Anti-Quiz # 8

New jobs are sent to a printer at the rate of 1 job every 3 minutes. It takes an average of 2 minutes to print one job. Assume Bernoulli single-server queuing process with 1-minute frames and unlimited capacity. If the printer is idle at 10:00, find the probability that it is busy printing some job at 10:02.

This problem is similar to Practice Problem 3a and Problems 2 and 5b of Homework 9.

It is given that: \( \lambda_A = 1/3 \text{ min}^{-1}, \lambda_S = 1/2 \text{ min}^{-1}, \Delta = 1 \text{ min} \).
We need to find \( P(X(2) > 0 | X(0) = 0) \), conditional probability that the printer is busy at 10:02, which is 2 frames away, given that it is idle now. For this, we start with the arrival and service probabilities,

\[
p_A = \lambda_A \Delta = (1/3)(1) = 1/3, \quad p_S = \lambda_S \Delta = (1/2)(1) = 1/2.
\]

Then find the transition probabilities,

\[
p_{00} = 1 - p_A = 2/3, \quad p_{01} = p_A = 1/3, \quad \text{ and for all } k \geq 1,
\]

\[
p_{k,k-1} = p_S(1-p_A) = 1/3, \quad p_{k,k} = p_A p_S + (1-p_A)(1-p_S) = 1/6 + 1/3 = 1/2, \quad p_{k,k+1} = p_A (1-p_S) = 1/6.
\]

Now,

\[
P(X(2) = 0 | X(0) = 0) = P(0 \to 0 \to 0) + P(0 \to 1 \to 0) = p_{00} p_{00} + p_{01} p_{10} = \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 = \frac{5}{9},
\]

so,

\[
P(X(2) > 0 | X(0) = 0) = 1 - \frac{5}{9} = \boxed{\frac{4}{9} \text{ or } 0.444}
\]
Anti-Quiz # 8

New jobs are sent to a printer at the rate of 1 job every 5 minutes. It takes an average of 4 minutes to print one job. Assume Bernoulli single-server queuing process with 1-minute frames and unlimited capacity. If the printer is idle at 10:00, find the probability that it is busy printing some job at 10:02.

This problem is similar to Practice Problem 3a and Problems 2 and 5b of Homework 9.

It is given that: \( \lambda_A = 1/5 \text{ min}^{-1}, \lambda_S = 1/4 \text{ min}^{-1}, \Delta = 1 \text{ min}. \)
We need to find \( P(X(2) > 0 | X(0) = 0) \), conditional probability that the printer is busy at 10:02, which is 2 frames away, given that it is idle now. For this, we start with the arrival and service probabilities,
\[
p_A = \lambda_A \Delta = (1/5)(1) = 0.2, \quad p_S = \lambda_S \Delta = (1/4)(1) = 0.25.
\]

Then find the transition probabilities,
\[
p_{00} = 1 - p_A = 0.8, \quad p_{01} = p_A = 0.2, \quad \text{and for all } k \geq 1,
\]
\[
p_{k,k-1} = p_S(1-p_A) = 0.2, \quad p_{k,k} = p_A p_S + (1-p_A)(1-p_S) = 0.65+0.6 = 0.65, \quad p_{k,k+1} = p_A(1-p_S) = 0.15.
\]
Now,
\[
P(X(2) = 0 | X(0) = 0) = P(0 \rightarrow 0 \rightarrow 0) + P(0 \rightarrow 1 \rightarrow 0) = p_{00} p_{00} + p_{01} p_{10}
\]
\[
= (0.8)^2 + (0.2)(0.2) = 0.64 + 0.04 = 0.68,
\]
so,
\[
P(X(2) > 0 | X(0) = 0) = 1 - 0.68 = \boxed{0.32}
\]