \( p \). Therefore, \( T \) is inadequate with respect to the path coverage criterion.

Example VII (5)

To cover \( p \) we construct the following test:

\[ T' = \{ \langle \text{request} = 2, x = 4 \rangle, \langle \text{request} = 1, x = 2, y = 3 \rangle, \langle \text{request} = 3 \rangle \} \]

When the values in \( T' \) are input to our example program in the sequence given, the program correctly outputs 24 as the factorial of 4 but incorrectly outputs 192 \((24 \times 2 \times 2 \times 2)\) as the value of \( 2^3 \).

This happens because \( T' \) traverses our “tricky” path which makes the computation of the exponentiation begin without initializing product. In fact the code at line 14 begins with the value of product set to 24.
Example VII (7)

- In our effort to increase the path coverage we constructed $T'$. Execution of the test program on $T'$ did cover a path that was not covered earlier and revealed a bug in the program.
- This example has illustrated a benefit of test enhancement based on code coverage.

Multiple Executions (1)

- In the previous example we constructed two test sets $T$ and $T'$. Notice that both $T$ and $T'$ contain three tests one for each value of the variable request. Should $T$ (or $T'$) be considered a single test or a sequence of three tests?
- $T' = \{ <\text{request} = 2, x = 4 >, <\text{request} = 1, x = 2, y = 3 >, <\text{request} = 3 > \}$
Multiple Executions (2)

- We assumed that all three tests, one for each value of request, are input in a sequence during a single execution of the test program. Hence we consider \( T \) as a test set containing one test case and write it as follows:

\[
T = \left\{ t_1 : \begin{array}{c}
<< \text{request} = 1, x = 2, y = 3 >> \rightarrow \text{< request = 2, x = 4 >} \\
< \text{request} = 2, x = 4 > \rightarrow \text{< request = 3 >>}
\end{array} \right\}
\]

\[
T'' = \left\{ t_1 : \begin{array}{c}
<< \text{request} = 1, x = 2, y = 3 >> \rightarrow \text{< request = 2, x = 4 >} \\
< \text{request} = 3 >>
\end{array}
\right\}
\]

\[
T''' = T \cup T''
\]