No books or notes! You may use a scientific calculator provided it does not allow for access to the internet. Show all work and give complete explanations. This 2 hours 45 mins exam is worth 100 points.

(1) [10 pts]
(a) Find the area of the parallelogram with vertices (1, 1), (3, 4), (5, 6) and (7, 9).

(b) Calculate the vector projection of $\mathbf{u} = (1, 2, -4)$ onto $\mathbf{v} = (3, -2, 1)$.
(2) [12 pts] Let \( C \) be the curve in \( \mathbb{R}^2 \) parametrized by \((x, y) = r(t) = (3 \cos t, 4 \sin t)\) for \(0 \leq t \leq \pi/2\).

(a) Sketch the curve \( C \).

(b) Calculate \( \int_C f \, ds \) where \( f(x, y) = xy \).

(c) Let \( \mathbf{F}(x, y) = yi + x^2j \). Find a function \( g = g(t) \) and numbers \( a \) and \( b \) so that \( \int_C \mathbf{F} \cdot d\mathbf{r} = \int_a^b g(t) \, dt \).
(3) [8 pts] Find the limit if it exists, or show that the limit does not exist.

(a) \( \lim_{(x,y) \to (0,0)} \frac{xy}{x^2+y^2} \)

(b) \( \lim_{(x,y) \to (0,0)} \frac{x^2y}{x^2+y^2} \)
(4) [8 pts] Let \( z = f(x, y) = 3x^2 + 4xy + 5y^2 \).

(a) Calculate the equation of the tangent plane to the graph of \( f \) at \((x_0, y_0) = (2, -1)\).

(b) Suppose that an ant is walking on a hot plate in the \( xy \)-plane and that the function \( z = f(x, y) \) given above is the temperature of the hot plate at the point \((x, y)\). Suppose that at time \( t = 0 \) the position of the ant is \( \mathbf{x} = (2, -1) \) and the velocity of the ant is \( \mathbf{v} = (4, 3) \). What is the rate of change of the temperature of the ant’s feet at time \( t = 0 \)?
(5) [12 pts] Find the absolute maximum and absolute minimum of the function \( f(x, y) = x + y - xy \) on the triangle in the \( xy \)-plane with vertices (0, 0), (4, 0), and (0, 2).
(6) [10 pts] Use spherical coordinates to calculate the triple integral \( \iiint_E z \, dV \), where \( E \) is the solid region inside the sphere \( x^2 + y^2 + z^2 = 4 \) and above the cone \( z = \sqrt{x^2 + y^2} \).
(7) [10 pts]
Let $E$ be the solid region in $\mathbb{R}^3$ bounded by the surfaces $z = 1 - y^2$, $y = x - 1$, $x = 0$, and $z = 0$.

(a) Sketch $E$. Is $\iiint_E y \, dV$ positive or negative? Why?

(b) Calculate $\iiint_E y \, dV$. 
(8) [10 pts] Use the Change of Variables Theorem to evaluate the integral \( \iint_R y \, dA \), where \( R \) is the quadrilateral region bounded by the lines \( x + 2y = 2 \), \( x + 2y = 4 \), \( x = 0 \), and \( y = 0 \). **Hint:** Let \( u = x + 2y \) and \( v = y \).
(9) [12 pts] Let \( \mathbf{F} \) be the vector field in the plane given by \( \mathbf{F}(x, y) = x^2y\mathbf{i} + (x^2 - y^2)\mathbf{j} \).

(a) Calculate the divergence of \( \mathbf{F} \).

(b) Calculate the curl of \( \mathbf{F} \).

(c) Is \( \mathbf{F} \) conservative? Why?

(d) Suppose that the vector field \( \mathbf{F} \) given above is the velocity vector field of a fluid flowing in the plane. On average is the fluid flowing in or out of a small disk centered at the point \((-1, 2)\)? Why?
(10) [8 pts]
(a) Define what it means for a vector field to be conservative.

(b) Define what it means for the integral of a vector field to be independent of path.

(c) Prove that if $\mathbf{F}$ is a conservative vector field then $\int_{C} \mathbf{F} \cdot d\mathbf{r}$ is independent of path.

Pledge: I have neither given nor received aid on this exam

Signature: ________________________________