Convolution-Example-Solutions

Question 1

\[
\begin{array}{ccc}
1 & -1 \\
A \\
1 & 0 \\
B \\
1 & 0 \\
C \\
1 & 3 & 2 \\
D
\end{array}
\]

Cross correlation of a picture with the mask in A, then cross correlation of the result with the mask in B, then cross correlation of the result with the mask in C, then cross correlation of the result with the mask in D, is the same as cross correlation of the same picture with the following mask:

\[
\begin{array}{ccc}
1 & -1 \\
4 & -2 & -2 \\
3 & -1 & -2 \\
\end{array}
\]

\[
D \otimes (C \otimes (B \otimes (A \otimes P))) = D' \star (C' \star (B' \star (A' \star P)))
\]

\[
= (D' \star C' \star B' \star A') \star P
\]

\[
= (D' \star (C' \star (B' \star A'))) \star P
\]

\[
= (D \otimes (C \otimes (B \otimes A'))) \star P
\]

\[
= (D \otimes (C \otimes ((1,0) \otimes (-1,1)))) \star P
\]

\[
= (D \otimes (C \otimes ((-1,1)))) \star P
\]

\[
= (D \otimes ((1 \ 1) \otimes ((-1,1)))) \star P
\]

\[
= (D \otimes \begin{pmatrix} -1 & 1 \\ -1 & 1 \end{pmatrix}) \star P
\]

\[
= \begin{pmatrix} -2 & -1 & 3 \\ -2 & -2 & 4 \\ 0 & -1 & 1 \end{pmatrix} \star P
\]