Practice Problems for Homework 10. MM1 Queuing Process - SPR
1. For an M/M/1 queuing system with the average interarrival time of 5 minutes and the average service time of 3 minutes, compute

   a) the expected response time;

   b) the fraction of time when there are fewer than 2 jobs in the system;

   c) the fraction of customers who have to wait before their service starts.
2. Cars arrive at fast food drive through window according to a Poisson process with the average rate of 1 car every 10 minutes. The time each customer spends ordering and getting food is Exponential with the average time of 3 minutes. When a customer is served, the other arrived customers stay in a line waiting for their turn. Compute

    a) the expected number of cars in the line at any time.

    b) the proportion of time when nobody is served at the drive through window.

    c) the expected time it takes to follow the drive through lane, from arrival till departure.
3. Jobs sent to a printer are held in a buffer until they can be printed. Jobs are printed sequentially on a first-come, first-serve basis. Jobs arrive at the printer at the rate of four per minute. The average time to print a job is 10 seconds. Assuming an M/M/1 system,

a) Find the expected value and standard deviation of the number of jobs in this system at any time.

b) When a job is submitted, what is the probability that it will begin printing immediately?
4. The arrival rate is 10 customers in a unit of time. Suppose that it is important to have no more than 2 customers in the system. What must the service rate be so that the probability that there are two or fewer customers in the system is 0.90?
5. A server is designed to have the average service time of 3 minutes. How many arrival per hour can it tolerate to guarantee the average waiting time of no longer than 5 minutes? Assume the M/M/1 model.
6. In queuing theory, the Little’s Law states that for any queuing system or its part,

\[ E(X) = \lambda A E(T) \]

i.e., the expected number of customers equals the product of the arrival rate and the expected service time. Use our known equations to verify this Law

a) For the whole M/M/1 queuing system.

b) For the server of the M/M/1 queuing system.

c) For the waiting queue of the M/M/1 queuing system, excluding the job receiving service, if there is any.
7. You arrived to a bank at 9:00 am. There were 6 customers at the bank at that time and one teller on duty. You noticed that customers arrive at the average rate of one customer every 5 minutes. Assuming the M/M/1 queuing system and that X=6 is the average number of customers at this bank, when should you expect to leave the bank?