Thresholding by quantization

Let \( p \) be the picture histogram, so that \( p(x) \) is the number of pixels of value \( x \), for \( x = 0, \ldots, M \). We are looking for a threshold value \( t \) and two values \( q_1, q_2 \), such that all pixels in the range \( 0 \leq x < t \) are replaced with \( q_1 \), and all pixels in the range \( t \leq x \leq M \) are replaced with \( q_2 \). Define the following expression as the total error:

\[
E(t, q_1, q_2) = \sum_{x=0}^{t-1} (x - q_1)^2 p(x) + \sum_{x=t}^{M} (x - q_2)^2 p(x).
\]

For each \( t \) we can compute the minimum of \( E \) by choosing the “best possible” values for \( q_1, q_2 \). These are computed by taking the derivatives of \( E \) with respect to \( q_1, q_2 \).

Taking the derivative of \( e \) with respect to \( q_1 \) we have:

\[
\frac{\partial e}{\partial q_1} = 2 \sum_{x=0}^{t-1} xp(x) - 2q_1 \sum_{x=0}^{t-1} p(x).
\]

The requirement that \( \frac{\partial e}{\partial q_1} = 0 \) gives:

\[
q_1 = \frac{\sum_{x=0}^{t-1} xp(x)}{\sum_{x=0}^{t-1} p(x)}
\]

and similarly:

\[
q_2 = \frac{\sum_{x=t}^{M} xp(x)}{\sum_{x=t}^{M} p(x)}
\]

Therefore, we can compute the value of \( E \) for any given value of \( t \) by first computing \( q_1, q_2 \) and then substituting their values in the above expression for \( E \). Since there are only 255 possible values for \( t \) the minimizer of \( t \) can be determined by examining all values of \( E(t) \) for \( t = 1..255 \).