MATH 251 (Fall 2009) Exam III, Nov 25th

No calculators, books or notes! Show all work and give complete explanations. This is 65 min exam is worth 50 points.

(1) [10 pts] Calculate $\iint_D x \, dA$, where $D$ is the triangle in the $xy$-plane with vertices $(0,0)$, $(1,0)$, and $(1,2)$. 
(2) [10 pts] Evaluate the integral

\[ \int_{x=0}^{x=2} \int_{y=-\sqrt{4-x^2}}^{y=\sqrt{4-x^2}} x \, dy \, dx, \]

by converting it to polar coordinates.
(3) [10 pts] Let $\mathbf{F}$ be the vector field $\mathbf{F}(x, y, z) = xy\mathbf{i} + 3z\mathbf{j} + y\mathbf{k}$ and let $C$ be the curve parametrized by $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + \mathbf{k}$, where $0 \leq t \leq 1$. Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$. 


(4) [10 pts] Consider the two vector fields
\[
\mathbf{F}_1(x, y) = (2xy - 2y^2 \sin x)\mathbf{i} + (x^2 + 4y \cos x)\mathbf{j} \\
\mathbf{F}_2(x, y) = (2xy^2 - 2y \sin x)\mathbf{i} + (x^2 + 4y^2 \cos x)\mathbf{j}
\]

One of these vector fields is conservative.

(a) Which vector field is conservative and which is not? Why?

(b) For the vector field that is conservative, evaluate the line integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \), where \( C \) is any curve from \((0, 0)\) to \((0, 1)\).
(5) [10 pts] Find a double integral equal to the volume of the solid bounded by the surfaces $y = x$, $x = 2$, $z = 0$, and $z = y$, and evaluate this integral.

Pledge: I have neither given nor received aid on this exam

Signature: ______________________________