Ante-Quiz # 10

A sample (1, 0, 0, 0, 1, -1, 0, 1, 0, 0) is collected from a discrete distribution with the probability mass function

\[
\begin{array}{c|c}
 x & P(x) \\
 \hline
-1 & \theta \\
 0 & 0.6 \\
 1 & 0.4 - \theta \\
\end{array}
\]

(a) Compute the method of moments estimator of parameter \( \theta \).

(b) Estimate the standard error of your estimator \( \hat{\theta} \).

(a) This problem is similar to Practice Problems 5(a), 6, 7(a-g), and 9 of Homework-11.

To find the method of moments estimator of \( \theta \), we solve the equation \( M_1 = \mu_1 \), where

\[
M_1 = \bar{X} = \frac{1 + 0 + 0 + 0 + 1 - 1 + 0 + 1 + 0 + 0}{10} = 0.2 \\
\mu_1 = E(X) = \sum x P(x) = (-1)(\theta) + (0)(0.6) + (1)(0.4 - \theta) = 0.4 - 2\theta.
\]

Solving the equation \( \bar{X} = 0.6 - 2\theta \), we get \( \hat{\theta} = (0.4 - \bar{X})/2 = 0.1 \).

(b) This problem is similar to Practice Problems 2(b) and 3(d) of Homework-11.

Find the standard error of \( \hat{\theta} \)

\[
Std(\hat{\theta}) = Std((0.4 - \bar{X})/2) = Std(\bar{X})/2 = \frac{\sigma}{2\sqrt{n}}
\]

which we estimate by

\[
\hat{Std}(\hat{\theta}) = \frac{S}{2\sqrt{n}} = \frac{1}{2\sqrt{10}} \sqrt{\frac{\sum (X_i - \bar{X})^2}{n-1}} = \frac{1}{2\sqrt{10}} \sqrt{\frac{(1 - 0.2)^2 + (0 - 0.2)^2 + ... + (0 - 0.2)^2}{9}}
\]

\[
= \frac{1}{2\sqrt{10}} \sqrt{\frac{(0 - 0.2)^2(6) + (1 - 0.2)^2(3) + (-1 - 0.2)^2}{9}} = \frac{1}{2} \sqrt{\frac{3.6}{90}} = 0.1
\]