Modeling Random Phenomena and Making Decisions under Uncertainty

Uncertainty in computer environment

- Arrival of jobs
- Execution time
- Memory requirement
- Failure of components
- Exposure to viruses
- Errors in codes
- Etc., etc., etc.
Random phenomena elsewhere

• Economy:
  – stock prices
  – number of jobs
  – price of oil

• Environment
  – temperature
  – pollution
  – natural disasters
Introduction: Uncertainty

• Going to UTD
  – number of green lights
  – available parking

• This class
  – quiz problems
  – time spent on each topic
  – grades

This course: Quantify uncertainty, model uncertainty, make decisions
What we’ll learn

• Uncertainty; probabilities of events

• Random variables

• Monte Carlo simulation

• Stochastic processes
• Application: queuing systems

What we’ll learn
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- Statistical inference
- Estimation
- Testing hypothesis
Probability

= chance (common sense)

= odds (gambling)

= long-term proportion (relative frequency)

= likelihood (forecasting)

= finite measure (mathematics)
Probability

= function of an event = $P(E)$

**Domain:** events

Event = combination of outcomes

Consider an experiment, results are outcomes,

Sample space $S = \{\text{all possible outcomes}\}$

Event $E$ = a set of outcomes

= subset of $S$ ($E \subset S$)

**Range:** $[0, 1]$
“Every event” $E$ has probability $P(E)$,

$$0 \leq P(E) \leq 1$$

If

$$E = \{O_1, \ldots, O_n\} = \{\text{outcomes}\}$$

then

$$P(E) = \sum_{k=1}^{n} P(O_k) = P(O_1) + \cdots + P(O_n)$$

An empty event $\emptyset$ is an event,

$$P(\emptyset) = 0$$

Also, $P(S) = 1$. 
Set operations

\( E_1, E_2, \ldots, E_n \) - events (sets of outcomes)

**Union** of \( E_1, \ldots, E_n \) is an event

- consists of **all** outcomes of \( E_1, \ldots, E_n \)
- occurs if **any** of \( E_1, \ldots, E_n \) occurs

\[ \{ E_1 \cup \ldots \cup E_n \} = \{ E_1 \ \text{or} \ \ldots \ \text{or} \ E_n \} \]
Set operations

**Intersection** of $E_1, ..., E_n$ is an event

- consists of **common** outcomes of $E_1, ..., E_n$

- occurs if **each** $E_1, ..., E_n$ occurs

\[
\{E_1 \cap ... \cap E_n\} = \{E_1 \text{ and } ... \text{ and } E_n\}
\]
Complement of $E$ is an event

- consists of outcomes that are not in $E$
- occurs if $E$ does not occur

\[ \bar{E} = \{ \text{not } E \} \]
Disjoint, or mutually exclusive events

- cannot occur together

- \( A \cap B = \emptyset \)

Exhaustive events

- their union is Sample Space

- at least one occurs for sure

- \( A \cup B \cup C = S \)
Basic rules of Probability

- $0 \leq P(E) \leq 1$ for all $E$

- $P(\emptyset) = 0$ and $P(S) = 1$

- **Disjoint events:**
  \[
P(E_1 \cup \ldots \cup E_n) = P(E_1) + \ldots + P(E_n)
  \]

- **Any events:**
  \[
P(A \cup B) = P(A) + P(B) - P(A \cap B)
  \]
• **Complement rule**

\[ P(E) + P(\overline{E}) = 1 \]

(disjoint and exhaustive)

\[ \Rightarrow \quad P(\overline{E}) = 1 - P(E) \]

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**Question:**
What should I compute: \( P(E) \) or \( P(\overline{E}) \)?
Independence

- Independent events:
  \[ P(A \cap B) = P(A)P(B) \]
  \[ P(E_1 \cap \ldots \cap E_n) = P(E_1) \times \ldots \times P(E_n) \]

Questions:
Can disjoint events be independent?
Can exhaustive events be independent?